Mean Drift Forces on a Vertical Porous Cylindrical Body

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1 INTRODUCTION

This paper presents a theoretical investigation of the second-order steady horizontal and vertical drift forces acting on a vertical porous cylindrical body which is exposed to the action of regular plane waves propagating in finite depth waters. The examined body consists of a truncated cylinder with an upper porous sidewall and an inner cylindrical column. Porous- surfaced bodies constitute an important class of maritime structures since they can reduce the influence of wave-body interaction through pores on the body surface. Thus, a porous sidewall surrounding an inner column of a truncated cylindrical body can be used to effectively reduce both the transmitted and reflected wave trains. In past decades, numerous works have been presented concerning the behavior of porous bodies in waves. To raise some of them as examples, Teng et al., (2001) studied the wave diffraction from a bottom seated cylinder with porous upper wall and an inner column, whereas Ning et al., (2017) extended the previous work for a floating truncated cylinder with an upper sidewall. Other similar studies on concentric porous cylinder systems are those from Wang & Ren, (1994); Song & Tao, (2007); Liu et al., (2018).

2 THEORETICAL BACKGROUND

We consider a free floating truncated cylindrical body with an upper porous sidewall as depicted in Fig. 1. The body's outer and inner radius, draught of the impermeable cylindrical body, draught of the upper sidewall and water depth are denoted by the symbols *α*, *b*, *h_w-h*, *d-h_w*, *d*, respectively. The fluid domain is divided into three regions, region I ($r \geq a$; $0 \leq z \leq d$); region II ($b \leq r \leq \alpha$; $h_w \leq z \leq d$); and region III ($0 \leq r \leq \alpha$; $0 \leq z \leq h$) and it is assumed incompressible, inviscid and its motion irrotational. A cylindrical coordinate system (r, θ, z) is assumed with origin located at the center of the cylinder on the sea bottom. The flow is governed by the velocity potential $\Phi^k, k = I, II, III$, which can be decomposed into the diffraction potential φ_D^k and the motion–radiation potential φ_j^k, induced around the body due to its forced oscillation in the *j*-th mode of motion, *j*=1,3,5 with unit velocity amplitude, i.e.:

$$\Phi^k(r,\theta,z;t) = Re[\varphi^k(r,\theta,z)e^{-i\omega t}] = Re\left[\left(\varphi_D^k(r,\theta,z) + \sum_{j=1,3,5} \dot{x}_{j0}\varphi_j^k(r,\theta,z)\right)e^{-i\omega t}\right] \tag{1}$$

In Eq. (1) $\dot{x}_{j0}$ denotes the complex velocity amplitude of the body motion in the *j*-th direction. It holds $\varphi_D^I = \varphi_0^I + \varphi_7^I$, where φ_0^I is the velocity potential of the incident harmonic waves and φ_7^I the scattered potential around the body.

The complex velocity potentials $\varphi_j^k, j = 1,3,5, D; k = I, II, III$ have to fulfill the Laplace differential equation in the entire fluid domain and the proper boundary conditions on the free water surface and the seabed. Furthermore, $\varphi_j^I, j = 1,3,5,7$ have to satisfy an appropriate radiation condition as $r \to \infty$. Also, the following kinematic conditions on the horizontal and vertical boundaries of the body (i.e., porous and impermeable surfaces) should be satisfied, i.e.

$$\frac{\partial \varphi_j^{II}}{\partial z} = V_j, z = h_w;\ b \leq r \leq \alpha;\ \text{and}\ \frac{\partial \varphi_j^{III}}{\partial z} = V_j, z = h; 0 \leq r \leq \alpha \tag{2}$$

$$\frac{\partial \varphi_j^{II}}{\partial r} = u_j, r = b;\ h_w \leq z \leq d;\ \text{and}\ \frac{\partial \varphi_j^{I}}{\partial z} = u_j, r = a; h \leq r \leq h_w \tag{3a}$$

$$\frac{\partial \varphi_j^{II}}{\partial r} = u_j + ikG\left(\varphi_j^{II} - \varphi_j^{I}\right), r = \alpha;\ h_w \le z \le d \tag{3b}$$

In Eq. (2) $V_D = V_1 = 0$; $V_3 = 1$; $V_5 = -r\cos\theta$, whereas in Eqs. (3a, $3b$) $u_D = u_3 = 0; u_1 = 1$; $u_5 = z - e$. In this formulation the body's forced pitch motion (j=5) is performed about a horizontal axis lying at the distance $z{=}e$ from the seabed. Also, in Eq. ($3b$) k denotes the wave number and G a dimensionless porous coefficient (Sankar & Bora, 2020).

Moreover, both the velocity potentials and their derivatives must be continuous at the vertical boundaries of neighboring ring elements.

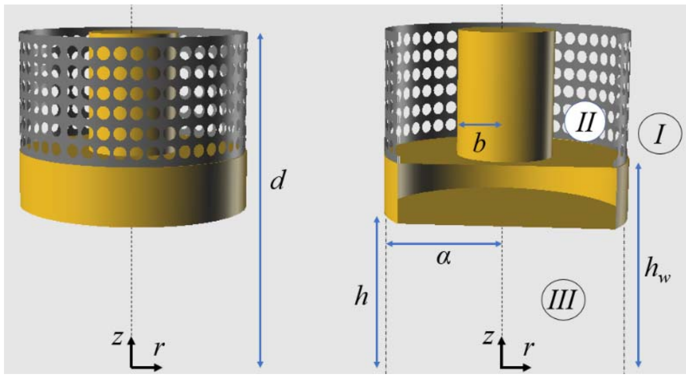

Fig. 1. 3-D representation of the examined truncated cylindrical body

Applying the method of separation of variables in the Laplace equation both the diffraction and the radiation potentials can be expressed as a superposition of eigenfunction solutions satisfying the corresponding kinematic conditions at the body's surface boundaries. Detailed expressions for the diffraction and radiation potentials in each type can be found in Kokkinowrachos et al., (1986). Indicatively, here the relevant expressions for the potential functions in the II fluid domain ($b \le r \le \alpha$; $h_w \le z \le d$) are given. It holds:

$$\varphi_D^{II} = -\frac{i\omega H}{2} \sum_{m=0}^{\infty} \varepsilon_m i^m \Psi_{Dm}^{II}(r,z)\cos(m\theta)\,; \text{and } \varphi_j^{II} = \Psi_{jm}^{II}(r,z)\cos(m\theta)\,, j = 1,3,5 \tag{4}$$

where, ω is the wave frequency; H the wave height; and ε_m the Neumann's symbol. The unknown coefficients $\Psi_{Dm}^{II}, \Psi_{jm}^{II}$ can be written as:

$$\frac{1}{\delta_j}\Psi_{jm}^{II}(r,z) = g_{jm}^{II}(r,z) + \sum_{l=0}^{\infty}\left(R_{ml}^{II}(r)F_{ml}^{II} + R_{ml}^{*II}(r)F_{ml}^{*II}\right) Z_l(z-h_w);\ j = 1,3,5,D \tag{5}$$

Where: $\delta_D = \delta_1 = \delta_3 = d$; $\delta_5 = d^2$; and $g_{Dm}^{II}(r,z) = g_{11}^{II}(r,z) = 0$; $g_{30}^{II}(r,z) = \frac{z}{d} - 1 + \frac{g}{\omega^2}$; $g_{51}^{II}(r,z) = -\frac{r}{d^2}\left[(z-d) + \frac{g}{\omega^2}\right]$

In Eq. (5) the $F_{ml}^{II}, F_{ml}^{*II}$ are the unknown Fourier series expressions; Z_l are orthonormal functions defined by:

$$Z_l(z-h_w) = \left[\frac{1}{2}\left(1 + \frac{\sin\left(2a_l(d-h_w)\right)}{2a_l(d-h_w)}\right)\right]^{-1/2} \cos\left(a_l(z-h_w)\right) \tag{6}$$

whereas $R_{ml}^{II}, R_{ml}^{*II}$ can be written in the form:

$$R_{ml}^{II} = \frac{I_m(a_l r)K_m(a_l b) - I_m(a_l b)K_m(a_l r)}{I_m(a_l a)K_m(a_l b) - I_m(a_l b)K_m(a_l a)};\ R_{ml}^{*II} = \frac{I_m(a_l a)K_m(a_l r) - I_m(a_l r)K_m(a_l a)}{I_m(a_l a)K_m(a_l b) - I_m(a_l b)K_m(a_l a)} \tag{7}$$

Here, I_m, K_m are the m-th order modified Bessel function of first and second kind, respectively. The a_l terms are roots of the equations: $\omega^2 + g a_l \tan\left(a_l(d-h_w)\right) = 0$, with the imaginary one $a_l = -ik$ considered as first.

3 MEAN DRIFT FORCES

Two principally different approaches have been presented in the literature for the determination of the mean drift forces, namely the momentum principle and the direct integration method. The latter method is applied herein, based on the direct integration of the fluid pressure over the instantaneous wetted surface of the body, keeping all relevant terms up to second order. This method which has been introduced by Pinkster & VanOortmersen (1977), has been elaborated and extended by several investigators adding some missing terms in their analysis concerning the vertical drift force components and the pitch and roll drift moments (Molin, 1983; Papanikolaou & Zaraphonitis, 1987). Analytical presentation of the mean drift forces acting on a truncated cylindrical body using the direct integration method can be found in Konispoliatis & Mavrakos (2014, 2020).

4 NUMERICAL RESULTS

This subsection is dedicated to the presentation of the horizontal and vertical mean drift forces acting on a truncated cylinder with an upper porous sidewall and an inner cylindrical column fixed to the incoming wave trains. The examined body consists of an outer radius α and inner radius b=0.25α, whereas the distance between the bottom of the body and the seabed is h=3.75α. The water depth equals to d=6.25α and the distance of the bottom of the porous sidewall from the seabed is h_w=4.25α (see Fig. 1). Several real and complex dimensionless porous coefficients are examined, i.e., G=0; 1.0; 3.0; 10.0 and G=0.5+i1.0; 1.0+i1.0; 3.0+i1.0; 10.0+i1.0 It should be noted that for G=0 the sidewall is assumed impermeable (i.e., no water is getting inside or outside the upper part of the body), as well as its thickness is considered negligible. Furthermore, for G>>0 the sidewall is considered fully permeable. The presented numerical results (i.e., horizontal, and vertical mean drift forces) are normalized by the factors $\pi\rho g\alpha(H/2)^2$ and $-\pi\rho g\alpha^2(H/2)^2/(2d)$, respectively, where ρ is the water density and g the gravity acceleration.

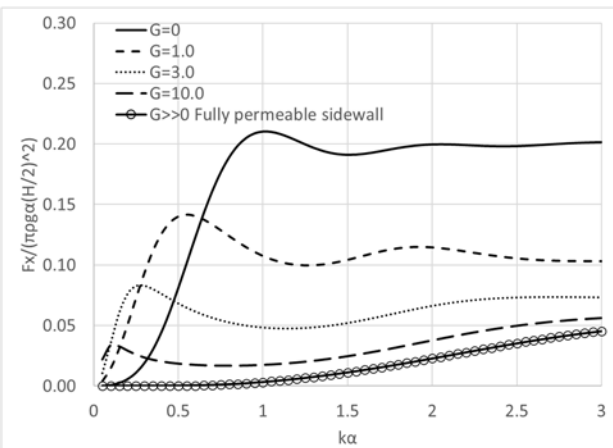

Fig. 2. Horizontal mean drift forces on the truncated cylindrical body for real porous coefficients

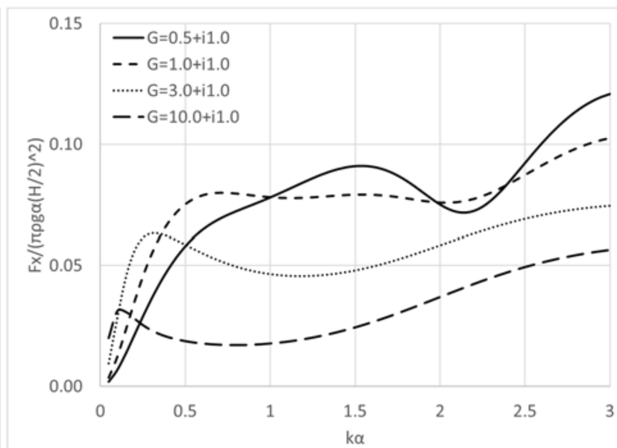

Fig. 3. Horizontal mean drift forces on the truncated cylindrical body for complex porous coefficients

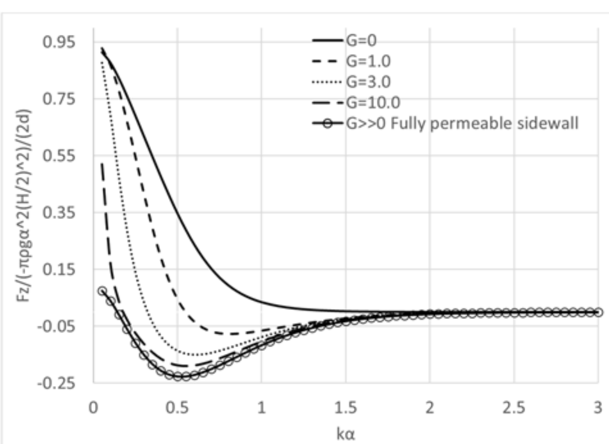

Fig. 4. Vertical mean drift forces on the truncated cylindrical body for real porous coefficients

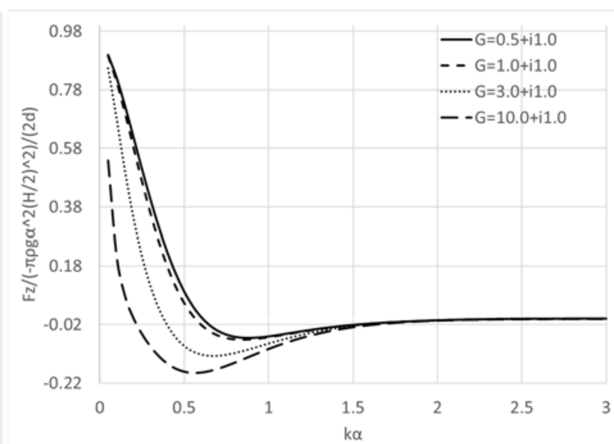

Fig. 5. Vertical mean drift forces on the truncated cylindrical body for complex porous coefficients

5 CONCLUSIONS

Based on the presented theoretical formulation the horizontal and vertical mean drift forces acting on a vertical porous cylindrical body have been evaluated. By comparing the results for different values of porous coefficients the following conclusions can be drawn:

- The results of the mean drift forces keep a similar trend for a real or a complex porous coefficient. However, the imaginary part of G causes a decrease of the values of the horizontal and vertical drift forces. Physically, the real and the imaginary part of the porous coefficient represent the drag term and the inertia term, which lead to the wave energy loss and the phase change, respectively (Teng, B. et al., 2001);
- The presented numerical results were obtained applying l=50 terms for the *II* ring element, whereas 40 and 100 terms were used for *I* and *III* ring elements, respectively. Also, the considered modes equal to m=7. Nevertheless, this study will be further continued by examining how the increase of the latter quantities affects the accuracy of the mean drift forces.

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