36th International Workshop on Water Waves and Floating Bodies, April 25 - 28, 2021, SNU, South Korea

Acoustic-gravity wave generation as a result of bottom oscillation of compressible ocean

Santu Das[1,*] **and Michael H. Meylan**[2]

[1]Mathematical and Computational Sciences Division, Institute of Advanced Study in Science and Technology, Guwahati 781035, India

[2]School of Mathematical and Physical Sciences, University of Newcastle, NSW 2308, Australia

[*]Corresponding author, e-mail: *santudas20072@gmail.com / d.santu@iasst.gov.in*

Highlights

- The generation of AGW (Acoustic-gravity waves) due to bottom oscillation of a compressible ocean is studied based on linearized water wave theory.
- generation of AGW modes due to water compressibilty is shown graphically.
- Fourier transformation method is used to solve the boundary value problem and to obtain a closed form solution for the velocity potential.

1. Introduction

Acoustic-gravity waves are high–frequency acoustic waves that propagate in the ocean analogous to modes in a waveguide. They are somewhat mysterious, and while they are well attested to, theoretically they have not been confirmed to exist experimentally to the best of the authors' knowledge. They are expected to be generated when there is a large displacement of the seafloor [1], so they have been proposed as a way to detect a tsunami. It should be noted that despite their well–attested physical properties they have yet to be measured. Acoustic gravity waves travel much faster than the surface gravity tsunami waves and, if identified, have been proposed as a method to produce an early warning for the arrival of a Tsunami. Normal modelling of ocean waves assumes the water is incompressible, due to the high value of the sound speed in the water. However, if the scale of the fluid displacement becomes large, then the compression needs to be accounted for. The ocean water compressibility plays an important role in the generation of Tsunami ([2, 3]). In incompressible water, the most widely used model for linear water wave mechanics, there are infinitely many evanescent modes that decay away from the disturbance source and only one propagating mode except for the exceptional cases of wave blocking when multiple propagating modes exist. However, when the notion of low compressibility is introduced in the formulation, multiple propagating modes may arise along with the already existing gravity modes [4]. These newly generated propagating modes are called Acoustic-gravity waves in literature. These waves have a sinusoidal vertical profile that creates a pressure signature at ocean bottom and causes a microseism [5]. These waves are also responsible for deepwater transport [6] and can interact with ice-sheets [7] and ice-shelves [8]. This study aims to find a closed-form solution for the velocity potential that includes the AGW modes generated due to the bottom oscillation of the compressible ocean.

2. Mathematical formulation

We consider free surface gravity wave propagation in an incompressible ocean of finite depth h. The physical problem is formulated in a two-dimensional Cartesian coordinate system having z- axis pointing upwards and x-axis horizontal. The ocean bed is characterised as rigid. A wave propagation due to the ocean floor disturbance is realized both towards towards the positive and negative x- direction (see Fig. 1) under the assumption of linearised water wave theory. The flow is considered irrotational. There exists velocity potential $\phi(x, z, t) = f(x, z)\exp(\mathrm{i}\omega t)$ which obey the following equations:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2}, \tag{1}$$

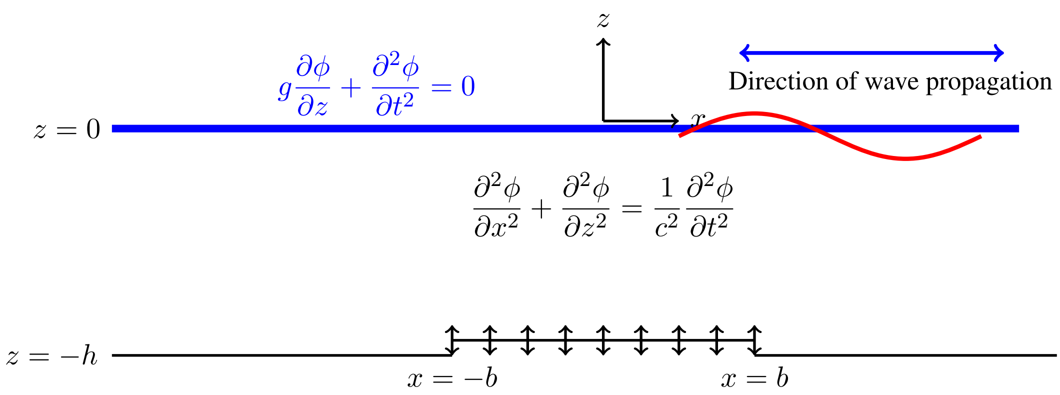

Figure 1: Schematic diagram of the physical problem. The ocean bottom in $-b<x<b$ is oscillating vertically.

where c is the speed of sound in the water, and given by $c=\sqrt{K_0/\rho_0}$ (K_0 is the bulk modulus and ρ_0 is the undisturbed density of water). The surface condition is given by

$$g\frac{\partial\phi}{\partial z}+\frac{\partial^2\phi}{\partial t^2}=0. \tag{2}$$

Now we consider a block of the ocean bottom centered at $(0,0)$ with a flat surface is oscillating vertically with an unit amplitude and angular frequency ω in the following form:

$$\zeta(x,t)=\mathcal{H}(b^2-x^2)e^{\mathrm{i}\omega t}, \tag{3}$$

where $\mathcal{H}(\cdot)$ represents Heaviside unit step function. Hence the vertical fluid displacement at the ocean bottom is given by

$$\phi_z=\mathcal{H}(b^2-x^2)\exp(\mathrm{i}\omega t). \tag{4}$$

Applying Fourier transformation of the form

$$\mathcal{F}(k,z)=\int_{-\infty}^{\infty}f(x,z)\exp(-\mathrm{i}kx)dx, \tag{5}$$

whose inverse Fourier transformation is given by

$$f(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathcal{F}(k,z)\exp(\mathrm{i}kx)dk, \tag{6}$$

3. Characteristics of wave motion

Applying the Fourier transformation defined above, solving the transformed equation using the separation of variables method, and applying the inverse Fourier transform, the velocity potential can be written as

$$f(x,z)=\mathbb{F}(x+b,z)-\mathbb{F}(x-b,z). \tag{7}$$

where

$$\mathbb{F}(\xi,z)=\int_{-\infty}^{\infty}M(k,z)e^{\mathrm{i}k\xi}dk, \quad \text{and } M(k,z)=\frac{1}{2\pi\mathrm{i}}\frac{\frac{\omega^2}{g}\sinh\mu z+k\cosh\mu z}{\mu k\left(\frac{\omega^2}{g}\cosh\mu h-\mu\sinh\mu h\right)}, \tag{8}$$

with $\mu^2=k^2-k_s^2$ for $Re(\mu)>0$ where $k_s^2=\omega^2/c^2$ and μ satisfies the dispersion relation (obtained from the singularities of the function $M(k,z)$)

$$\frac{\omega^2}{g}-\mu\tanh\mu h=0. \tag{9}$$

The roots of the above dispersion relation are given by

$$k=\pm k_0\pm k_n(n=1,2,\dots N),\ \text{and}\ \pm\mathrm{i}\lambda_n(n=1,2,\dots).$$

where

$$k_0 = \sqrt{\mu_0^2 + \frac{\omega^2}{c^2}},$$
$$k_n = \sqrt{\frac{\omega^2}{c^2} - \mu_n^2}, \quad \left(\frac{\omega}{c} > \mu_n\right) \qquad \text{for} \quad n = 1, 2, \ldots, N,$$
$$\lambda_n = \sqrt{\mu_n^2 - \frac{\omega^2}{c^2}}, \quad \left(\frac{\omega}{c} < \mu_n\right) \qquad \text{for} \quad n = N+1, N+2, \ldots.$$

The roots $\pm k_0$ and $\pm \lambda_n$ represent the gravity (propagating) and evanescent wave modes, respectively. The roots k_n are the acoustic-gravity wave modes which are generated when the wavenumber (k_s) of the sound wave becomes larger than the wavenumber of the evanescent waves. In the absence of water compressibility, μ can be replaced by k to obtain the equivalent dispersion relation for incompressible ocean. Figure (2) provides a comparative study on the location of the roots of the dispersion relation in both the cases. The movement of the first few evanescent modes of incompressible ocean is readily seen to move into the real axis to provide the AGW.

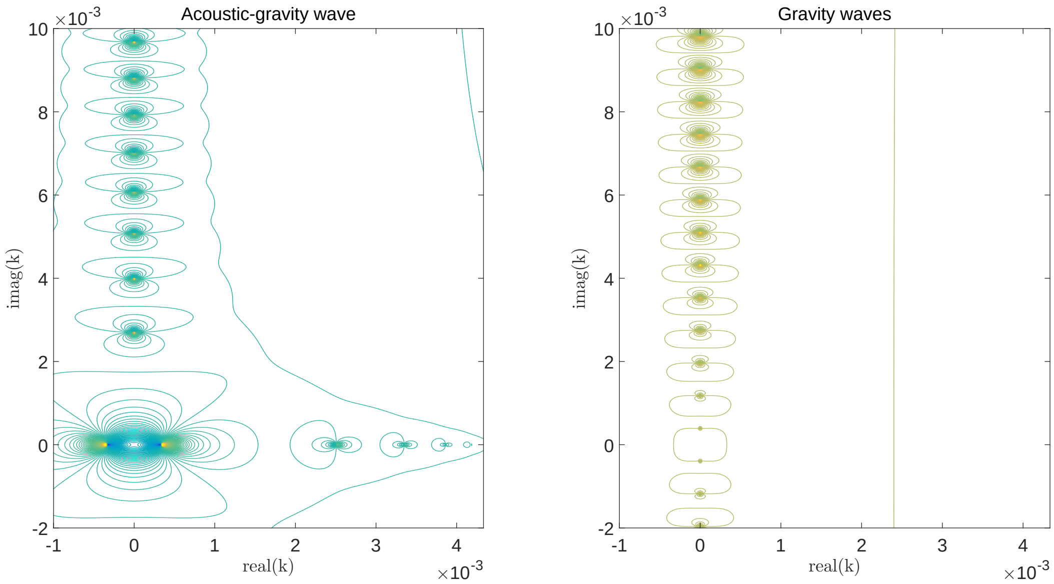

Figure 2: Generation of AGW from the evanescent modes when water compressibility is taken into account, is presented.

Residue calculus has been used to obtain the integral $M(k, z)$, and the region-wise potential functions are obtained by using inverse Fourier transform. The expression for $f(x, z)$ for $x > b$ can be obtained as

$$f(x,z) = -\sum_{n=N+1}^{\infty} \frac{4\mu_n \cos\mu_n(z+h)\sinh(\lambda_n b)}{\lambda_n^2\,(2\mu_n h + \sin 2\mu_n h)} e^{-\lambda_n x} + \frac{4\mu_0 \cosh\mu_0(z+h)\sin(k_0 b)}{k_0^2\,(2\mu_0 h + \sinh 2\mu_0 h)} e^{-\mathrm{i}(k_0 x - \pi/2)}$$
$$\sum_{n=1}^{N} \frac{4\mu_n \cos\mu_n(z+h)\sin(k_n b)}{k_n^2\,(2\mu_n h + \sin 2\mu_n h)} e^{-\mathrm{i}(k_n x - \pi/2)}.$$

Now, the expression for $f(x, z)$ for $x < -b$ can be obtained as

$$f(x,z) = -\sum_{n=N+1}^{\infty} \frac{4\mu_n \cos\mu_n(z+h)\sinh(\lambda_n b)}{\lambda_n^2\,(2\mu_n h + \sin 2\mu_n h)} e^{\lambda_n x} + \frac{4\mu_0 \cosh\mu_0(z+h)\sin(k_0 b)}{k_0^2\,(2\mu_0 h + \sinh 2\mu_0 h)} e^{\mathrm{i}(k_0 x + \pi/2)}$$
$$+ \sum_{n=1}^{N} \frac{4\mu_n \cos\mu_n(z+h)\sin(k_n b)}{k_n^2\,(2\mu_n h + \sin 2\mu_n h)} e^{-\mathrm{i}(k_n x - \pi/2)}.$$

The expression for $f(x,z)$ in the region $-b < x < b$ can be evaluated as

$$f(x,z) = \sum_{n=N+1}^{\infty} \frac{4\mu_n \cos\mu_n(z+h)e^{-\lambda_n b}}{\lambda_n^2\,(2\mu_n h + \sin 2\mu_n h)} \cosh\lambda_n x - \frac{4\mu_0 \cosh\mu_0(z+h)e^{-ik_0 b}}{k_0^2\,(2\mu_0 h + \sinh 2\mu_0 h)} \cos k_0 x$$
$$- \sum_{n=1}^{N} \frac{4\mu_n \cos\mu_n(z+h)e^{-ik_n b}}{k_n^2\,(2\mu_n h + \sin 2\mu_n h)} \cos k_n x + \frac{k_s \cos k_s z + \dfrac{\omega^2}{g}\sin k_s z}{k_s\left(\dfrac{\omega^2}{g}\cos k_s h + k_s \sin k_s h\right)}.$$

Figure (3) represents the water surface elevation for different values of length of ocean floor (b) that is oscillating periodically. Higher amplitude of surface wave away from the oscillating bottom is observed when b is smaller.

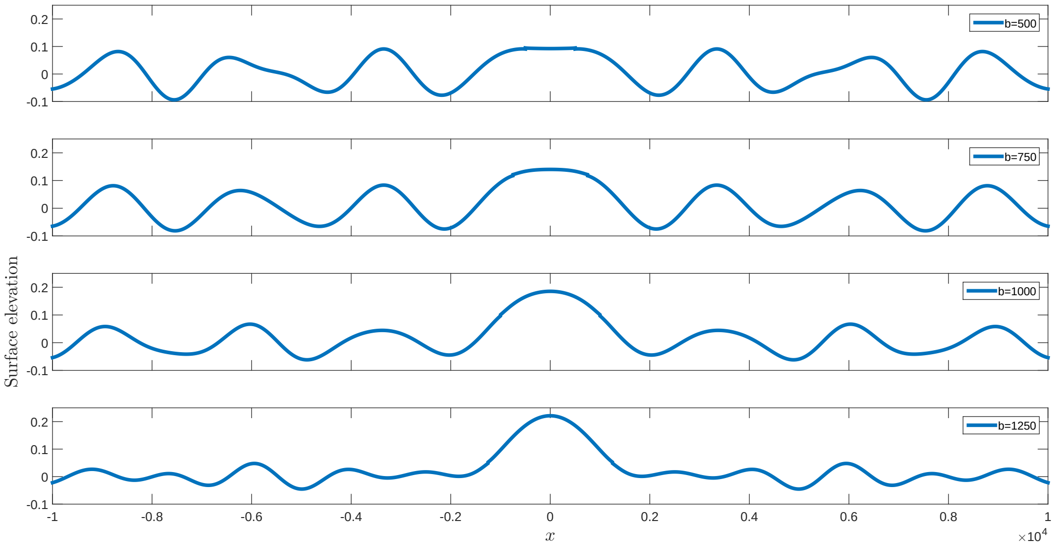

Figure 3: Free surface profile for different values of elevation width b against the horizontal space x. The parameter values are: $m = 0$, $g = 9.81\,\mathrm{m\cdot s^{-2}}$, $\rho = 1020\,\mathrm{Kg\cdot m^{-3}}$, $\omega = 2\pi$, $c_l = 1450\,\mathrm{m\cdot s^{-1}}$, $h = 3000\,\mathrm{m}$ and the number of acoustic gravity waves are 5.

More results on the free water surface elevation and pressure signature will be presented during the workshop.

References

[1] M Stiassnie. Tsunamis and acoustic-gravity waves from underwater earthquakes. *J. Eng. Math.*, 67(1-2):23–32, 2010.

[2] H Miyoshi. Generation of the tsunami in compressible water (part i). *J. Oceanogr. Soc. Japan*, 10(1):1–9, 1954.

[3] CC L Sells. The effect of a sudden change of shape of the bottom of a slightly compressible ocean. *Philos. Tr. R. Soc. S-A*, 258(1092):495–528, 1965.

[4] U Kadri. Wave motion in a heavy compressible fluid: Revisited. *Eur. J. Mech. B-Fluid.*, 49:50–57, 2015.

[5] MS Longuet-Higgins. A theory of the origin of microseisms. *Philos. Tr. R. Soc. S-A*, 243(857):1–35, 1950.

[6] U Kadri. Deep ocean water transport by acoustic-gravity waves. *J. Geophys. Res- Ocean*, 119(11):7925–7930, 2014.

[7] A Abdolali, U Kadri, W Parsons, and JT Kirby. On the propagation of acoustic-gravity waves under elastic ice sheets. *J. Fluid Mech.*, 837:640–656, 2018.

[8] U Kadri and M Stiassnie. Acoustic-gravity waves interacting with the shelf break. *J. Geophys. Res- Ocean*, 117(C3), 2012.