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On drift motion of deformable ice sheets by nonlinear waves

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The ice cover near the polar regions has been strongly affected as a result of global warming, particularly on its margins, such that it consists of a mixture of discrete ice floes and open water. The ocean waves propagating in such regions contribute not only to the break-up of the ice, but also stimulate their drift. Therefore, in addition to melting, surface waves play a determining role in shaping the marginal ice zones [1].

The drift motion and hydroelastic response of a set of floating elastic ice sheets to incident nonlinear waves in shallow water is studied by use of the Level I Green-Naghdi (GN hereafter) equations. The set of N deformable ice sheets is presented by thin elastic plates of variable properties. The resulting governing equations together with the appropriate boundary and matching conditions are solved in two-dimensions by the finite difference method. Free surface elevation and ice sheet deformations are calculated. The effect of multiple plates on the wave-induced loads and velocity field is investigated. Drift motion of the ice sheets is determined by calculating the instantaneous wave-induced force on the bodies, and by solving Newton's second law. Only horizontal motion is considered.

Problem formulation. The plane flow of inviscid fluid of constant depth h is considered in the Cartesian coordinate system Oxy with the horizontal axis lying on the undisturbed free surface and vertical axis directed upwards. Incident waves propagate in the positive x–direction and excite the motion of a set of floating deformable sheets being initially at rest. The sketch of the problem is shown in Fig. 1. The ice sheets are homogenous and have arbitrary length L_k, mass per unit width m_k and flexural rigidity D_k, where subscript $k = 1, 2, \ldots, N$ indicates each ice sheet.

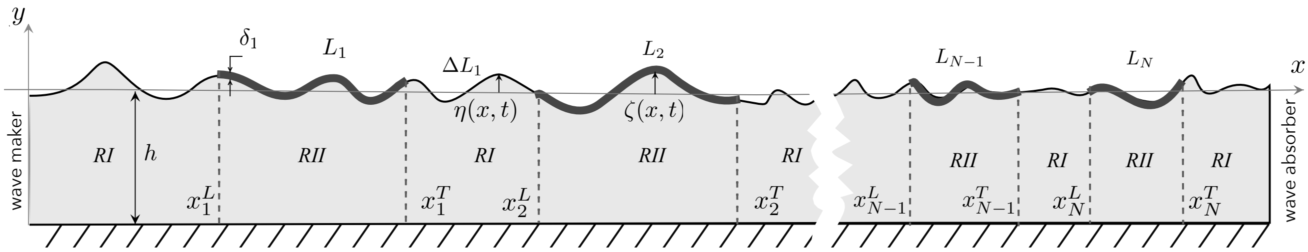

Fig. 1. Schematic of the problem of nonlinear wave interaction with N number of deformable plates of arbitrary size and location. The plates may have horizontal motion due to the wave loads. Also shown in this figure are the RI and RII regions referred to in the text.

In the context of using the GN equations, the problem is best studied by dividing the fluid domain into two types of regions. Region I (RI) is formed by a flat floor at the bottom and by a free surface at the top, where the fluid pressure is the constant atmospheric pressure. Region II (RII) is formed by a flat floor at the bottom and by an elastic plate at the top, where, as opposed to Region RI, the fluid pressure is unknown. Solutions, obtained in each region, are connected through the proper matching conditions at the interfaces.

The governing equations for the motion of the fluid in RI are provided by the Level I GN theory for a flat and stationary seafloor [2, 3]. Using density, ρ, gravitational acceleration, g, and fluid depth h, the Level I GN equations can be written in dimensionless and compact form as [4]:

$$\eta_{,t} + [(1+\eta)u]_{,x} = 0, \qquad \dot{u} + \eta_{,x} = -\frac{1}{3}[2\eta_{,x}\ddot{\eta} + (1+\eta)\ddot{\eta}_{,x}], \tag{1}$$

where η is the surface elevation measured from the still-water level and u is the horizontal particle velocity. Subscripts after comma indicate differentiation, and superposed dots are the two-dimensional total derivatives. Similarly, the governing equations for the coupled motion of the fluid and the elastic sheet in RII Regions can be written as:

$$\zeta_{,t} + [(h_k + \zeta)u]_{,x} = 0, \qquad \dot{u} + \zeta_{,x} + \hat{p}_{,x} = -\frac{1}{3}[2\zeta_{,x}\ddot{\zeta} + (h_k + \zeta)\ddot{\zeta}_{,x}], \tag{2}$$

where ζ is the plate deformation, $h_k = 1 - d_k$ is dimensionless fluid depth under the k-th plate. The wave-induced pressure $\hat{p}$ under the plate is given by the thin elastic plate theory [5], usually adopted to model ice floes and very large floating structures (VLFS), when they interact with surface waves:

$$\hat{p} = m_k\zeta_{,tt} + D_k\zeta_{,xxxx} + m_k. \tag{3}$$

Here, the flexural rigidity is defined as $D_k = E_k\delta_k^3/12(1 - \nu_k^2)$, with δ_k, E_k, and ν_k being the thickness, Young's modulus and Poisson's ratio of the k-th plate, respectively. Equations (1)–(3) are solved for free surface elevation η, plate deformation ζ, horizontal component of fluid velocity u and pressure under the plates $\hat{p}$.

Ertekin [6] provided explicit relations for the vertical velocity of the fluid along the water column and pressure on the bottom ($y = -1$), given as:

$$v(x,y) = \begin{cases} \dot{\eta}\,(1+y)/(1+\eta), & (x,y) \in RI \\ \dot{\zeta}\,(h_k+y)/(h_k+\zeta), & (x,y) \in RII, \end{cases} \qquad \bar{p}(x) = \begin{cases} \frac{1}{2}(1+\eta)(\ddot{\eta}+2), & (x,y) \in RI \\ \frac{1}{2}(h_k+\zeta)(\ddot{\zeta}+2) + \hat{p}, & (x,y) \in RII. \end{cases} \tag{4}$$

In order to obtain continuous solution, suitable matching conditions at the leading ($x = x_k^L$) and trailing ($x = x_k^T$) edges of each floating sheet must be specified. Since the elastic sheets are floating freely, the bending moment and the shear stress should vanish at the edges, i.e. $D_k\eta_{,xx} = D_k\eta_{,xxx} = 0$. Moreover, we assume no gap between the bottom surface of the sheets and the top surface of the fluid layer, and hence the mass continuity equation (2) together with vanishing bending moment condition imply:

$$3\zeta_{,x}u_{,xx} + (h_k + \zeta)u_{,xxx} = 0, \qquad 4\zeta_{,x}u_{,xxx} + (h_k + \zeta)u_{,xxxx} + \zeta_{,xxxx}u = 0. \tag{5}$$

In the approach discussed above, the floating elastic surfaces cause discontinuities of the fluid layer and velocity at the interfaces between regions. Consequently, the derivatives of η and u are also discontinuous. Appropriate jump conditions should be called to provide the matching of the solution at the interfaces between regions. The theory demands the conservation of mass and momentum (achieved by continuity of bottom pressure across the discontinuity curves).

On the left side of the domain, numerical wavemaker capable of generating GN solitary and cnoidal waves is installed. On the right side of the domain, Orlanski condition is used to minimise the wave reflection back to the domain.

The system of equations of the entire domain, subject to appropriate boundary conditions in each region, along with the matching and jump conditions, is solved simultaneously for the unknowns. Spatial discretization of the equations is carried out by a central-difference method, second order in space, and time marching is obtained by use of the modified Euler's method. The system of equations are solved by use of a Guassian Elimination method. Hayatdavoodi & Ertekin [7] applied successfully the same model to nonlinear problem of the wave scattering by a submerged rigid plate.

In two-dimensions, the horizontal wave-induced force on the floating plate is calculated by considering the pressure differential at the leading and trailing edges of each plate

$$F_k(t) = \hat{p}(x_k^L, t) - \hat{p}(x_k^T, t). \tag{6}$$

We note here that since the plate is thin, the two-dimensional horizontal force is the force per unit width (into the page) and per unit thickness, and thus has the same dimension as pressure. The force F_k has positive and negative components. Drift motion of the floating ice sheets is determined by solving the following equation of motion:

$$m_k a_{k,tt} = F_k(t), \tag{7}$$

where $a_{k,tt}$ is the instantaneous horizontal acceleration of k-th plate. Spatial location of each plate is determined by integrating the horizontal acceleration twice.

Discussion of results. Results of the level I GN model for wave interaction with N number of floating and deformable ice sheets are presented here. The results include plate deformations, time series of wave induced forces, and variation of the forces with wave conditions. In these preliminary cases shown here, we assume that the plates are fixed in their horizontal locations.

Figure 2 shows velocity vectors of fluid particles plotted on contours of fluid velocity module $(u^2 + v^2)^{1/2}$ in case of interaction of a solitary wave with two plates, floating at a short distance $\Delta L = 5$ from each other, at three successive time moments. Small-amplitude leading waves propagate with higher speed along the elastic surfaces, and hence the fluid domain feels the wave motion long before the wave itself reaches the end of the second plate. These elastic waves cause greater wave attenuation in a longer plate with higher rigidity. Another contribution to the wave attenuation is due to the effect of multiple plates (see [8]).

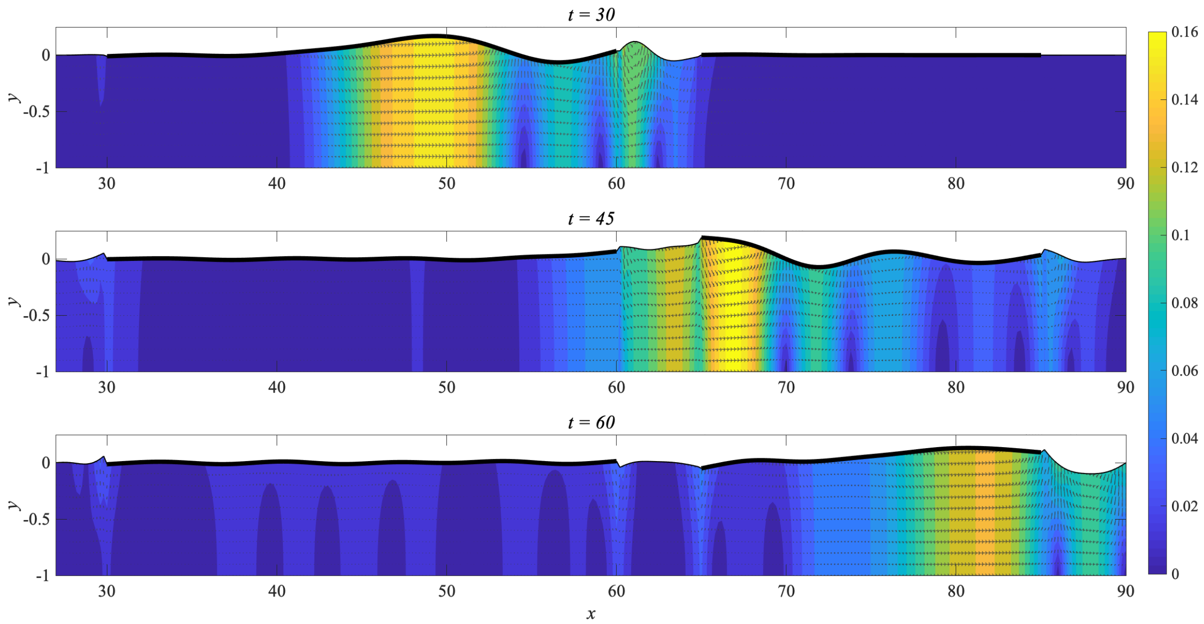

Fig. 2. Vector field and module contours of dimensionless fluid velocity for the interaction of a solitary wave ($A = 0.25$) with the set of two plates with the same properties ($L_k = 30$, $m_k = 0.1$, $D_k = 5$) separated by the fluid gap ($\Delta L = 5$).

Figure 3 shows the interaction of a cnoidal wave with a set of three elastic plates of the same properties for two values of plate rigidity. Also shown in Fig. 3 are the time series of the wave-induced horizontal cnoidal forces on these deformable ice sheets. The amplitudes of plates deformation and horizontal force reduces downwave. The higher rigidity D_k of each plate contributes to the higher wave attenuation by the floating system and to the lower drift of its elements.

Figure 4 shows the variation of the peak cnoidal wave horizontal force on the ice sheets F_k^+ with the incoming wave length λ and wave height H for the cases of a single plate and two plates located at different distances ΔL from each other. The effect of multiple plates is greater for shorter distance. The first plate in the group of plates experiences greater drift force than a single plate of the same properties if the distance is small. For larger distance, the second plate has less impact on the first plate and experiences less horizontal force itself. Figure 4 also shows that drift force varies almost linearly with the wave height and slight deviation of force from the straight line in the case of two plates can be observed.

More detailed discussion of results, including the interplay between of each plate in more complex plate systems and the horizontal mean motion of the plates, will be presented at the workshop.

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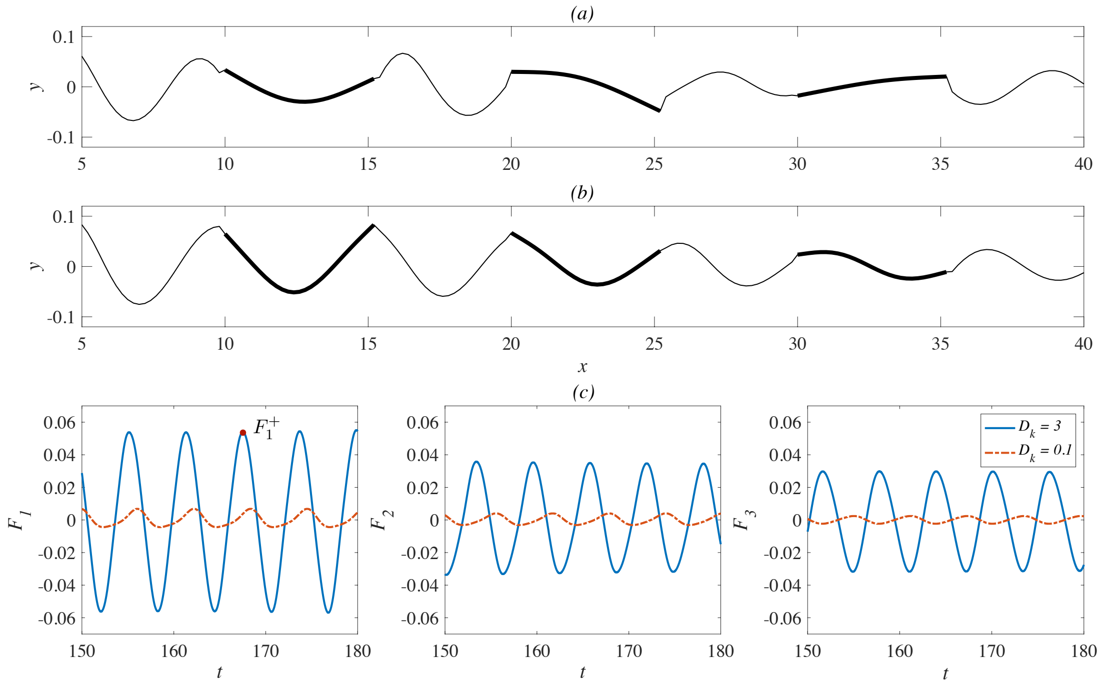

Fig. 3. Surface elevation and plate deformation for the interaction of the cnoidal wave ($H = 0.2$, $\lambda = 5$) with a set of three identical plates ($L_k = 3$, $m_k = 0.01$), equally spaced, having the rigidity of (a) $D_k = 3$ and (b) $D_k = 0.1$ at time $t = 200$. (c) Time series of horizontal force acting on each plate in the set for different values of rigidity D_k. The peak F_1^+ of horizontal force F_1 is indicated in the figure.

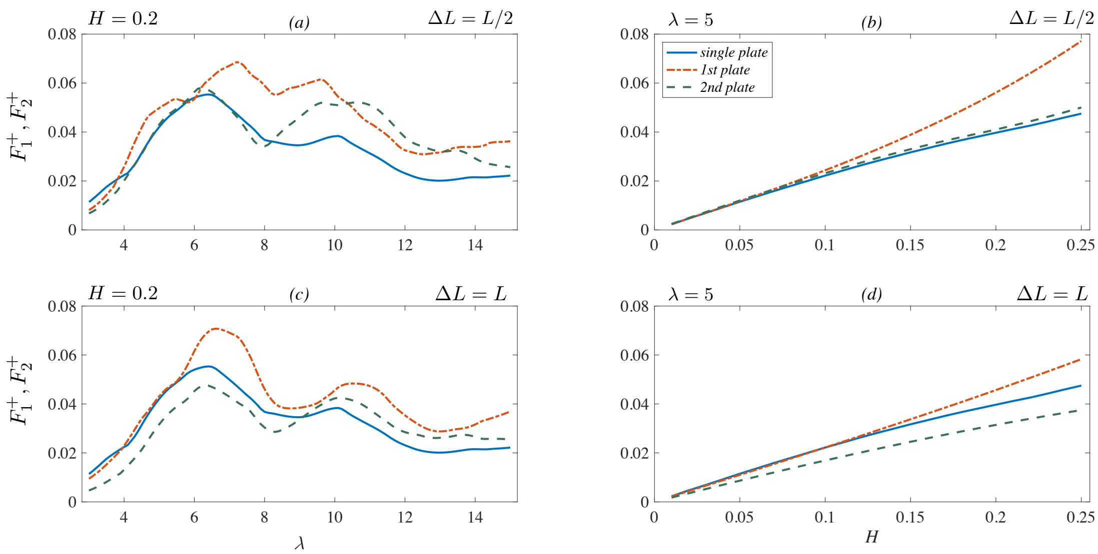

Fig. 4. Variation of the peak of cnoidal wave horizontal force on floating ice sheets with wave length and wave height, for two configurations of a single plate ($L = 5$, $m = 0.01$, $D = 0.1$) and two identical plates of the same properties. In sub-plots (a) and (b), the distance between the two plates is $\Delta L = L/2$, and in (c) and (d) $\Delta L = L$.

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