

# Heading and spreading effects on wave run-up due to tertiary interactions

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## Highlights

- A series of experiments is conducted to investigate the large wave run-up on the front of a fixed box, following the results reported at the previous workshop (Zhao et al. 2018);
- We explore the structure of the wave run-up over a large frequency range using a realistic JONSWAP-type input spectrum.
- The focus of this study is to examine the effects of heading and spreading for tertiary wave-structure interactions;
- It is observed that the surface amplification is greatly reduced with realistic spreading in the incident waves, while for uni-directional waves the amplification is very sensitive to the heading condition of the structure.

## 1. Introduction

Molin et al. (2003) reported a large run-up phenomenon on a fixed vertical plate in regular wave tests, where the local wave surface elevation reached up to 5x the amplitude of the incident waves. It was proposed by Molin et al. (2005a) that the reflected wave fields (from the body) ‘slow down’ the incident waves (as a shoal would) locally, through the tertiary wave interactions of Longuet-Higgins and Phillips (1962). This results in a local lensing on the weather side of the structure, leading to significant wave amplitude enhancement. Further studies (Bingham et al. 2004 and Molin et al. 2010) were conducted, again based on regular waves, emphasizing the important role of the tertiary interactions in the large wave run-up phenomenon.

Running uni-directional irregular waves, Zhao et al. (2019) identified similar amounts of wave run-up as those based on regular waves. This emphasizes the practical importance of tertiary wave-structure interactions in uni-directional wave with normal incidence. Molin et al. (2005b) looked at the heading effect using regular waves and found that the water surface amplification disappears when the wave approach angle exceeds 20°.

However, there are some important open questions remaining particularly for waves in a random sea, e.g. (1) how the wave run-up will vary with heading of the structure; (2) whether tertiary interactions play an important role for directionally spread sea states. To address these questions, tank testing is conducted at a scale of 1:60.

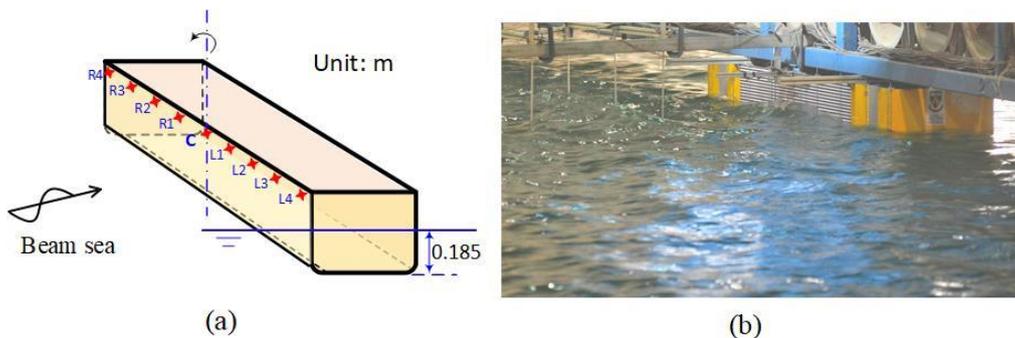


Fig. 1 Experimental setup: (a) sketch of the fixed box in the wave basin, with red symbols showing the location of the wave probes; (b) a snapshot of the model rigidly connected to the gantry (blue) in the basin.

## 2. Experimental set-up

The experiments were performed in the Deepwater Wave Basin at Shanghai Jiao Tong University. The wave basin is 50 m long, 40 m wide and the water depth was set to 10 m. Flap-hinged wavemakers are fixed along two neighbouring sides of the basin and wave absorbing beaches are installed on the opposite sides to minimize reflected waves.

The experimental set-up is illustrated in Fig. 1, with a rectangular box fixed in the wave basin. The model hull is 3.333 m long, 0.767 m wide and 0.425 m high, with immersed depth in still water being 0.185 m. As shown in Fig. 1, we deployed a series of wave gauges along the weather side of the model, measuring the surface elevations.

Both uni-directional and directional waves are used to explore the effects of heading and spreading on wave run-up due to tertiary wave-structure interactions. The wave field was calibrated prior to the actual model tests in the absence of the model. Then, with the model in place, the same paddle signal was used to produce identical incident waves. Each random wave test was run for  $\sim 1800$  s in the basin, and the wave surface elevations were sampled at 40 Hz. Fig. 1 shows typical wave elevations in the uni-directional wave test, measured in front of the fixed model with and without the model in place. The incident waves were based on a JONSWAP spectrum with  $H_s=52.5$  mm,  $T_p=0.9$  s and a shape factor of  $\gamma=1.5$  in the lab. One can see that the spectrum has a large frequency range of 0.6 - 2.7 Hz.

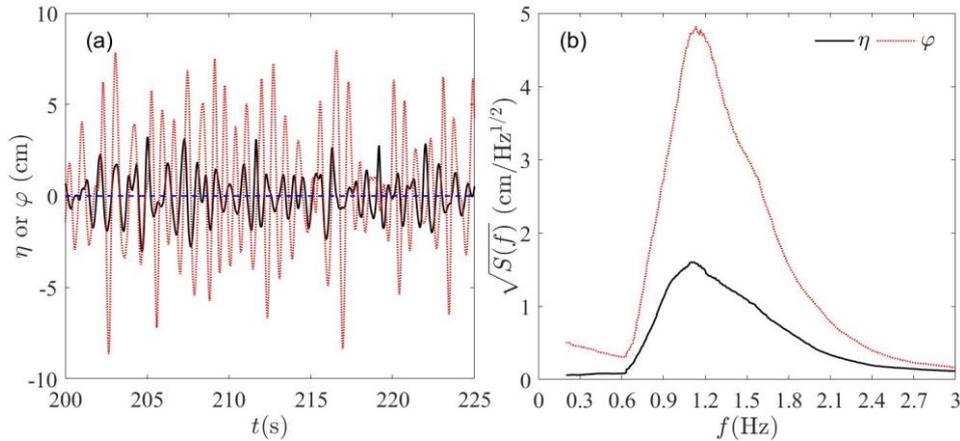


Fig. 2 Measured data from the wave gauge in front of and at midship of the fixed model in the uni-directional irregular wave. (a) representative time histories of the measured surface elevations with (the responses  $\varphi$  – red dashed line) and without (the incident waves  $\eta$  – black solid line) the model, (b) the corresponding amplitude spectra calculated based on the steady state time histories.

### 3. Structure of the wave run-up phenomenon in frequency domain

Response amplitude operators (RAOs) are obtained from Fourier analysis of the measured signals over individual windows each approximately 205 s long (8192 data points), giving a total of 8 time segments over the whole measured signal (1800 s). For the sake of clarity, we only plot in Fig. 3 (a) the beginning of the first time segment measured by wave probes located at L2 and R2 (see Fig. 1) which are symmetrically positioned either side of the centre-point. One can see that the time histories match initially, confirming the quality of the experimental set-up. As time goes on, the agreement between these two time histories deteriorates. This might be related to asymmetry introduced by wave breaking upstream of the model. Accordingly, there is some lack of perfect symmetry in the long-term averaged RAOs either side of the central point, as shown in Fig. 3 (b).

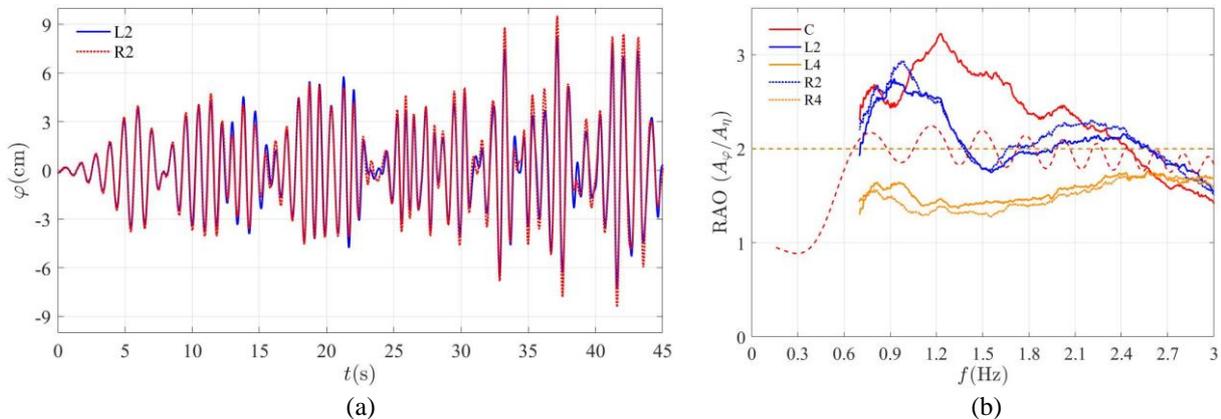


Fig. 3 Demonstration of the symmetry of the experimental set-up: (a) time history; (b) mean of RAOs at various locations, the red dashed curve is the linear prediction at the centre of the front face.

To provide a better understanding of the large wave run-up phenomenon due to tertiary interactions, we now focus on the RAOs at the midship in front of the fixed box. Fig. 4 shows the range boundaries (grey shaded) and the mean (red line) of the RAOs obtained from all the segments. In the previous study (Zhao et al. 2019), we looked at the frequency

range from 0.2 Hz to 1.8 Hz, where the surface amplifications grew with increasing frequency. However, waves will eventually break if they grow sufficiently. To explore the limit of this run-up amplification due to tertiary interactions, the incident wave frequencies cover a large 0.6 - 2.7 Hz, much larger than that in the previous study. Understanding the structure of the wave run-up at very large frequency is of some interest, although for realistic spectra the high frequencies compared to the spectral energy peak contain little energy to be locally amplified.

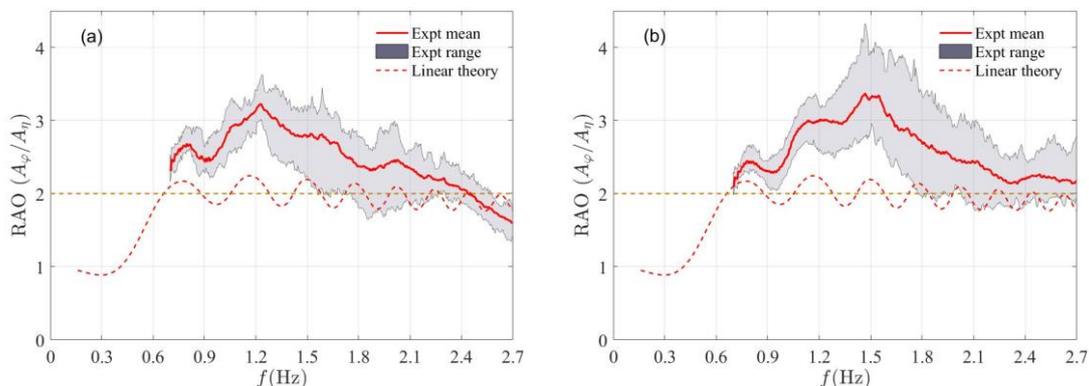


Fig. 4 Surface elevation RAOs in front of the model under uni-directional irregular waves: (a) the incident wave is the same as in Fig. 2; (b) the incident wave is generated using the same wave paddle signal as in (a) but  $\times 0.5$ .

To facilitate the demonstration of the wave run-up phenomenon due to tertiary interactions, we provide linear predictions of the surface RAOs based on HydroStar, as shown in Fig. 4. The RAOs obtained from linear theory show an oscillation pattern around a value of 2, which is known to be a result of the Fresnel diffraction effect (e.g. Grice et al. 2013). Fig. 4 (a) shows that the experimentally-determined elevation RAOs are significantly amplified, compared to those from linear potential flow theory. The Fresnel diffraction pattern is also visible in the experimental results at a certain range of frequencies ( $\sim 0.7 - 1.2$  Hz), though the RAOs have been amplified. This run-up amplification has been demonstrated to be consistent with tertiary wave interactions in the previous study (Zhao et al. 2019), and so we do not repeat this. However, we are interested in the structure of the surface amplifications across a larger frequency range than has been explored previously. As frequency increases, the experimentally-determined RAOs reduce towards the linear predictions. This may be attributed to the following reasons. The wave field in front of the model kept growing due to the ‘Molin lensing’ effect until there was significant breaking of apparently mostly short wave components, impeding further surface amplifications; considerable wave breaking was observed (and recorded on video) in front of the fixed model in the experiment shown in Fig. 4 (a). It also is possible that the short waves could be ‘over’-lensed, where the ‘Molin lens’ could be focussing the short wave components well up-wave of the weather side of the model.

We repeated this experiment by generating the wave field using the same random wave paddle signal but with the amplitude halved, now without observing visible breaking in the wave field. The results are plotted in Fig. 4 (b). It is clear that the RAOs are larger and the maximum RAO value shifts towards higher frequencies with smaller incident waves, though the Molin lensing mechanism is itself amplitude dependent so reduces with smaller waves.

#### 4. Heading and spreading effects

It is of practical interest to explore how the wave run-up will behave (i) when the heading of the structure is changed or (ii) when the irregular incident waves are directionally spread.

To investigate the heading effect, we used the same waves as in Fig. 4 (b), but with the model being rotated  $8^\circ$  and  $15^\circ$  respectively in the anti-clockwise direction. To examine the spreading effect, testing is conducted with directional waves whose main direction has a normal incidence to the fixed model. The directional factor of the spread sea is selected to be  $s = 30$  as a starting point. We do not provide details of the results for directionally spread waves here but will present them at the workshop.

The wave surface elevation RAOs at different locations along the front face of the fixed model are given in Fig. 5, with linear predictions also being given for comparison. The wave run-up is very sensitive to the heading conditions. For example, when the heading of the model is rotated by  $8^\circ$  in the anti-clockwise direction, the RAOs are significantly reduced and the maximum RAO is shifted away from the centre towards the left side, i.e. occurring at the location of wave probe L3, which is now downwave of midships. When the heading of the model is changed to  $15^\circ$ , the RAOs tend to the linear predictions. It is noted that the RAOs measured by the wave probes on the downwave side only are provided in Fig. 5 (c) for the sake of clarity. In the case of spread waves, the RAOs are reduced as shown in Fig. 5 (d). It is interesting to note that the largest wave run-up RAO now occurs away from the centre, though the main wave direction is still normal incidence onto the fixed model.

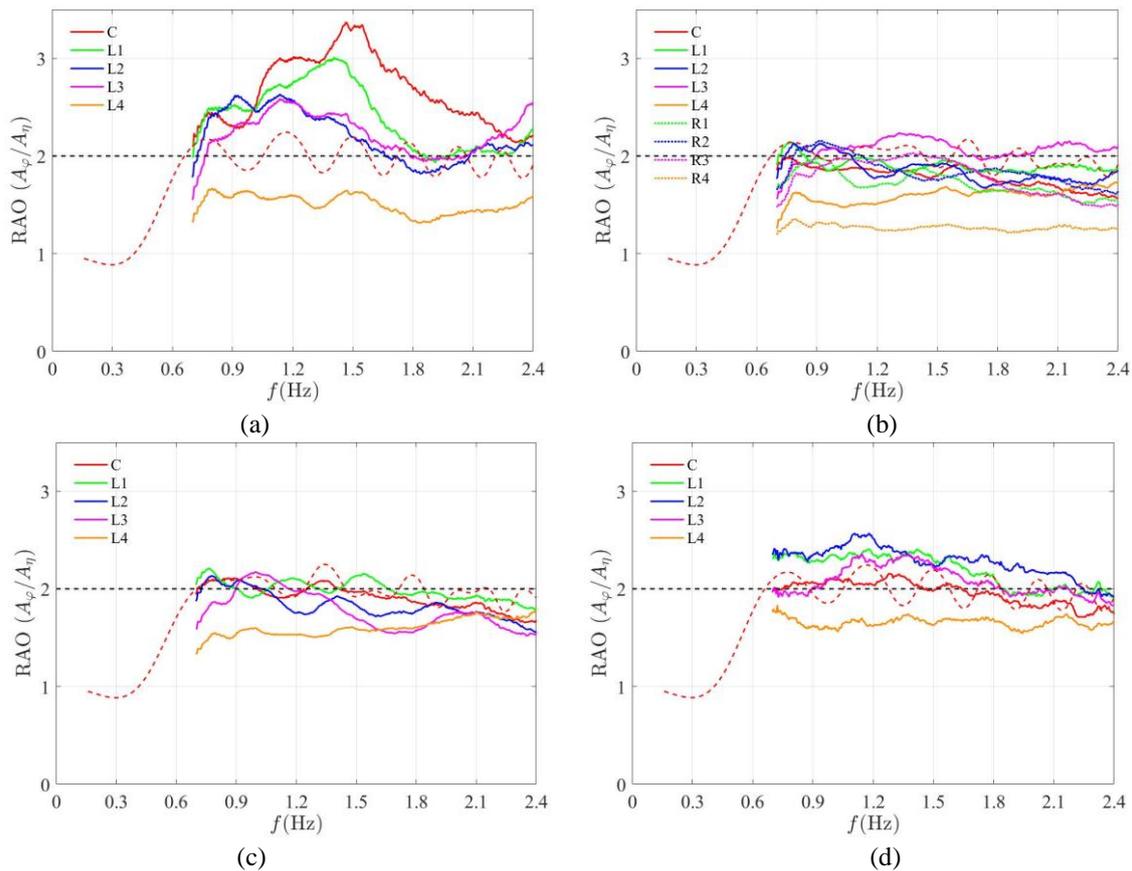


Fig. 5 Heading and spreading effects on wave run-up: (a) uni-directional waves with normal incidence; (b) uni-directional waves with incidence of  $8^\circ$  (e.g. model is rotated by  $8^\circ$  anti-clockwise relative to Fig. 1); (c) uni-directional waves with incidence of  $15^\circ$ ; (d) directional waves with normal incidence.

## 5. Concluding remarks

Previously we showed that the surface elevation in front of a highly reflective structure can be significantly amplified at high frequencies in uni-directional irregular waves (Zhao et al. 2019), reaching up to 4x the amplitude of the incident waves. To explore the effects of heading and spreading, we conducted new experiments on a single box fixed in the wave basin. We find:

- Tertiary wave interactions contribute to large surface amplifications starting from a specific frequency dependent on the size of the scattering box. However there is a peak RAO with frequency and above this the interaction weakens and eventually appears to approach the simple linear RAO values.
- It is striking that the surface elevation RAOs are very sensitive to the nonlinearity of the incident waves but also to wave breaking in the Molin lensing region up-wave of the box.
- Heading and spreading show strong effects on the surface RAOs: the maximum response is shifted away from the centre of the box when the heading of the model is changed, and even small changes in heading or a small degree of directional spreading can reduce the surface amplifications.

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