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Iterative multiple cluster scattering

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Highlights

- Theory for computing hydrodynamical interaction between large numbers of oscillating bodies is presented.
- The method combines features of the direct matrix and the iterative multiple scattering methods to compute interaction between subgroups of floating structures in an iterative way.
- High accuracy and low computational cost is achieved with the method as compared to full multiple scattering theory as well as numerical benchmark software.

1 Introduction

Accurate methods for computing the hydrodynamical interaction between a large number of floating bodies have multiple applications, such as very large floating platforms for airports, or offshore renewable energy farms consisting of many wind energy turbines or wave energy converters. To compute the dynamics and loads on the structures, it is necessary to solve the wave field within the park, resulting from incident waves and hydrodynamical interaction due to scattered and radiated waves.

In addition to accurate, the method should be computationally efficient. In particular, this is a requirement if the hydrodynamical interaction is to be computed multiple times in an optimization routine, for instance if the locations of the floats are to be optimized to minimize destructive loads on the platform legs, or to maximize power absorption of the wave energy converters.

Several analytical methods have been developed to solve the wave-structure interaction in arrays of structures. By applying multi-body diffraction theory for accoustics to ocean waves, an iterative multiple scattering theory was presented in [1] and extended in [2, 3] to various floating body geometries. The direct matrix method was used to develop a non-iterative multiple scattering theory in [4], and later combined with the single-body diffraction solution [5] and extended to independent radiation [6]. Whereas in the first approach, the scattered waves within an array are added iteratively until convergence is reached, in the non-iterative multiple scattering theory the single-body diffraction solution is inserted in a large diffraction matrix used to solve the scattering problem within the array. In addition to analytical models, there are several numerical tools available to solve the wave-structure interaction, mostly relying on the boundary element method [7, 8]. However, currently none of the analytical or numerical methods is fast enough to be implemented in optimization algorithms for a large number of interacting floating bodies.

In the present work, features of both iterative and non-iterative multiple scattering are used and combined. More specifically, the hydrodynamical interaction is solved non-iteratively within subgroups of floating objects, and the interaction between subgroups is included iteratively. This reduces drastically the high computational cost involved with inverting the large diffraction matrix in the multiple scattering theory: the matrix will contain only the unknowns of the present subgroup, as the remaining coefficients of other subgroups are known from previous iteration. Simultaneously, a high accuracy is retained, as the hydrodynamical interaction between bodies in close proximity are included exactly, within the assumptions of linear potential flow theory.

The proposed method will be analysed by studying the hydrodynamical interactions within wave energy parks; a natural application as wave energy converters will typically be deployed in subgroups, or *clusters*.

2 Theory

Wave energy parks Ocean waves contain large amounts of untapped energy, and much research has been invested to develop approaches and technologies to harvest this energy. Many wave energy converters (WECs) have been developed and differ fundamentally in absorption principle, size, and dynamics. Here, we will focus on small point-absorbing wave energy converters, consisting of a cylinder float connected to a bottom-mounted generator through a stiff line.

The energy absorption from a single point-absorber is typically not sufficient to motivate the investment in sea cables and other related infrastructure, and the WECs can be deployed in arrays, or parks, to deliver an

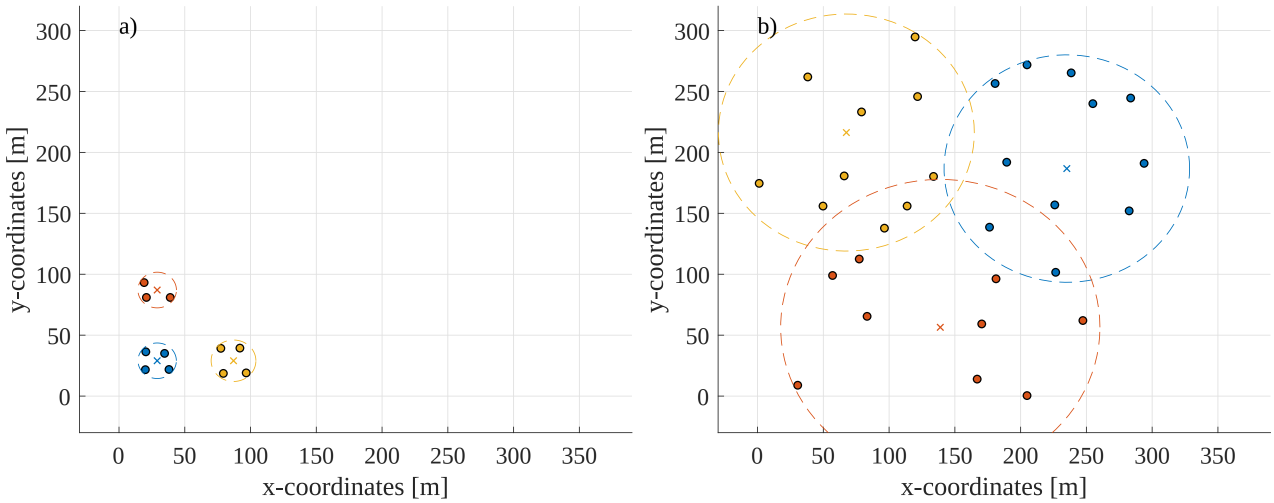

Figure 1: Examples of two wave energy parks. The circumferences of the clusters are illustrated by dotted circles. a) 11 WECs separated into three clusters. b) 30 WECs separated into three clusters.

average power of a few MW. The installation of WECs in parks will also reduce the unwanted power fluctuations [9], which is necessary to obtain an electricity quality compatible with grid connection.

The floats will interact hydrodynamically by scattered and radiated waves throughout the park, which will affect their dynamics and thereby the delivered power. Already in pioneering works on wave energy, it was investigated how such park effects would affect the total power of the park [10, 11, 12]. As a contrast to wind energy arrays, in wave energy arrays the park effect can be constructive, meaning that the WECs absorb more energy in the array than they would in isolation, and much work has been invested in finding optimal design parameters to achieve high energy absorption, low power fluctuations, and low costs. Parameters that can affect the performance are, for instance, the layout, separation distance, control configurations, and the dimensions of the WECs. In state-of-the-art wave energy park optimization studies, several objectives and parameters are often optimized simultaneously, using global metaheuristic optimization algorithms [13]. However, to find optima in the large solution space of wave energy parks, many configurations need to be evaluated, which calls for fast yet accurate methods of computing the hydrodynamics. This is the objective of the current work.

Wave energy parks will typically be deployed in clusters; groups of WECs sharing some infrastructure like a substation and sea cables. A park will consist of several clusters, separated by sufficient space to allow for installation and maintenance vessels. This configuration has been the inspiration for the proposed method, even if the method can be applied to any system of fixed or floating structures, installed in clusters or not. Examples of two wave energy parks consisting of three clusters each is shown in figure 1.

Multiple cluster scattering theory The proposed method combines both iterative [1, 2, 3], and non-iterative multiple scattering theory [4, 5, 6], and the reader is referred to these original works for further details.

Linear potential flow theory will be assumed, i.e. the fluid is assumed to be incompressible and irrotational, such that the fluid velocity can be written as the gradient of a potential function which satisfies the Laplace equation, $\nabla^2\Phi = 0$. Viscosity is neglected and the wave height is assumed to be small compared to the wave length, so that the boundary constraints at the free surface can be linearized.

Consider an array of N point-absorbing wave energy converters, each consisting of a floating truncated cylinder buoy connected to a linear generator at the seabed. At the origin of each buoy (x^i, y^i) we define local cylindrical coordinate systems (r, θ, z). Due to the linearity of the problem, the fluid potential can be decomposed into incident, radiated, and scattered waves. The general solution to the Laplace equation and the linear boundary constraints at the seabed, sea surface and all rigid boundaries is given in terms of Bessel functions and vertical eigenfunctions. Graf's addition theorem for Bessel functions is used to write the scattered and radiated waves from one buoy as incident waves on the other buoys. The fluid potential in the exterior region of a float i in a cluster A can thus be written

$$\phi^{i,(\mathrm{ext})} = \phi_0^i + \phi_S^i + \phi_R^i + \sum_{\substack{j\neq i\\ j\in A}} \left(\phi_S^j + \phi_R^j\right)\Big|_i + \sum_{B\neq A}\sum_{j\in B} \left(\phi_S^j + \phi_R^j\right)\Big|_i, \tag{1}$$

where index $0, S, R$ denotes incident waves, scattered waves, and radiated waves, respectively. Note that we distinguish between waves coming from buoys in the same cluster A and different clusters $B \neq A$.

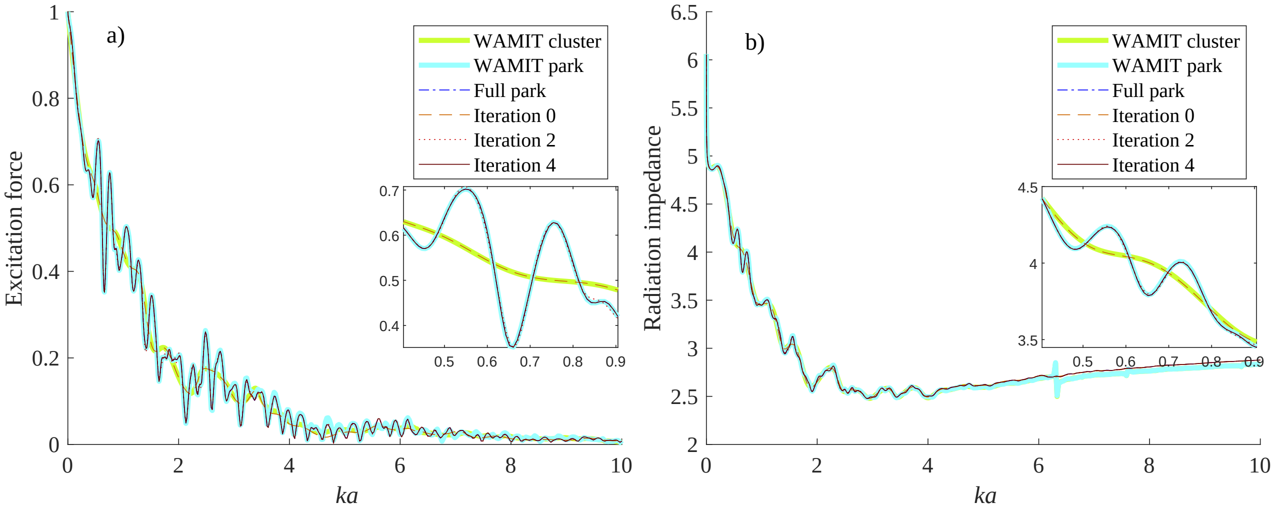

Figure 2: Absolute value of the non-dimensionalized excitation force and radiation force computed with the multiple cluster scattering method, as compared to full multiple scattering (dash-dotted line) and WAMIT (thick lines). Iteration 0 represents isolated clusters, corresponding to WAMIT cluster simulation.

By requiring continuity for the potentials at each buoy boundary, the unknown coefficients in the potentials can be found by solving the diffraction equation,

$$\begin{pmatrix} \mathbf{D}^{A-A} & \mathbf{D}^{A-B} & \cdots & \mathbf{D}^{A-Z} \\ \mathbf{D}^{B-A} & \mathbf{D}^{B-B} & \cdots & \mathbf{D}^{B-Z} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{D}^{Z-A} & \mathbf{D}^{Z-B} & \cdots & \mathbf{D}^{Z-Z} \end{pmatrix} \begin{pmatrix} \alpha^A \\ \alpha^B \\ \vdots \\ \alpha^Z \end{pmatrix} = \begin{pmatrix} \mathbf{B}^A \\ \mathbf{B}^B \\ \vdots \\ \mathbf{B}^Z \end{pmatrix} \tag{2}$$

where α^A are the unknown coefficients and $\mathbf{B}^A$ the known single-body diffraction or radiation matrices for cluster A. The submatrices $\mathbf{D}^{A-B}$ in the diffraction matrix involve the hydrodynamical interaction between floats in cluster A and B. The diffraction matrix is square of size $N(\Lambda_z + 1)(2\Lambda_\theta + 1)$, where Λ are truncation cut-offs of the infinite matrices in the fluid potentials. Hence, for a park of 20 structures, the diffraction matrix contains millions of entries, and solving the diffraction equation (2), in particular repeatedly, is time consuming. In [14], the approach was to include only those terms in the diffraction matrix that belong to floats within a user-defined interaction distance, which produces a sparse diffraction matrix which is more easily invertible. Here, a different strategy will be employed.

Assume that the coefficients α_t^B for all clusters $B \neq A$ are known from a previous iteration t and rewrite the diffraction equation (2) as Z matrix equations of the form

$$\mathbf{D}^{A-A}\alpha_{t+1}^A = \mathbf{B}^A - \sum_{B \neq A} \mathbf{D}^{A-B}\alpha_t^B. \tag{3}$$

The unknown coefficients of the next iteration, α_{t+1}^A, can then be found by solving the much smaller diffraction equations (3) within each cluster. For $t = 0$, the coefficients are assumed zero, so that hydrodynamical interaction between clusters are are only included for iterations $t \geq 1$.

3 Results

The proposed method has been implemented in MATLAB code and the results are evaluated here and compared to the multiple scattering theory (exact interactions within the full park) and with the numerical software WAMIT. In figure 2, the absolute value of the non-dimensionalized hydrodynamical forces are shown for one of the WECs in figure 1a. The excitation force is $F_{\text{exc}}/(\rho g \pi a^2)$, where ρ is the water density and a the radius of the buoy; the radiation impedance is the velocity independent part of the radiation force, $Z_{\text{rad}}/(\rho V \omega)$ where V is the submerged volume of the buoy at equilibrium and ω the angular frequency. The forces are shown as functions of the non-dimensionalized wave number ka. As can be seen from the figure, the forces computed with the multiple cluster scattering theory agree very well to the benchmark results. Iteration 0 agrees with the results for the isolated cluster, as expected, and higher iterations agree with the full park computations.

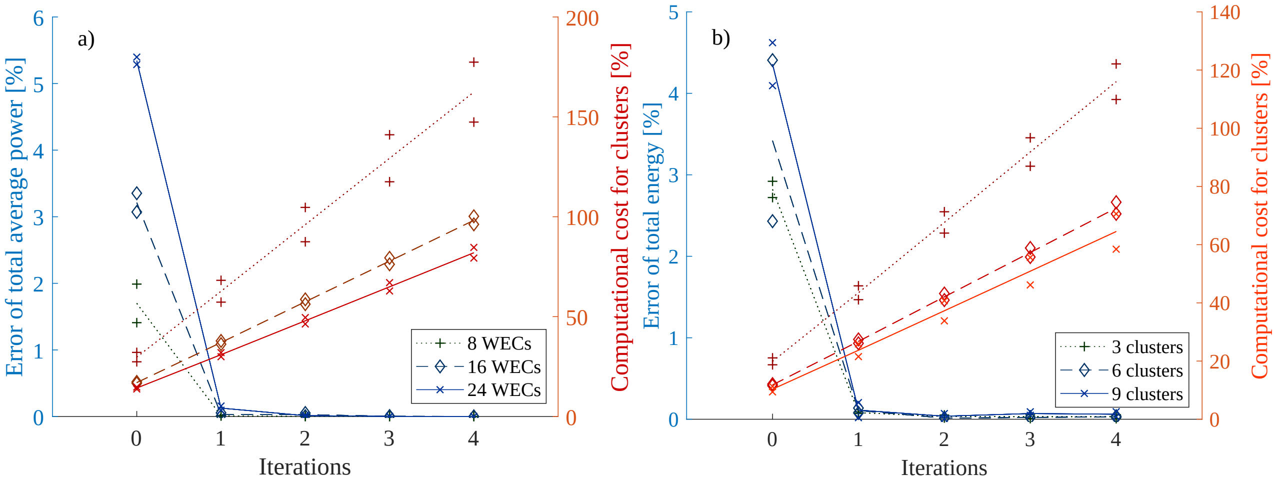

Figure 3: Accuracy and computational efficiency of the multiple cluster scattering method, for increased number of iterations. a) Park with 4 clusters of 2, 4, and 6 WECs each. b) 30 WECs separated into 3, 6, and 9 clusters. Two simulations are shown for each park size, with the lines showing the average between them.

As the next step, the accuracy and computational cost is evaluated quantitatively. In figure 3, the difference in the total absorbed energy computed by the multiple cluster method and the exact multiple scattering method is shown. Neglecting cluster interactions (iteration 0) gives an error of 1-6% (largest errors for large parks and many clusters), but already at the first iteration the error is reduced to $< 0.2\%$. The computational cost as compared to the multiple scattering method increases linearly with the number of iterations.

4 Conclusions

The proposed iterative multiple cluster scattering method is a simple yet effective method of computing the hydrodynamical interaction within large arrays of fixed or floating structures. As compared to the multiple scattering method, the method produces very accurate results already after one iteration. The cost savings are largest for large parks and many clusters; and it is for these parks that the method will be most useful.

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