Nonlinear Stochastic Prediction of Extreme Deck Slamming on Offshore Platform in Irregular Waves

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1 INTRODUCTION

Deck slamming occurrence and resultant loads are one of crucial elements for offshore platform design. The primary concern is the extreme slamming loads in harsh ocean environment which the nonlinearity of water waves is highly nonlinear, so that the statistical behavior of motion responses and wave run-up around offshore platform also exhibit strong non-Gaussian characteristics. Model-scale experiments and CFD simulation are preferred to deal with such strongly nonlinear problem, but those methods are very time consuming and/or costly. Furthermore, the number of ocean environment scenarios to be considered is dramatically increased as the numbers of wave, current, and win conditions become high. Therefore, we need an efficient method to filter extreme ocean environment conditions for heavy computation or model test. To this end, an appropriate scheme to predict the slamming loads up to a certain degree of nonlinearity is essential.

In this study, the statistical behavior of the air gap and deck slamming of a TLP is investigated, considering all the second-order components of wave elevation and including the radiated and diffracted waves. The start of this analysis is to model the second-order wave elevation as a two-term Volterra series, whose statistical moments can be obtained analytically from the eigenvalue problem. Then, the Hermite-moment method is applied to calculate the probability distribution and the upcrossing rate of the wave elevation. A rational formulation for the upcrossing rate of the relative wave elevation is suggested, taking into account the non-Gaussian nature of the nonlinear wave elevation and platform set-down. For the TLP model, the air-gap and hence the possibility of occurrence are predicted at different locations below of horizontal deck.

2 MATHEMATICAL MODELING

2.1 Statistical Modeling

The relative wave elevation between incident wave and the horizontal deck of offshore platform can be written as follows:

$$\eta_R(x,y,t) \equiv a_0 - a = \eta(x,y,t) - \delta(x,y,t) = \eta(x,y,t) - (\xi_3 + y\xi_4 - x\xi_5) \tag{1}$$

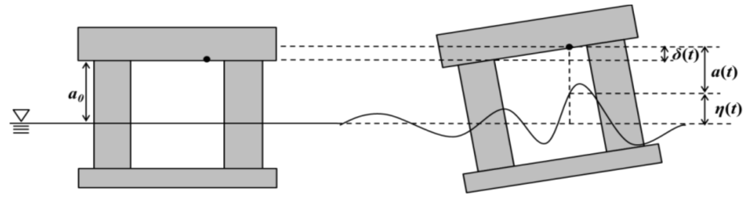


Fig. 1 Definitions of relative wave elevation

where ξ_j means the motion of j-th mode. This can be approximated by using a two-term Volterra series, s.t.

$$\eta_R(x,y,t) = \mathrm{Re}\begin{bmatrix} \sum_{j=1}^{\infty} A_j H_R^{(1)}(\omega_j) e^{i\omega_j t} \\ + \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} A_j A_k H_R^{(2)}(\omega_j,\omega_k) e^{i(\omega_j+\omega_k)t} + \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} A_j A_k^{*} H_R^{(2)}(\omega_j,-\omega_k) e^{i(\omega_j-\omega_k)t} \end{bmatrix} \tag{2}$$

where A_j is complex wave amplitude and $H^{(i)}$ indicates the i-th order transfer function of relative wave elevation.

Kac and Sieggart(1947) showed that Eq.(2) can be simplified using eigenvalue analysis as follows:

$$\eta_R(x,y,t) = \eta_R^{(1)}(t) + \eta_R^{(2)}(t) = \sum_{j=1}^{\infty}\left[c_j W_j(t) + \lambda_j W_j(t)^2\right] \tag{3}$$

where c_j and λ_j are eigenvalues of the linear and nonlinear signals, and $W_j(t)$ is a real stochastic process which is independent, zero-mean, stationary Gaussian process. c_j is defined simply as

$$c_j = \left|\int_{-\infty}^{\infty} H_R^{(1)}(\omega)\sqrt{S(\omega)}\psi_j^*(\omega)d\omega\right| \tag{4}$$

where $S(\omega)$ is wave spectrum and $\psi_j^*(\omega)$ is the complex conjugate of eigenfunction. The second-order eigenvalue, λ_j, must be obtained by solving the following equation (Naess, 1990):

$$\int_{-\infty}^{\infty} K(\omega_1,\omega_2)\psi_j(\omega_2)d\omega_2 = \lambda_j\psi_j(\omega_1) \tag{5}$$

where $K(\omega_1,\omega_2) = \sqrt{S(\omega_1)S(\omega_2)}H_R^{(2)}(\omega_1,-\omega_2)$.

The benefit of the eigenvalue expansion is that the statistical moments can be obtained analytically with those eigenvalues and eigenfunctions. The first four statistical moments of $\eta(t)$ are given by (Winterstein et al., 1994)

$$m = \sum_{j=1}^{\infty}\lambda_j,\ \sigma^2 = \sum_{j=1}^{\infty}\left(c_j^2 + 2\lambda_j^2\right),\ \alpha_3 = \sum_{j=1}^{\infty}\left(\frac{6c_j^2\lambda_j + 8\lambda_j^3}{\sigma^3}\right),\ \alpha_4 = 3 + \sum_{j=1}^{\infty}\frac{48\left(c_j^2\lambda_j^2 + \lambda_j^4\right)}{\sigma^4} \tag{6}$$

where m, σ, α_3, and α_4 are the mean, standard deviation, skewness, and kurtosis of $\eta(t)$, respectively.

2.2 Probability Analysis Based on Hermite-Moment Approach

The Hermite-moment method for the probability distribution of non-Gaussian variables using the first four statistical moments was introduced by Winterstein (1988). The key idea of Hermite-moment method is to transform the non-Gaussian variable into a standard normal variable by a mapping function. Winterstein (1988) expanded this mapping function with the Hermite polynomials, whose unknown coefficients are related to the statistical moments. The equation of the mapping function $g(u)$ is given by

$$\eta_R = g(u) = m_{\eta_R} + \kappa\sigma_{\eta_R}\left[u + \sum_{n=3}^{4}\hat{h}_n He_{n-1}(u)\right] \qquad \text{where } \mathrm{He}_n(u) = (-1)^n e^{\frac{u^2}{2}}\frac{d^n}{du^n}e^{-\frac{u^2}{2}} \tag{7}$$

Here, He_n is the n-th order Hermite polynomial, κ is the scaling factor, and h_3 and h_4 are the coefficients reflecting the shape of the distribution. The unknown coefficients κ, h_3, and h_4 are related to the first four statistical moments by a system of nonlinear equations. Detailed equations and the solution process used in this study can be found in Yang et al. (2013).

Since deck slamming occurs when the air gap is less than zero, i.e. a<0 (see Fig.1) and the relative velocity is positive, the joint probability of η_R and $\dot{\eta}_R$ must be considered. Nam(2019) showed that the joint probability density function of η_R and $\dot{\eta}_R$ can be written as below:

$$\begin{aligned} P(\eta_R,\dot{\eta}_R) &= P_1(u,\dot{u})\left|\frac{\partial(u,\dot{u})}{\partial(\eta_R,\dot{\eta}_R)}\right| = P_1\left(g^{-1}(\eta_R), \frac{1}{g'\left(g^{-1}(\eta_R)\right)}\dot{\eta}_R\right)\times\left[\frac{1}{g'\left(g^{-1}(\eta_R)\right)}\right]^2 \\ &= \frac{1}{2\pi\sigma_{\dot{u}}g'\left(g^{-1}(\eta_R)\right)^2}\exp\left[-\frac{1}{2}\left(\left(g^{-1}(\eta_R)\right)^2 + \frac{1}{\sigma_{\dot{u}}^2 g'\left(g^{-1}(\eta_R)\right)^2}\dot{\eta}_R^2\right)\right] \end{aligned} \tag{8}$$

where $|\partial(u,\dot{u})/\partial(\eta_R,\dot{\eta}_R)|$ is the Jacobian matrix and $g^{-1}(u)$ is the inverse function of $g(u)$. Then, the probability of deck slamming occurrence can be written as

$$\Pr\left[\max\left(\eta_R(t)\middle|0\le t\le T_R\right)>a_0\right]=1-\left[\Pr\left(\text{wave crest}<a_0\right)\right]^n=1-\left[1-\exp\left\{-\frac{1}{2}\left[g^{-1}(a_0)\right]^2\right\}\right]^n \tag{9}$$

where n means the number of wave crest during the time window T_R.

2.3 Estimation of Deck-Slamming Pressure

A typical approximation of deck-slamming pressure can be written as below:

$$P_s=\frac{1}{2}\rho C_V\left[\dot{\eta}_R\middle|\eta_R=a_0\right]^2 \tag{10}$$

where C_V is an empirical coefficient. DNV-GL (2010) proposed to apply 5.0 for decks-slamming in head sea condition and 10.0 for 45-deg wave heading, but the value of C_V is not important in this study since we focus on the methodology of analysis, not the accuracy of pressure.

Extending the above approach, the pressure at the moment of deck slamming occurrence can be written as foolow:

$$P\left(P_s\middle|\eta_R=a_0\right)=\frac{1}{\rho C_V\sigma_{\dot{u}}^{\,2}\left[g'\left(g^{-1}(a_0)\right)\right]^2}\exp\left[-\frac{1}{\rho C_V\sigma_{\dot{u}}^{\,2}\left[g'\left(g^{-1}(a_0)\right)\right]^2}P_s\right] \tag{11}$$

3 APPLICATION & RESULTS

3.1 Computational Model

The computational model is a tension-leg platform shown in Fig. 2. Its column has 19.52m diameter and pontoon length is 41.58m. Its draft is 31.42m and displacement is 35,290 tons. Five points, P1~P5, are chosen to observe deck slamming occurrence using the present method. The hydrodynamic coefficients, i.e. linear and second-order quadratic transfer functions, are computed using the WADAM program. Fig. 2 shows the solution panels for QTF computation and five locations to be considered. In this application case, the irregular ocean waves are represented by JONSWAP spectrum and 13m significant wave height and 14sec modal period are assumed.

Fig. 3 shows the PDF of relative wave elevation and the probability of exceedance of wave crest at P4. In this case, the signal of η_R was generated from linear and second-order transfer functions, and the results from rainflow counting are observed with the present statistical method and the approximation based on Rayleigh distribution. It is natural that the current nonlinear statistical method has a better correspondence with the results of rainflow counting. Fig. 4 compares the probability of deck slamming in 3 hours at five locations. It should be mentioned that the severest condition may occur at P4, i.e. near column in lee side. Fig. 5 shows the maximum averaged slamming pressure at P4, showing the largest peak pressure may occur when the deck height is about 13m.

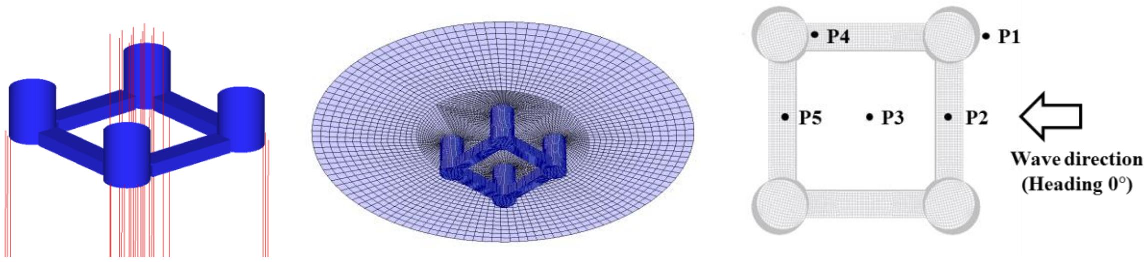
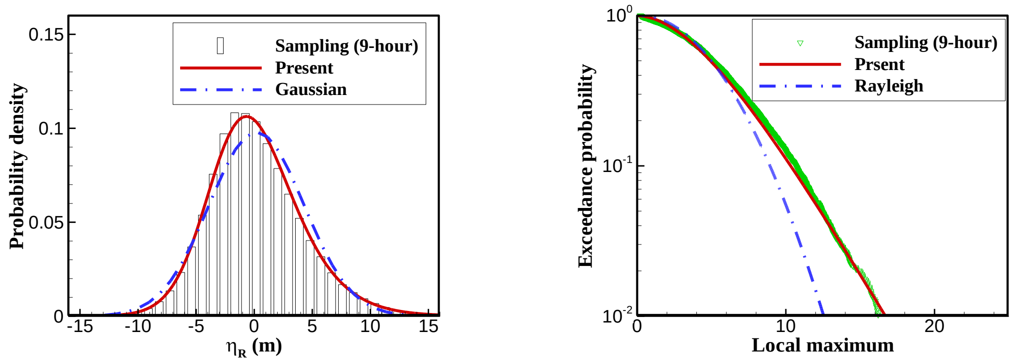


Fig. 2 Computational TLP model, solution panels for QTF computation and observation points



(a) PDF of η_R (b) Probability of exceedance of wave crest

Fig. 3 Probability density function and probability of exceedance of wave crest at P4

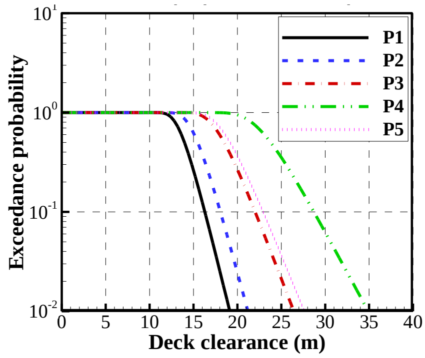


Fig. 4 Exceedance probability distribution of deck slamming occurrence in 3-hour

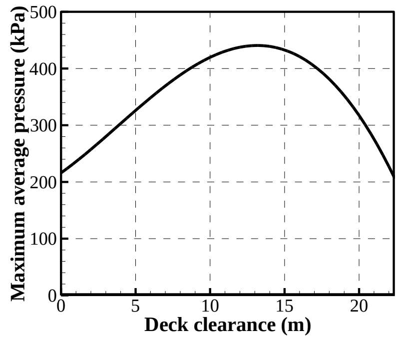


Fig. 5 Maximum average slamming pressure at P4

4 CONCLUSIONS

Based on the present study, the following conclusions are possible:

- The relative wave velocity is statistically dependent with the relative wave elevation when the second-order components are included.
- The proposed statistical model well follows the change of probability distribution of relative wave velocity depending on the value of relative wave elevation.
- It can be seen that the second-order component of relative wave elevation causes the increase of deck slamming pressure, and the sum-frequency component of relative wave elevation provides a main contribution

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