A note on a trial to improve linearized solution in water wave problems

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1. Introduction

In an attempt to improve the linearized solution in classical water waves and wave-structure interaction problems based on linear potential flow theory, we introduced a new concept of kinematic assumptions. In contrast to conventional vertical perturbation approach, so-called Stokes' expansion method, present approach deals with not only the vertical motions, but also horizontal ones at the free surface.

Not only that, when it comes to wave-structure interaction problem, we separately evaluate the kinematics of each fluid and structure domain by differentiating the structural motions from the fluid ones when the body is interacted with ocean waves under slip-condition. Also, the change of the normal vector is derived by employing the Nanson's formula which is commonly used in the field of continuum-mechanics especially when dealing with structural deformation. However, in this research, we apply the same formula to the deformation of the fluid, not structure. In this way, we obtain new hydrodynamic force and moment. In particular, when we derive the new external moment, we newly introduce a different definition of the moment arm which results from the collaboration of the fluid and structure displacement fields.

In this paper, our attention is restricted to the first-order harmonic problem in the frequency-domain. However, it is straightforward to extend to high-order problems, so it is expected that the proposed method is applicable to various wave-body interaction problems.

2. The outline of a proposed approach

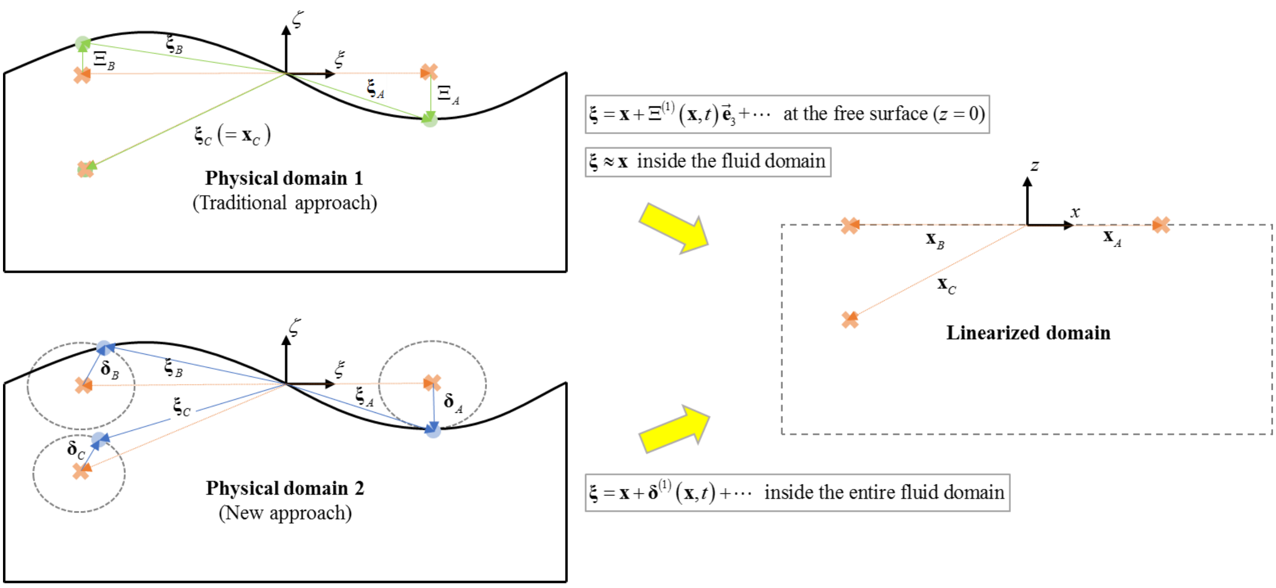

Fig. 1. Linearization from two physical domains to a linearized domain: Wave only

In the Fig. 1., the left and upper picture shows conventional vertical perturbation expansion method while the lower one shows our new approach. We call each physical domain as physical domain 1 and 2, respectively.

Whereas the right picture shows a linearized domain onto which the two physical domains are linearized. Here, we introduce position vectors, $\mathbf{x}$ in the linearized domain and $\boldsymbol{\xi}$ in the physical domain. And we selected three representative position vectors with subscript A, B, and C, to clarify the distinction of position vectors, $\boldsymbol{\xi}$, between the two physical domains. As shown in the figure, Ξ denotes wave elevation, while $\boldsymbol{\delta}$ the fluid displacement vector. It is worthwhile to mention that even though the $\boldsymbol{\delta}$ can be defined over whole fluid domain, vertical component of it at the linearized free-surface, $z=0$, is identical to the wave elevation function, i.e., $\delta_3 = \Xi$. In addition, as illustrated in the physical domain 2, one can see that the motion of the fluid at the free surface is able to move in the horizontal direction not only in the vertical one (See points with subscript A and B). While in the domain 1, it only behaves in the vertical direction. Meanwhile, the position vector with subscript C tells us well that the fluid displacement vector, $\boldsymbol{\delta}$, is not restricted to the motion of the fluid at the free-surface.

On the other hand, in the Fig. 2. we illustrate two physical domains where a floating structure is interacted with ocean waves. As shown in our new approach (See left lower picture), we introduced two different displacement fields over one body surface. Here, $\mathbf{u}$ is the structural displacement vector, and $\boldsymbol{\delta}$ is the fluid displacement one. By introducing these two different displacement vectors, we can separately evaluate the structure and the fluid motions when they are interacted on the body surface. As a result, it leads us to obtain the new hydrodynamic force and moment, which will be detailed in the next section.

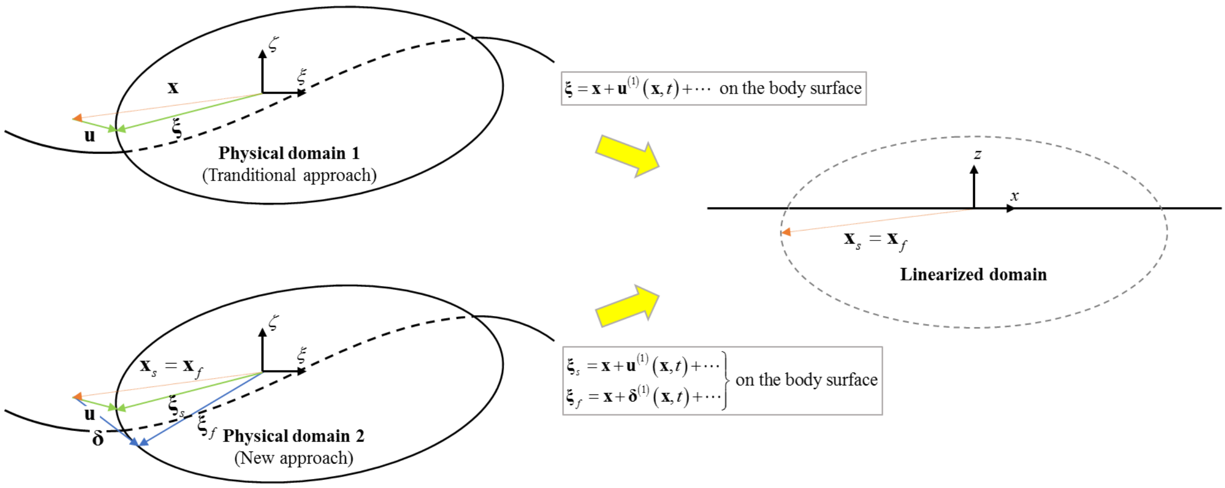

Fig. 2. Linearization from two physical domains to a linearized domain: Wave-structure interaction

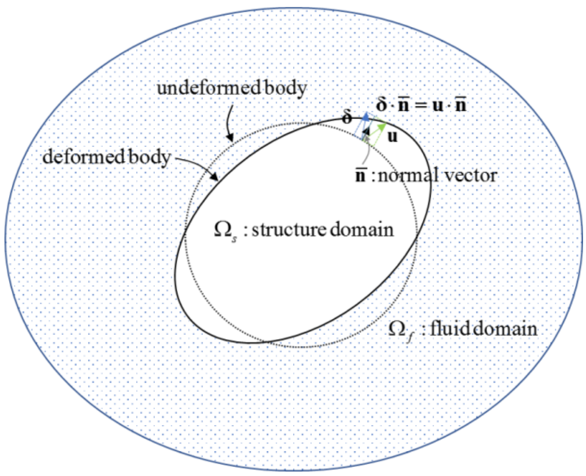

Fig. 3. Fluid and structure displacement fields

Now then, it is very important to see how these two distinct kinematic motions could be matched with each other over the body surface. Let us see Fig. 3. The two fluid and structure displacement vectors in each domain, i.e., the fluid (Ω_f) and the structure (Ω_s) domains, are matched together along the normal direction of the body surface since both two vectors satisfy one body boundary condition that appears in the boundary value problem of the velocity potential. However, it is not necessary that their tangential motions are equal if they are assumed to be under slip-condition.

3. Hydrodynamic Force and Moment

The definition of the hydrodynamic force and moment are given by

$$f_i = \int_{\partial\Omega_f} p n_i dS \,, \tag{1}$$

$$m_i = \int_{\partial\Omega_f} p \varepsilon_{ijk} \rho_j n_k dS \,, \tag{2}$$

where ε_{ijk} is permutation symbol, n_i is the normal vector of the deformed body surface, and ρ_j is the new moment arm that is given in the Eq. (8). The fluid pressure is obtained up to first-order by expanding the Bernoulli's equation with respect to the fluid displacement vector, instead of structural one, as

$$\frac{p(\boldsymbol{\xi},t)}{-\rho} = gz + g\delta_3^{(1)} + \frac{\partial \Phi^{(1)}}{\partial t} + \cdots, \tag{3}$$

where $\delta_3^{(1)}$ is the vertical component of the fluid displacement, $\delta_i^{(1)}$:

$$\delta_i^{(1)}(\mathbf{x},t) = \int \frac{\partial \Phi^{(1)}}{\partial x_i}(\mathbf{x},t)\, dt = \mathrm{Re}\left\{ \frac{1}{i\omega} \frac{\partial \phi^{(1)}}{\partial x_i}(\mathbf{x}) e^{i\omega t} \right\}, \tag{4}$$

where imaginary number $i = \sqrt{-1}$, and ω denotes wave frequency.

In a similar way, infinitesimal fluid surface can be linearized by employing the Nanson's formula as

$$n_i dS = \bar{n}_i d\bar{S} + Q_{ij}^{(1)} \bar{n}_j d\bar{S} + \cdots, \tag{5}$$

where $Q_{ij}^{(1)}$ is given by

$$Q_{ij}^{(1)} = \delta_{ij} \frac{\partial \delta_k^{(1)}}{\partial x_k} - \frac{\partial \delta_j^{(1)}}{\partial x_i} = -\frac{\partial \delta_j^{(1)}}{\partial x_i} \,, \tag{6}$$

where δ_{ij} denotes Kronecker delta symbol.

In the above, the divergence-free condition, i.e., the Laplace equation, enables us to obtain the above result:

$$\frac{\partial \delta_k^{(1)}}{\partial x_k} = \mathrm{Re}\left\{ \frac{1}{i\omega} \frac{\partial^2 \phi^{(1)}}{\partial x_k \partial x_k} e^{i\omega t} \right\} = 0 \,. \tag{7}$$

The Nanson's formula has been commonly used in the field of continuum-mechanics especially when dealing with structural deformation. For instance, applications of the hydroelasticity analysis of a floating body could be good examples (See more details in [1]). However, in this research, we apply the same formula to the deformation of the fluid, not structure.

Meanwhile, the moment arm given in Eq. (2) should be carefully evaluated. It can be represented by collaborating the fluid and structure displacement vectors as

$$\begin{aligned}\rho_j(\mathbf{x},t) &= \xi_{f,j}(\mathbf{x},t) - \xi_{s,j}(\mathbf{x}_{OR},t) = x_j + \delta_j^{(1)}(\mathbf{x},t) - x_{OR,j} - u_j^{(1)}(\mathbf{x}_{OR},t) + \cdots \\ &= r_j + \delta_j^{(1)}(\mathbf{x},t) - u_j^{(1)}(\mathbf{x}_{OR},t) + \cdots\end{aligned}, \tag{8}$$

where $\mathbf{x}_{OR}$ denotes a position vector from origin to center of rotation in the linearized domain. If we assume that the body is rigid, $\mathbf{u}(\mathbf{x}_{OR},t)$ becomes simply translational displacement of the body.

Then, we finally obtain the new hydrodynamic force and moment for the first-order as

$$f_i^{(1)} = -\rho g \int_{\partial\bar{\Omega}_f} z Q_{ij}^{(1)} \bar{n}_j d\bar{S} - \rho g \int_{\partial\bar{\Omega}_f} \delta_3^{(1)} \bar{n}_i d\bar{S} - \rho \int_{\partial\bar{\Omega}_f} \frac{\partial \Phi^{(1)}}{\partial t} n_i dA, \tag{9}$$

$$\begin{aligned} m_i^{(1)} &= -\rho g \int_{\partial\bar{\Omega}_f} z\varepsilon_{ijk} r_j Q_{kl}^{(1)} \bar{n}_l d\bar{S} - \rho g \int_{\partial\bar{\Omega}_f} z\varepsilon_{ijk} \delta_j^{(1)} \bar{n}_k d\bar{S} + \rho g \int_{\partial\bar{\Omega}_f} z\varepsilon_{ijk} u_{OR,j}^{(1)} \bar{n}_k d\bar{S} \\ &\quad - \rho g \int_{\partial\bar{\Omega}_f} \delta_3^{(1)} \varepsilon_{ijk} r_j \bar{n}_k d\bar{S} - \rho \int_{\partial\bar{\Omega}_f} \frac{\partial \Phi^{(1)}}{\partial t} \varepsilon_{ijk} r_j \bar{n}_k d\bar{S} \end{aligned}, \tag{10}$$

where $u_{OR,j}^{(1)} = u_j^{(1)}(\mathbf{x}_{OR},t)$.

4. Results and discussion

Based on the proposed scheme, it is straightforward to derive high-order potential models with small modification on existing formulations (See, e.g., [2] and [3]). However, computed results only for the first-order examples will be presented at the workshop site.

Acknowledgements

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