Wave-Induced Drift Motion during Ship Turning

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1. Introduction

The ship turning in calm water reaches to a steady turning after time has elapsed, and the trajectory becomes a circle. On the other hand, the turning circle in waves is deformed mainly due to the effect of the wave-induced steady forces, and during the turning the ship drifts to a direction different from the incident wave direction (Ueno, et al., 2003)(Yasukawa and Nakayama, 2009). This paper presents a theoretical treatment with respect to the deformation of the turning circle in waves and the ship drift motion during turning by extending the linear maneuvering motion theory by Nomoto et al. (1957).

2. A Theoretical Treatment of Ship Turning in Waves

2.1 Motion equations

In this theoretical analysis, the following assumptions apply:

- Rudder angle δ, ship's lateral velocity v, and ship's yaw rate r are small.
- Surge-coupling effects on maneuvering are neglected. The ship speed U is given.
- Wave-induced steady lateral force Y_W and yaw moment N_W acting on the ship are small.

Therefore, the motion equations of the ship is simplified to the equations with respect to sway and yaw.

In the framework of the ship-fixed coordinate system, the motion equations in the non-dimensional form are expressed as

$$(m' + m'_y)\dot{v}' + (m' + m'_x)r' = Y', \tag{1}$$

$$(I'_{zz} + J'_{zz})\dot{r}' = N', \tag{2}$$

where m is the ship's mass, I_{zz} is the moment of inertia for yaw, m_x is the added mass for surge, m_y is the added mass for sway, J_{zz} is the added moment of inertia for yaw, Y is the lateral force acting on the ship, and N is the yaw moment around the center of gravity acting on the ship. These equations are non-dimensionalized by using the water density ρ, ship length L, ship draft d, and ship speed U. The dot notation denotes the ordinary differential with respect to non-dimensionalized time $t'(= tU/L)$.

Y' and N' are expressed as

$$\left.\begin{array}{lcl} Y' &=& Y'_v v' + Y'_r r' + Y'_\delta \delta + Y'_W(\chi_r) \\ N' &=& N'_v v' + N'_r r' + N'_\delta \delta + N'_W(\chi_r) \end{array}\right\} \tag{3}$$

Y'_v, Y'_r, N'_v, and N'_r are linear hydrodynamic derivatives on maneuvering. Y'_δ and N'_δ are rudder force coefficients. $Y'_W(\chi_r)$ and $N'_W(\chi_r)$ are coefficients of the wave-induced steady lateral force and the yaw moment in waves, respectively; are each functions of the relative wave direction $\chi_r(= \chi - \psi)$; and are expressed as follows:

$$Y'_W(\chi_r) = \frac{2}{F_n^2}\frac{H_{1/3}^2}{Ld}C_Y(\chi_r), \quad N'_W(\chi_r) = \frac{2}{F_n^2}\frac{H_{1/3}^2}{Ld}C_N(\chi_r), \tag{4}$$

where F_n is the Froude number based on L, and $H_{1/3}$ is the significant wave height. C_Y and C_N are wave-induced steady lateral force coefficient and wave-induced steady yaw moment coefficient, respectively.

v', r', and heading angle ψ are assumed to be expressed as follows:

$$\left.\begin{aligned} v' &= v'_0 + \Delta v' \\ r' &= r'_0 + \Delta r' \\ \psi &= \psi_0 + \Delta\psi \end{aligned}\right\} \tag{5}$$

The subscript 0 implies the quantity in calm water; substituting Δ implies the change in quantity due to the wave effect. ψ_0 is assumed to be $O(1)$, and the other terms are assumed to be $O(\varepsilon)$, where ε is a small quantity.

By substituting eq. (5) into eqs. (1) and (2) and linearizing the equations, we obtain two sets of motion equations: one set gives the motion equations in calm water and the other set gives the equations for motion change due to the wave effect. The motion equations in calm water coincide with the formulas derived by Nomoto et al.(1957). The equations for the motion changes due to the wave effect are expressed as

$$(m' + m'_y)\Delta\dot{v}' + (m' + m'_x)\Delta r' = Y'_v\Delta v' + Y'_r\Delta r' + Y'_W(\chi_r), \tag{6}$$

$$(I'_{zz} + J'_{zz})\Delta\dot{r}' = N'_v\Delta v' + N'_r\Delta r' + N'_W(\chi_r). \tag{7}$$

For simplicity, the Taylor expansion is applied to $Y'_W(\chi_r)$ at $\psi = \psi_0$ as follows:

$$Y'_W(\chi_r) \simeq Y'_W(\chi_0) + \Delta\psi\frac{\partial Y'_W}{\partial\psi} + ... = Y'_W(\chi_0) + O(\varepsilon^2), \tag{8}$$

where χ_0 is defined as $\chi - \psi_0$. Therefore, the following motion equations are obtained as

$$(m' + m'_y)\Delta\dot{v}' + (m' + m'_x)\Delta r' = Y'_v\Delta v' + Y'_r\Delta r' + Y'_W(\chi_0), \tag{9}$$

$$(I'_{zz} + J'_{zz})\Delta\dot{r}' = N'_v\Delta v' + N'_r\Delta r' + N'_W(\chi_0). \tag{10}$$

If the heading angle in calm water ψ_0 is given, χ_0 is known when χ is given, and $Y'_W(\chi_0)$ and $N'_W(\chi_0)$ are also known. By eliminating $\Delta v'$ in eqs. (9) and (10), the following equation is obtained:

$$T'_1T'_2\Delta\ddot{r}' + (T'_1 + T'_2)\Delta\dot{r}' + \Delta r' = F'_W(\chi_0), \tag{11}$$

where

$$F'_W(\chi_0) = \left[N'_vY'_W(\chi_0) - Y'_vN'_W(\chi_0)\right]/C. \tag{12}$$

The constants T'_1 and T'_2 are expressed using the linear derivatives (details are skipped here). Eliminating $\Delta r'$ in eqs. (9) and (10), the following equation is obtained:

$$T'_1T'_2\Delta\ddot{v}' + (T'_1 + T'_2)\Delta\dot{v}' + \Delta v' = F'_V(\chi_0), \tag{13}$$

where

$$F'_V(\chi_0) = \left[N'_rY'_W(\chi_0) - (Y'_r - m' - m'_x)N'_W(\chi_0)\right]/C. \tag{14}$$

Eq. (11) for $\Delta r'$ and eq. (13) for $\Delta v'$ are base equations for the motion changes due to the wave effect.

2.2 Approximate solution of turning change due to wave effect

Next we will consider the solution of eq. (11), where the absolute wave direction χ is assumed to be zero. This means that the head wave is assumed at the time of approaching before steering is initiated. In addition, for analytical treatment of the problem, F'_W and F'_V are assumed to be expressed using the sine function, and considering the condition after time has elapsed, we can approximate as follows:

$$F'_W(\chi_0) = -A_W\sin(r'_St'), \tag{15}$$

$$F'_V(\chi_0) = -A_V\sin(r'_St'). \tag{16}$$

where r'_S is the non-dimensional yaw rate during steady turning in calm water.

The motion equation (11) is then rewritten as

$$T'_1 T'_2 \Delta\ddot{r}' + (T'_1 + T'_2)\Delta\dot{r}' + \Delta r' = -A_W \sin(r'_S t'). \tag{17}$$

Here the particular solution for $\Delta r'$ is assumed to be

$$\Delta r' = A_W \Im[r_C \exp(i r'_S t')], \tag{18}$$

where $\Im$ is obtained by taking the imaginary part of the complex number, and i is $\sqrt{-1}$. By substituting eq. (18) into eq. (17), the following is obtained:

$$r_C = \frac{-1}{i(T'_1 + T'_2)r'_S + 1 - T'_1 T'_2 r'^2_S} = -1 + i(T'_1 + T'_2)r'_S + O(r'^2_S). \tag{19}$$

Therefore, the solution is expressed as

$$\Delta r' = A_W C_W \sin(r'_S t' + \epsilon_W), \tag{20}$$

where

$$C_W = \sqrt{1 + (T'_1 + T'_2)^2 r'^2_S} \simeq 1, \tag{21}$$

$$\epsilon_W = \tan^{-1}\left[\frac{(T'_1 + T'_2)r'_S}{T'_1 T'_2 r'^2_S - 1}\right] \simeq \tan^{-1}\left[(T'_1 + T'_2)r'_S\right]. \tag{22}$$

When eq. (20) is integrated by t', the heading change due to the wave effect $\Delta\psi$ can be expressed as

$$\Delta\psi = -\frac{A_W}{r'_S}\cos(r'_S t' + \epsilon_W) + \psi_I, \tag{23}$$

where ψ_I is an integration constant. Similarly, $\Delta v'$ is obtained as

$$\Delta v' = A_V \sin(r'_S t' + \epsilon_W). \tag{24}$$

2.3 Deformation of turning circle in waves

In the space-fixed coordinate system, the equation for the ship position (x', y') is expressed as

$$\left.\begin{aligned} \dot{x}' &= \cos\psi - v'\sin\psi \\ \dot{y}' &= \sin\psi + v'\cos\psi \end{aligned}\right\} \tag{25}$$

Substituting eq. (5) into eq. (25) and linearizing the equation, the followings are obtained:

$$\left.\begin{aligned} \dot{x}' &= \cos\psi_0 - v'_0\sin\psi_0 + \Delta\dot{x}' \\ \dot{y}' &= \sin\psi_0 + v'_0\cos\psi_0 + \Delta\dot{y}' \end{aligned}\right\} \tag{26}$$

where

$$\Delta\dot{x}' = -(\Delta\psi + \Delta v')\sin\psi_0, \tag{27}$$

$$\Delta\dot{y}' = (\Delta\psi + \Delta v')\cos\psi_0. \tag{28}$$

$(\Delta x', \Delta y')$ expresses the change in the ship's position due to the wave effect.

Here, we consider the turning condition after time has elapsed. By substituting eqs. (23) and (24) into eqs. (27) and (28), the following are obtained:

$$\Delta\dot{x}' = -\left[A_V \sin(r'_S t' + \epsilon_W) - \frac{A_W}{r'_S}\cos(r'_S t' + \epsilon_W) + \psi_I\right]\sin(r'_S t'), \tag{29}$$

$$\Delta\dot{y}' = \left[A_V \sin(r'_S t' + \epsilon_W) - \frac{A_W}{r'_S}\cos(r'_S t' + \epsilon_W) + \psi_I\right]\cos(r'_S t'). \tag{30}$$

When eqs. (29) and (30) are integrated by t', the turning trajectory change due to the wave effect ($\varDelta x'$, $\varDelta y'$) are obtained as

$$\begin{aligned}
\varDelta x' &= -\frac{A_W}{4r_S'^2}\cos(2r_S' t' + \epsilon_W) - t'\frac{A_W}{2r_S'}\sin\epsilon_W + \frac{\psi_I}{r_S'}\cos(r_S' t') \\
&\quad +\frac{A_V}{4r_S'}\sin(2r_S' t' + \epsilon_W) - t'\frac{A_V}{2}\cos\epsilon_W + x_{0I}', \qquad (31)\\
\varDelta y' &= -\frac{A_W}{4r_S'^2}\sin(2r_S' t' + \epsilon_W) - t'\frac{A_W}{2r_S'}\cos\epsilon_W + \frac{\psi_I}{r_S'}\sin(r_S' t') \\
&\quad +\frac{A_V}{4r_S'}\cos(2r_S' t' + \epsilon_W) + t'\frac{A_V}{2}\sin\epsilon_W + y_{0I}'. \qquad (32)
\end{aligned}$$

Eqs. (31) and (32) are composed of three terms: a varying term with frequency $2r_S'$, a varying term with frequency r_S', and a term that is proportional to t'. Since the first two terms vary periodically with t', it can been seen that the drift motion that occurs during the turning of ships in waves comes from the term that is proportional to t'. The term that is proportional to t' emerges from the time integration in terms of $\sin^2(r_S' t')$ and $\cos^2(r_S' t')$ in eqs. (29) and (30). Initially, this comes from the interaction between the ship's heading in calm water and the heading change due to the wave effect (see eqs. (27) and (28)).

Now, consider a condition in which the heading changes by 2π from a certain time $t' = t_0'$. When $\varDelta t'$ is denoted as the time it takes, $\varDelta t'$ is expressed as $2\pi/r_S'$. The coordinates of the trajectory change due to waves at $t' = t_0'$ are represented by ($\varDelta x_{p0}'$, $\varDelta y_{p0}'$), and the coordinates of the trajectory change after the heading changes by 2π are represented by ($\varDelta x_{p2\pi}'$, $\varDelta y_{p2\pi}'$). Consequently, the distance between the two coordinates (drifting distance) l_{01}' and the inclination (drifting direction) θ_{01} are calculated as follows:

$$l_{01}' = \sqrt{\left(\varDelta x_{p2\pi}' - \varDelta x_{p0}'\right)^2 + \left(\varDelta y_{p2\pi}' - \varDelta y_{p0}'\right)^2} = \frac{\varDelta t'}{2r_S'}\sqrt{A_W^2 + A_V^2 r_S'^2} \simeq \frac{\pi|A_W|}{r_S'^2}, \qquad (33)$$

$$\theta_{01} = \tan^{-1}\left[\frac{\varDelta x_{p2\pi}' - \varDelta x_{p0}'}{\varDelta y_{p2\pi}' - \varDelta y_{p0}'}\right] = \tan^{-1}\left[\frac{-A_W\sin\epsilon_W - A_V r_S'\cos\epsilon_W}{-A_W\cos\epsilon_W + A_V r_S'\sin\epsilon_W}\right] \simeq \epsilon_W. \qquad (34)$$

l_{01}' and θ_{01} are determined independently of time. l_{01}' is proportional to the square of the turning radius in calm water and is proportional to A_W. Therefore, as shown in eq. (4), l_{01}' is proportional to $H_{1/3}^2$ and is inversely proportional to F_n^2. θ_{01} coincides with ϵ_W as defined in eq. (22).

3. Conclusion

Extending the linear theory of ship maneuvering motions by Nomoto et al.(1957), a formula representing the deformation of the ship turning trajectory due to waves and formulae for calculating the drifting distance and the drifting direction due to waves were derived. These specific calculation examples will be presented at the workshop and compared with the tank test results.

References

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