Vertical impact on floating ice

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This study is concerned with a vertical impact of a rigid body onto a floating ice. The problem is unsteady, two-dimensional and coupled. The hydrodynamic loads and elastic ice response are determined at the same time. The ice deflection is described by the equation of Euler beam with constant thickness. In contrast to many other studies of ice response to impact on it, we do not assume the impact loads but calculate them as part of the solution together with the region of contact between the impacting rigid body and the elastic ice plate. It is known that such a contact region may consist of several intervals of contacts, which are determined by the condition that the surface of the rigid body is above the deformed ice plate at any time instant after the impact. The contact may occur at separate points within some simplified models of elasticity. The problems with concentrated unknown loads and inequalities for elastic deflections are challenging both theoretically and computationally. A practical approach to such problems is to introduce an elastic layer between the impacting body and the ice, see [1]. This layer can model either some physical properties of the ice surface as in [1] or be considered as a way of regularization of problems with concentrated loads, or as a penalty method to satisfy the inequality concerning the positions of the body surface and the floating ice plate.

To keep focus on the elastic impact model, we consider a simplified configuration with water of infinity depth, $y < -h_i$, bounded by two vertical walls at $x = \pm L$. An elastic plate of a thickness h_i is floating on the water surface without gaps between the plate and the walls, see figure 1. The edges of the plate can slide freely along the walls. The interval $y = 0$, $-L < x < L$, corresponds to the initial position of the upper surface of the ice plate. The ice plate is covered by an elastic layer of constant thickness h_e. The reaction force of the elastic layer is a given function of the current local thickness of the layer. The body is of parabolic shape with radius of curvature R. Initially, $t = 0$, the rigid body touches the upper surface of the elastic layer at a single point $y = h_e$, $x = 0$. Then the body instantly starts to move downwards at a constant speed V pushing the elastic layer and the ice plate into the water. The position of the body surface at time t is described by the equation $y = x^2/(2R) + h_e - Vt$, $-L < x < L$.

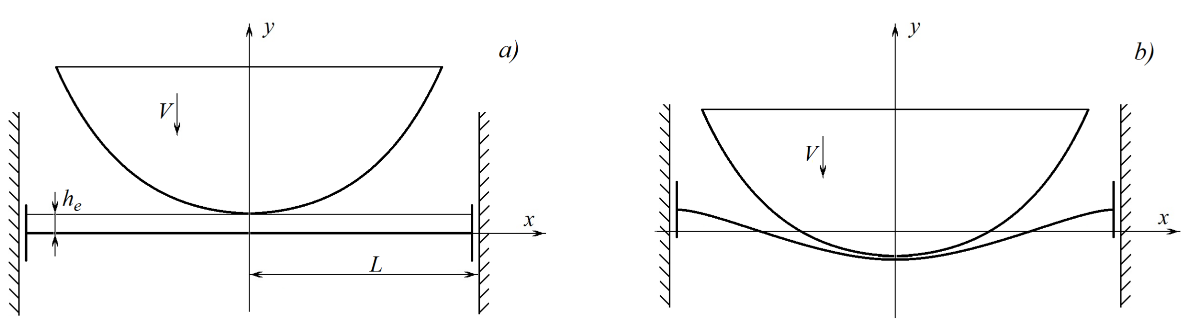

Fig. 1 Vertical impact of a rigid body onto a floating elastic plate between two vertical walls: (a) Initial position of the body and notation, (b) Sketch of the elastic plate deflection.

The problem is studied within the linear theory of hydroelasticity of potential flows. The ice plate deflection, $w(x,t)$, satisfies the Euler beam equation

$$m\frac{\partial^2 w}{\partial t^2} + EJ\frac{\partial^4 w}{\partial x^4} = p(x,0,t) - P_e(x,t) \quad (-L < x < L, t > 0), \tag{1}$$

where m is the mass of the beam per unit length, $m = \rho_i h_i$, ρ_i is the ice density, E is the Young modulus of the ice, $J = h_i^2/12$, $p(x,0,t)$ is the hydrodynamic pressure acting in the lower surface of the ice plate, $P_e(x,t)$ is the reaction force of the elastic layer, $w(x,t)$ is positive upwards. Initially the ice plate is at rest with

$$w(x,0) = 0, \qquad \frac{\partial w}{\partial t}(x,0) = 0. \tag{2}$$

The sliding conditions at the edges of the plate read

$$\frac{\partial w}{\partial x} = 0, \quad \frac{\partial^3 w}{\partial x^3} = 0 \qquad (x = \pm L,\ t > 0). \tag{3}$$

The hydrodynamic pressure at the ice/water interface is given by the linearised Bernoulli equation,

$$p(x,0,t) = -\rho\varphi_t(x,0,t) - \rho g w(x,t) \qquad (-L < x < L,\ t > 0), \tag{4}$$

where ρ is the water density, g is the gravitational acceleration, and $\varphi(x,y,t)$ is the velocity potential of the flow under the ice. The velocity potential is the solution of the following boundary-value problem,

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0 \quad (-L < x < L, y < 0), \qquad \frac{\partial\varphi}{\partial y} = \frac{\partial w}{\partial t}(x,t) \quad (y = 0),$$
$$\frac{\partial\varphi}{\partial x} = 0 \quad (x = \pm L, y < 0), \qquad \frac{\partial\varphi}{\partial y} \to 0 \quad (y \to -\infty). \tag{5}$$

We are searching for a bounded solution of the problem (5) in the flow region, $y < 0$. Note that the potential does not decays at the infinity in this problem. It is known that a solution of the problem (5) exists only if

$$\int_{-L}^{L} w(x,t)\mathrm{d}x = 0 \quad (t > 0), \tag{6}$$

which comes from the mass conservation law. The reaction force of the elastic layer, $P_e(x,t)$, in (1) is modelled by

$$P_e(x,t) = Kf(\delta), \quad \delta(x,t) = \frac{h_e - h(x,t)}{h_e}, \qquad h(x,t) = \frac{x^2}{2R} + h_e - Vt - w(x,t), \tag{7}$$

where $h(x,t)$ is the local thickness of the elastic layer at time t, $\delta(x,t)$ is the relative compression of the elastic layer, and K is the rigidity of the material of the elastic layer. The function $f(\delta)$ is zero, where $\delta \le 0$, linear for small positive δ, and tends to infinity as $\delta \to 1-0$. In the present simulations, we take $f(\delta) = \delta/(1-\delta)$. Integrating the inequality $h(x,t) \ge 0$ in x along the ice plate and using (6), we find the duration of the impact stage, $t \le h_e/V + L^2/(6RV)$. The computations below are performed for the shorter interval, $0 < t < L^2/(6RV)$, which is independent of the thickness h_e of the elastic layer. The strain $\varepsilon(x,t)$ of the upper surface of the ice plate due to the plate bending is calculated by the formula $\varepsilon(x,t) = -0.5h_i w_{xx}(x,t)$. Positive strain implies that this part of the upper surface of the ice is in tension, which could lead to cracking of ice if the strain is greater than the so-called yield strain ε_Y, see [2] where $\varepsilon_Y = 8 \times 10^{-5}$. If $h_e = 0$ and $w(x,t) = x^2/(2R) - Vt$ in the contact region, then $\varepsilon = -h_i/(2R)$ there. For example, for $R = 10$m and $h_i = 50$cm, this estimate gives $\varepsilon = 0.025$ in the contact region, which is much greater than the yield strain of the ice. The presence of the elastic layer on the top of the floating ice reduces the strains but not significantly.

The ice deflection and the flow under the ice are symmetric in the present formulation. The problem of vertical impact on ice is solved by the normal mode method. The symmetric ice deflection is sought in the form

$$w(x,t)=\sum_{n=1}^{\infty}a_n(t)\psi)n(x),\qquad \psi_n(x)=\cos(\lambda_n x),\qquad \lambda_n=\pi n/L, \tag{8}$$

where the coefficients $a_n(t)$ are to be determined. The deflection (8) satisfies the edge conditions (3) and the equality (6) for any $a_n(t)$, $n\geq 1$. The hydrodynamic problem (5) with account for (8) has the solution

$$\varphi(x,y,t)=\varphi_0(t)+\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\dot a_n(t)e^{\lambda_n y}\psi_n(x)\qquad (|x|<L, y<0), \tag{9}$$

where $\varphi_0(t)$ is defined by the reaction load $P_e(t)$. Equations (8), (9) and the beam equation (1) together with the orthogonality of the normal modes $\psi_n(x)$ provide the following equations for the coefficients $a_n(t)$, $n\geq 1$,

$$\ddot a_n+\omega_n^2 a_n=P_n(t)\quad (t>0),\qquad \dot a_n(0)=a_n(0)=0, \tag{10}$$

where

$$P_n(t)=-\frac{1}{L(m+\rho/\lambda_n)}\int_{-L}^{L}P_e(x,t)\mathrm{d}x,\qquad \omega_n^2=\frac{EJ\lambda_n^4+\rho g}{m+\rho/\lambda_n}, \tag{11}$$

$P_e(x,t)$ is calculated by (7) and (8). Note that $P_n(t)$ depends nonlinearly on the coefficients $a_n(t)$. The system of the ordinary differential equations (10) with (7), (8) and (11) is truncated and integrated in time by using the fourth-order Runge-Kutta scheme in the dimensionless variables. The dimensionless variables are denoted by tilde,

$$x=L\tilde x,\quad t=t_{sc}\tilde t,\quad w=w_{sc}\tilde w(\tilde x,\tilde t),\quad t_{sc}=\frac{1}{\omega_1},\quad w_{sc}=\frac{K}{\omega_1^2(m+\rho/\lambda_n)}. \tag{12}$$

The relative compression of the elastic layer, see the definition of $\delta(x,t)$ in (7), is given now by

$$\delta(x,t)=d_1\tilde w(\tilde x,\tilde t)+d_2\tilde t-d_3\tilde x^3,\quad d_1=\frac{w_{sc}}{h_e},\quad d_2=\frac{V}{\omega_1 h_e},\quad d_3=\frac{L^2}{2Rh_e}. \tag{13}$$

Non-trivial impact conditions are expected for $d_1, d_2, d_3=O(1)$. The system (10) for the dimensionless coefficients $\tilde a_n(\tilde t)=a_n/w_{sc}$ has the form

$$\frac{\mathrm{d}^2\tilde a_n}{\mathrm{d}\tilde t^2}+n^4\Omega(n)\tilde a_n=-2\mu_n\int_0^1 f(\delta)\cos(\pi n\tilde x)\mathrm{d}\tilde x,\quad \mu_n=\frac{1+u_1}{1+u_1/n},\quad \Omega(n)=\mu_n\frac{1+u_2/n^4}{1+u_2}. \tag{14}$$

The dimensionless time step of integration is taken to be smaller than 1/20 of the dimensionless period of the last mode, $\psi_{N_{mod}}$, retained in the series (8) and the system (14). The integrals in (14) are evaluated numerically. The interval of integration $[0,1]$ is divided into 1000 subintervals. Linear approximations of $f(\delta(\tilde x,\tilde t))$ in each subinterval and for each time step are used.

The results of numerical simulations are shown in figure 2 for the rigidity of the elastic layer $K=8$ MPa, thickness of the layer $h_e=2$ cm, radius of the rigid body $R=10$ m, speed of impact $V=5$ m/s, thickness of the ice plate $h_i=50$ cm, length of the plate $L=6$ m, the Young modulus of the ice $E=4.2\times10^9$ Pa, density of ice $\rho_i=917$ kg/m^3, density of water $\rho=1000$ kg/m^3, gravity acceleration $g=9.81$ m/s^2. The dimensionless duration of the impact stage is 4.74 with the time scale t_{sc} equal to 0.025 s. The distributions of strains, loads, and deflections are saved with the dimensionless step 0.2 from $\tilde t=0$ to $\tilde t=4.6$. The strains are calculated by the formula

$$\varepsilon(x,t)=\varepsilon_{sc}\sum_{n=1}^{N_{mod}}n^2\tilde a_n(\tilde t)\cos(\lambda_n x),\qquad \varepsilon_{sc}=\pi^2 w_{sc}h_i/(2L^2), \tag{15}$$

where $\varepsilon_{sc} = 0.79$ in the present impact conditions. Note that the scale of the elastic strains and the calculated strains are bigger than the yield strain ε_Y of ice. These numerical results should be considered as illustrative. Calculations were performed with $N_{mod} = 10, 20, 40, 60$. The strains are most sensitive to the number of retained modes. The strains calculated with 40 and 60 modes were found to be almost identical. The numerical dimensional results with $N_{mod} = 60$ are shown in figure 2 for the end of the impact stage. The ice deflection is very close to the position of the rigid body in the interval $|x| < 3.5$ m, see figure 2(a) but they are not equal to each other. The ice plate is displaced upwards towards the body, see figure 2(b), outside the contact intervals. The intervals of the elastic layer compression can be recognised in figure 2(c) as the intervals of positive loads $P_e(x,t)$. The strains, see figure 2(d), are large for the present impact conditions. This means that the ice plate is expected to crack shortly after the beginning of the impact. The maximum strain in the ice at $\tilde{t} = 0.2$ is already equal to 0.008 and is achieved at $x = 0$. The strains in the contact intervals can be estimated by assuming that the ice deflection follows the shape of the impacting body. This gives $\varepsilon = 0.025$, see above, which well corresponds to the calculated strains in figure 2(d).

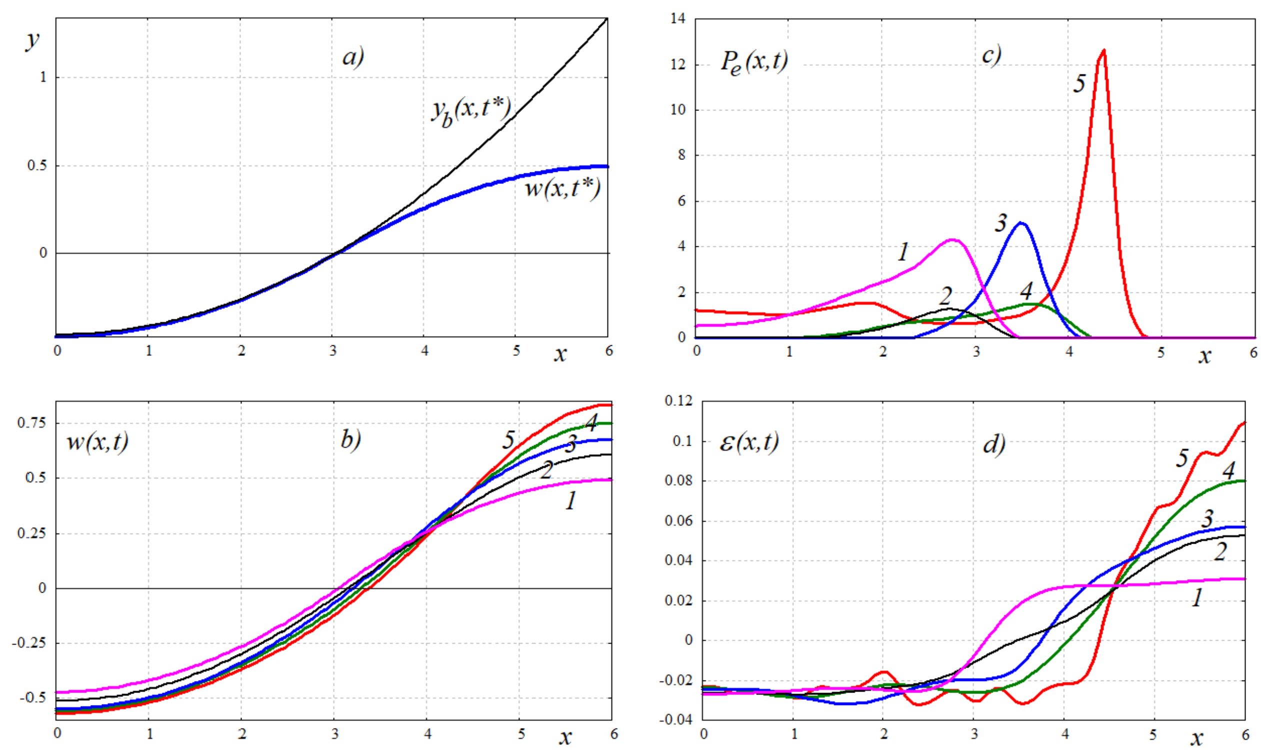

Fig. 2 Vertical impact of a parabolic body, $y_b(x,t) = x^2/(2R) - Vt$, onto a floating elastic plate between two vertical walls: (a) Relative positions of the body surface and the elastic ice plate at $t_* = 3.8t_{sc}$, (b) Deflection of the ice plate, (c) Reaction force of the elastic layer, and (d) Strains on the upper surface of the ice plate for $t/t_{sc} = 3.8(1), 4.0(2), 4.2(3), 4.4(4), 4.6(5)$.

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