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Wave groups in the ocean

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1 INTRODUCTION

More than half a century has passed since the paramount discovery of the fetch law of the ocean waves by Sverdrup and Munk, and nowadays, daily wave forecast is a routine for meteorological agencies worldwide. However, extensive researches done in the last few decades to understand the predictability of the freak waves led researchers to modify the quasi-Gaussian closure assumption of the Hasselmann-type that requires tens of thousands of wave periods. The ocean wave spectra are changing at a much shorter time scale and spatial extent, and as a result, detuned-resonant interaction becomes important. Subsequently, outstanding waves appear from nowhere. The key to understanding this nonlinear self-focusing mechanism is the modulational instability of a weakly nonlinear wave train. Here, we revisit the work of Marshall P. Tulin who has, in the middle of the 1990s, presented his unique view of the ocean waves that encompasses a wide variety of topics and questions to be answered. Some questions are answered by now, but some are not.

2 EVOLUTION AND STRUCTURE OF ENERGETIC WIND WAVES

Ocean waves are generated by the wind. In the balance of energy input from the wind and the energy loss due to the wave breaking, the waves grow with time and distance. The law that governs the growth of wind waves, or windsea after Sverdrup and Munk (1946), is called the fetch law. The fetch law tells us that, as the wave height becomes higher, the wavelength becomes longer. The wind waves are considered to be represented by a self-similar spectrum and as the waves grow, the spectral peak shifts to a lower frequency. This spectral downshifting is considered a result of the nonlinear wave-wave interaction whose stochastic representation was first derived by Hasselmann (1962) based on weak nonlinear assumption.

Tulin, Yao, and Magnusson (1996, TYM96) in their pioneering yet less well-known paper, touches on a variety of topics related to the evolution and structure of energetic wind waves, which, after a quarter of a century, poses relevant questions to be answered. Their research starts with a heuristic derivation of the coupled evolution equations of the peak energy density $\tilde{e}$ and the wave group velocity $\tilde{c}_g$ of the wind waves:

$$\frac{\partial \tilde{e}}{\partial t} + \tilde{c}_g \frac{\partial \tilde{e}}{\partial x} = -\tilde{e}\frac{\partial \tilde{c}_g}{\partial x} + \dot{e}_w - D_b \qquad (1)$$

$$\frac{\partial \tilde{c}_g}{\partial t} + \tilde{c}_g \frac{\partial \tilde{c}_g}{\partial x} = \gamma \tilde{c}_g \frac{D_b}{\tilde{e}} \qquad (2)$$

Here, $\dot{e}_w$ is the time rate of energy transfer from the wind, and D_b is the time rate of energy dissipation due to breaking. These dual equations correspond to the conservation of wave energy and momentum and will be formally derived by Fontaine (2013) later on. TYM96 first demonstrated that this dual equation can explain the fetch law under constant wind forcing and Toba's $3/2$ law which is considered to be one of the most robust relations of growing wind waves. Moreover, they attempt to explain the "freak waves" observed in the North Sea in relation to the wind squall where the first term on the right-hand side of (1), $-\tilde{e}\,\partial\tilde{c}_g/\partial x$, plays an important role in the subsequent dispersive focusing. The uniqueness of this approach is to consider a small temporal and spatial variations of the wind forcing which are disregarded in most other studies. Another important term is the right-hand side of (2), $\gamma\tilde{c}_g\, D_b/\tilde{e}$, representing the excess of momentum to the energy loss of breaking. The term derives from the assumption that waves lose energy not as a wave but in a different form of fluid motion and thereby the wave momentum is no longer in balance with the remaining energy. Therefore, the system inevitably changes the peak frequency, and the positive sign of the γ assures that the system downshifts (e.g. Tulin and Waseda 1999, TW99).

The uniqueness of the dual equation is its underlying assumption that the downshifting is related to wave breaking and hence disregarding the weakly nonlinear wave interaction. Indeed, Tulin cast doubt on the validity of the Hasselmann's equation. The idea is partially adopted by Donelan et al. (2012) where the downshifting is associated with the redistribution of the wave energy due to plunging breaker. The so-called Miami wave model was developed without the Hasselmann's nonlinear source term. Now, the value of γ needs to be empirically determined. TW1999 estimated γ to be around 0.4 for an unstable Stokes wave train with energetic breakers and for random directional seas.

Another visionary perspective presented in TYM96 is the notion of the significance of wave group modulation. Tulin related breaking to the modulated wave train and discovered that the low-grazing angle radar images well captured the breaking wave signature associated with the modulational instability. By now, the radar signatures are well-documented for the Benjamin-Feir unstable wave train (Fuchs et al. 1999), and for the wind-waves (Lamont-Smith et al. 2003). The long term evolution of the modulated wave train including breaking energy dissipation was studied numerically and experimentally exploring the parameter space of steepness and spectral bandwidth (TW1999). Whilst the initial evolution of the modulated wave train is well represented by a solution to the Nonlinear Schrödinger equation, the fate of its long-term evolution depended on the presence of energetic breakers with which the evolution deviated from the recursive evolution and led to a permanent downshifting.

Moreover, TYM96 addresses the significance of the wave deformation of the breaking wave on the engineering problems. A few of the studies using modulated wave trains with breaking waves are reported for interaction with floating structure (Welch et al. 1996) and for surface piercing column (Levi et al. 1998). Recently, Houtani et al. (2019) have investigated ship response to the modulated wave train and found that the vertical bending moment due to slamming impact depends on the geometrical properties of the wave train, which deforms largely in space and time due to nonlinear wave modulation.

How much have we learned since TYM1996? Perhaps one of the most notable development, since then, is related to the extensive study done on the generation mechanism of the freak waves, in which the modulational instability has drawn considerable attention (e.g. TW99). However, that was in a slightly different context as discussed in TYM96. Hasselman's theory was extended for a shorter time scale of $0(10^2)$ wave periods when the detuned-resonance becomes relevant (e.g. Janssen 2003). In TYM96, energetic wave breaking is considered as a consequence of modulational instability, but the downshifting is explained as due to the imbalance of the breaking energy and momentum loss, hence, not requiring Hasselmann's theory. In TYM96, the freak wave is explained by dispersive focusing associated with wind squall and not because of the modulational instability. In the following section, the author attempts to provide explanations to these seemingly contradictory prospects of TYM1996.

3 DETUNED-RESONANCE IN THE DIRECTIONAL SEA

According to the wave evolution equations (1) and (2), the spectral downshifting is considered a consequence of energy and momentum imbalance due to energetic breaking waves. And the wave breaking is a result of the modulational instability (TYM96). On the other hand, in the operational third-generation wave models, the nonlinear source term based on the resonant quartet interaction is essential for the successful wave forecasts. These seemingly contradictory viewpoints, however, are not totally unrelated. The answer relates to the theory of freak wave generation due to nonlinear self-focusing when the detuned-resonant interaction is at play (e.g. Janssen 2003). The detuned resonant quartet satisfies the following condition,

$$\begin{cases} \omega_1 + \omega_2 = \omega_3 + \omega_4 + \Delta_{1234} \\ \vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4 \end{cases}, \tag{3}$$

where $\Delta_{1234} \sim O(ak^2)$ is the resonance detuning. The so-called Benjamin-Feir instability is a degenerate case of the resonant quartet interaction in which the Stokes wave interacts with the two collinear sideband waves (e.g. TW99):

$$\begin{cases} \omega_- + \omega_+ = 2\omega_0 + \Delta \\ k_- + k_+ = 2k_0 \end{cases}, \text{ where } k_- = k_0 - \delta k, k_+ = k_0 + \delta k. \tag{4}$$

The first evidence of the freak wave generation due to nonlinear self-focusing in the random sea was discovered in a wave flume (Onorato et al. 2004). Since the wave field is collinear, exact resonance is not possible, and the only possible wave-wave interaction is the detuned-resonant interaction (3). The wavefield deviates from Gaussianity with large kurtosis and the occurrence probability of freak waves. However, it was soon discovered that the occurrence probability of freak wave for a directionally broad wave field compares well with the second-order theory (Socquet-Juglard et al. 2005). These seemingly contradictory results gave us a clue. When the wave field is directionally broad, exact resonance is possible and quasi-Gaussian closure of Hasslemann is valid. But when the wavefield is uni-directional, detuned-resonance is the only possible interaction. So, what happens for a slightly directional spectrum? The experiment conducted at the University of Tokyo (Ocean Engineering Basin, Institute of Industrial Science) revealed that the kurtosis gradually changes with directional spread and increases as the directionality narrows (Fig.1 left). It was also shown that the downshifting parameter $\Gamma \equiv \frac{1}{\omega}\frac{d\omega}{dt}/\frac{D_b}{E_T}$, which is equivalent to TYM96's γ, increases as the directional spectrum narrows (Waseda et al. 2009ab). In other words, the downshifting mechanism proposed by TYM96 is confirmed for the narrow spectrum but that of Hasselmann-type is also likely for a broader spectrum.

So what is happening in the ocean? Numerous studies were conducted investigating the spectral evolution during marine accidents, and the study near Japan revealed that the directional spectrum changes rapidly in time. To a surprise, most accidents occur when the directional spectrum was the narrowest (Waseda et al. 2012). This does not prove that the accidents were due to encounter with a freak wave, but the finding that the directional spread changes rapidly in time due to meteorological forcing is in itself an eye-opening discovery. And when the directional spectrum narrows, the occurrence probability of freak wave increases.

Now, this is not the end of the story, as a number of works conducting a Monte Carlo simulation of a phase-resolved model based on a given directional spectrum showed that the directional spreading in the realistic ocean is too broad for the nonlinear focusing to work (see review by Dudley et al. 2019). Hence, detuned resonance is not important in the real ocean. This conjecture is almost synchronous to the criticism raised against the work of TYM96 that the modulated wave train does not exist in the ocean. A quarter-century has passed and the same question recurred.

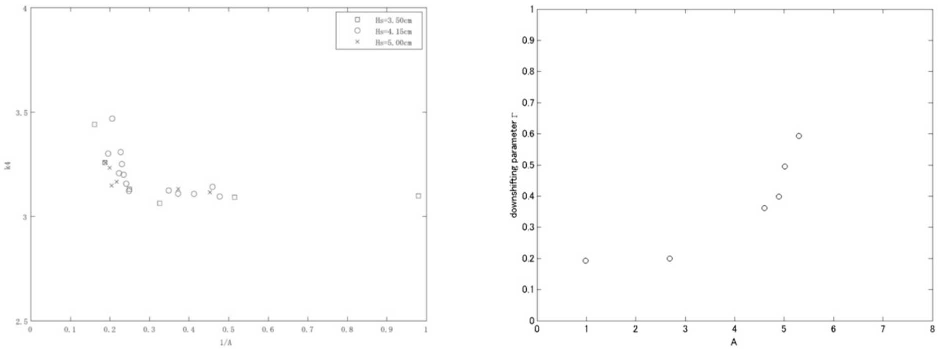

Fig. 1 (left) Kurtosis of the JONSWAP spectrum with different directional spread (narrower to the left); (right) Downshifting parameter with different directional spread (narrower to the right)

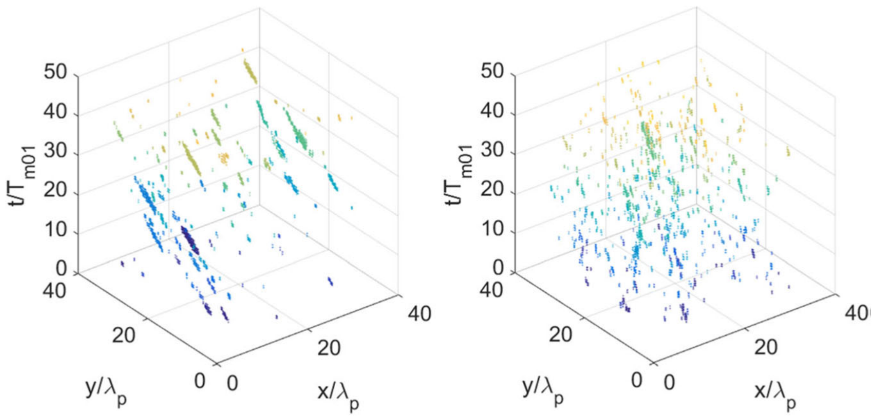

Fig. 2 Freak wave groups simulated by HOSM M=3. Left and right panels show JKEO-Narrow and JKEO-Broad cases, respectively. The color indicates indices assigned to each freak wave group. After Fujimoto et al. 2019.

The criticism that the modulated wave train does not exist in the ocean is based on the notion that the ocean waves are directionally broad. However, the ocean-wave spectrum changes rapidly in time and space and a study comparing a directionally broad mixed sea state and a directionally narrow sea state revealed that the population of the nonlinear freak wave groups increases as the directional spreading narrows (Fujimoto et al. 2019). The freak wave groups are identified from a reconstructed phase-resolved wavefield from a given directional spectrum (Fig. 2). For a directionally narrow wave field (left), elongated tubes appear among small dots in their background. The tube represents a freak wave group with a longer lifetime due to modulational instability. Whereas for a broad spectrum, these long-life wave groups seem missing and the space-time volume is filled with smaller dots representing freak wave groups spontaneously occurring due to linear focusing.

The message from these studies is that the nonlinear wave groups and linear-focus wave groups coexist, and their relative likelihood depends on the directional spread (see Fig. 9 of Fujimoto et al. 2019). Therefore, we believe that modulated wave groups exist in the ocean but their likelihood depends on the sea state. The ocean waves evolve not just because of the energetic breaking energy loss but concurrently by weak interactions among components of the directional spectrum. The wave spectrum changes dynamically under different wind forcing and at times the occurrence probability of modulated wave train may increase. This is one answer to the question raised by TYM96.

What seems left unanswered of the questions raised by TYM96 is to consider the effects of short-time wind forcing. Such sub-grid scale perturbation is not taken into consideration in an operational wave model and can be crucial in estimating the extreme wave statistics.

4 WAVE GROUPS IN THE OCEAN ~engineering relevance~

In TYM96, the kinematic and geometrical properties of waves prior to and after the energetic breaker are illustrated based on fully nonlinear wave model simulation. The notion of the relevance of accelerated particle velocity as well as deformed wave shape led to the evaluation of the impact loading on ocean structures (Welch et al. 1999 and Levi et al. 1998). In Levi et al. (1998), a strong correlation between rigid body loading and the local slope of the wave was found.

The important message from these works is that the waves in the modulated wave train deform in time and space and thereby creating asymmetric waveforms. Hence, depending on the encounter timing, the structure will feel distinct wave loading. A recent study by Houtani et al. (2019), investigated the vertical bending moment of the ship by the modulated wave train. Their study revealed that for a given encounter wave height, a distinct vertical bending moment is observed. When the rear wave trough is deeper than the front wave trough, the slamming impact load is much larger than when the rear trough is shallower than the front trough (see Fig. 10 of Houtani et al. 2019). Because the deformation occurs within a few wave periods (see Fig. 15 of Houtani et al. 2019), a slight difference in the encounter timing resulted in the completely different magnitude of the vertical bending moment.

It is also known that the shapes of extreme waves tend to be more symmetric when they are generated due to linear focusing. This was shown comparing the averaged wave shapes of freak waves between directionally broad and narrow spectral sea conditions (Fujimoto et al. 2019). Overall, these studies by Fujimoto et al. (2019) and Houtani et al. (2019) link meteorological causes and engineering impacts, which is an important viewpoint provided by TYM96.

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