Amplification of wave run-up in random waves driven by tertiary interactions

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Highlights

- Wave run-up is investigated on the front of a fixed box in uni-directional random waves with normal incidence;
- The wave surface elevations at the centre of the weather side can reach 4x the incident waves, much larger than the ~2x predicted from linear theory;
- The extra amplification builds up slowly and is believed to result from tertiary interactions between the incident and reflected wave fields in front of the box, as described by Molin et al (2003) for regular waves;
- Evidence is provided to support this in random waves using NewWave-type analyses at the centre of the box, to show that the amplification is localized.

1. Introduction

Based on regular wave tests, Molin et al. (2003) reported a large run-up phenomenon on a fixed vertical plate, where the local wave surface elevations reached 4 or 5 x the amplitude of the incident waves. It was proposed by Molin et al. (2005) that the reflected wave fields (from the body) 'slow down' the incident waves (as a shoal would) locally, through the tertiary wave interactions of Longuet-Higgins and Phillips (1962). The resulting local lensing induces wave focusing on the weather side of the structure, leading to significant amplitude enhancement. Two numerical models have been developed to predict the measured profiles along fixed vertical plates (Molin et al. 2010): the parabolic model of Molin et al. (2005) based on the tertiary theories of Longuet-Higgins and Phillips (1962) and a fully nonlinear numerical wave tank based on extended Boussinesq equations (Bingham et al. 2004). These studies identified the important role of the tertiary interactions in the large wave run-up phenomenon in regular waves.

It has been 15 years since the pioneering paper of Molin et al. (2003). Surprisingly limited attention has been paid to the tertiary interaction phenomena other than in generalizations of the Benjamin-Feir instability in undisturbed wave fields. We also note that all the literature available for enhanced run-up has focused on regular waves, both seeking a steady state solution and investigating the time evolution of the free surface elevation. However, there are some important open questions remaining particularly for waves in a random sea, e.g. (1) whether the tertiary interactions can occur and are the effects important; (2) if so, how to identify the time evolution of run-up magnification and time lag for large events in the random signals. A series of tank tests help us to address these questions. The theory of the average shape of large events in a random process (NewWave theory, Jonathan and Taylor (1997)) is applied to highlight the effects of tertiary interactions in a random sea.

2. Experimental set-up

The experiments were performed in the Deepwater Wave Basin at Shanghai Jiao Tong University. The wave basin is 50 m long, 40 m wide and the water depth was set to 10 m. Flap-hinged wavemakers are fixed along two neighbouring sides of the basin and wave absorbing beaches are installed on the opposite sides to minimize reflected waves.

The experimental set-up is the same as in Zhao et al. (2017), where tests were conducted to investigate the resonant fluid responses in narrow gaps between two side-by-side moored vessels. Two identical 3.333 m long and 0.767 m wide rectangular boxes were used, these were 0.425 m high and immersed such that the undisturbed draught was 0.185 m. The two boxes were rigidly mounted to a gantry near the centre of the wave basin in a side-by-side configuration, forming a narrow gap of 0.067 m (see figures 1 & 2 in Zhao et al. 2017). In addition to the wave gauges in the gap, we also deployed a few wave gauges outside the gap, allowing the investigation of the wave run-up in front of the two fixed boxes. Gap resonances do not seem to significantly affect the wave field in front of the boxes which will be demonstrated below. Therefore, the two identical boxes in side-by-side configuration with a narrow gap is equivalent to a single 'flat' box with width of $2\times0.767+0.067=1.601$ m, from the perspective of wave run-up on the weather side.

Uni-directional waves with a white noise spectrum over a frequency range 0.2 Hz to 1.8 Hz are used. The significant wave height at laboratory scale is 41.7 mm, so it is a very mild and broad-banded sea state. The white noise wave field was calibrated prior to the actual model tests in the absence of the model. Then, with the model in place, the same paddle signal was used to generate identical incident wave conditions. The random wave test was run for ~ 1600 s in the basin, and the wave surface elevations were sampled at 25 Hz. Fig. 1 shows typical wave elevations in front of the fixed boxes with and without the models in place. It can be seen in Fig. 1 (a) that the response magnitudes are more than twice those of the incident waves. The amplitude spectra in Fig. 1 (b) show that the amplitude ratio of the responses to the incident waves can reach up to ~ 4 at higher frequencies, much larger than linear theory predictions.

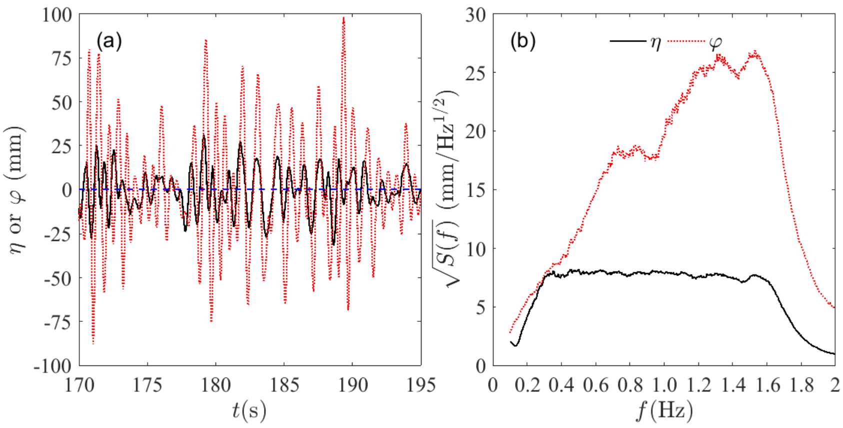

Fig. 1 Measured data from the wave gauge in front of and at midship of the fixed boxes in the white noise wave tests. (a) representative time histories of the measured surface elevations with (the responses φ – red dashed line) and without (the incident waves η – black solid line) the models, (b) the corresponding amplitude spectra calculated based on the steady state time histories.

3. Spectral analysis to explore the time evolution of amplitude magnification

Response amplitude operators (RAOs) are obtained from Fourier analysis of the measured signals over individual windows each about 82 s long (2048 data points), as shown in Fig. 2. There are a total of 19 time segments over the whole measured signal (~1600 s). We do not plot all 19 segment results here for clarity, instead we provide the range boundaries (grey shaded) and the mean (black line) of the results from segment 3 to 19, when the tertiary interactions have fully developed. For the first segment of the time history, we only used the first half (41 s with 1025 data points) rather than the whole to calculate the RAOs. This is because at the beginning, the tertiary interactions have not significantly developed, so the RAOs are likely to be in good agreement with linear theory predictions. Similarly, for the transient wave group tests (see Zhao et al. 2017 for more details), tertiary interactions do not occur due to the short test duration. As shown in Fig. 2, the result obtained from both the transient wave groups (dark green line) and the first half of the first segment signals (solid red line) agree very well with the linear theory predictions (dashed black line).

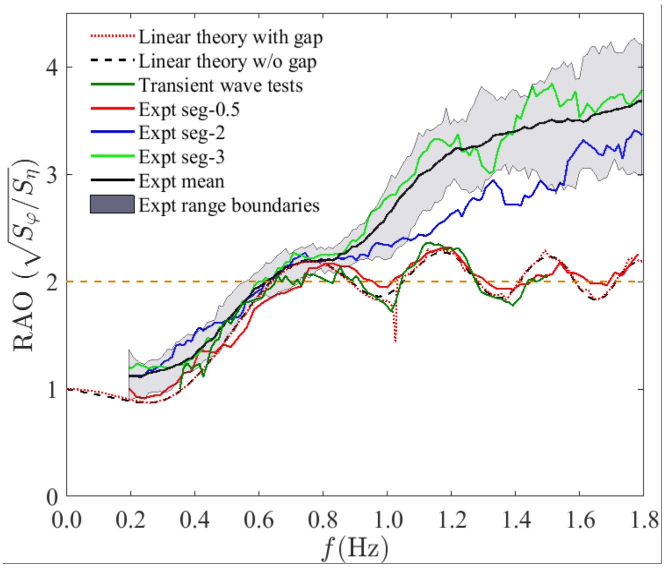

Fig. 2 RAOs in front of the model under transient wave groups and irregular waves. 'Linear theory with gap' refers to the two identical boxes in side-by-side configuration with a narrow gap and 'Linear theory w/o gap' for an equivalent 'flat' box. 'Expt seg-0.5' indicates the 1st half of the 1st segment signal, 'seg-2' and 'seg-3' the 2nd and 3rd segments, respectively. 'Expt mean' is the mean value from the 3rd segment to the last (19th).

One may have a concern about the gap resonance between the side-by-side fixed models affecting the wave run-up in front of the upwave box. To eliminate this complicating effect, we run DIFFRACT (Sun et al. 2015) providing linear diffraction predictions of the free-surface RAO at the centre of the upwave side for the set-up of Zhao et al. (2017) and for an equivalent 'flat' box with the gap 'filled in'. The linear predictions with and without the gap agree extremely well, apart from a small spike around 1.02 Hz where the 1st mode gap resonance occurs. However, the effect of this peak at the upwave box surface is very small and would be even smaller in the experiments because viscous damping would reduce the gap resonance peak below the linear diffraction results. Thus, the two identical boxes with a narrow gap used here can be regarded as entirely equivalent to a wider single box at least for wave run-up upwave of the models.

Fig. 2 shows the evolution of the RAOs over time. Initially the RAO matches the linear diffraction results, and it seems clear that the tertiary interactions are developing over the first two time segments but are fully developed from the third segment onwards. The most striking observation is that the wave run-up RAOs in front of the models reach up to 4x in such a broad-banded wave field, much larger than the linear predictions.

4. NewWave-type analysis to identify the phase lag and reciprocity

To identify the phase lag, which is another characteristic of the tertiary interaction theory of Longuet-Higgins and Phillips (1962), we run transient wave group tests based on NewWave, as developed by Jonathan and Taylor (1997).

4.1 Identifying the time delay

To facilitate the analysis, the localized focused NewWave-type transient group test results measured at the centre of the weather side of the upwave box are given in Fig. 3. The incident wave group was generated with a Gaussian spectrum with peak frequency of 0.5 Hz and the measured crest wave elevation of the time history is 50 mm. Four time series which are 90º out of phase were run in the wave basin, which in combination allow for the separation of the first four harmonics as we did for the gap resonance problem (Zhao et al 2017). There were no visible higher harmonics other than the second harmonic component. Therefore, we only show the first two harmonic signals in Fig. 3.

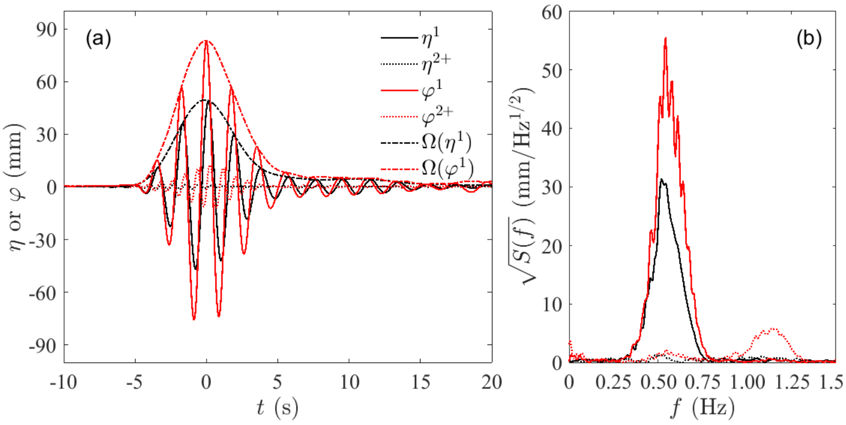

Fig. 3 NewWave-type results obtained from the transient wave group test. (a) the first and second harmonic components of the measured signal, (b) the corresponding spectra. The dashed-dot lines $\Omega(\eta^1)$ and $\Omega(\varphi^1)$ are the envelopes.

An important observation from Fig. 3 (a) is that the (linearized) incident wave signal (η^1 – solid black line) is in phase with the response signal (φ^1 in red). This is understandable given the short duration of the transient wave group tests - it takes time for incident waves to reflect to produce the outgoing field in front of the upwave box for later incident waves to interact with, leading to the tertiary interactions.

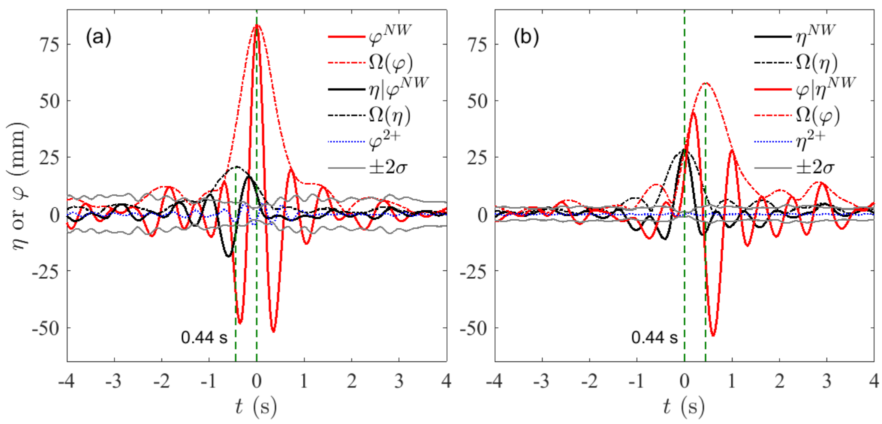

Fig. 4 NewWave-type results of the undisturbed waves and the wave fields with the models in place. (a) the NewResponse profile (φ^{NW}) and the corresponding undisturbed wave time history ($\eta|\varphi^{NW}$), (b) the NewWave profile (η^{NW}) in the incident field and the correspondingly generated response time history ($\varphi|\eta^{NW}$).

Fig. 3 provides a straightforward method to explore the relative phase information between the input and output signal for deterministic transient group tests. To be able to explore the phase lag due to tertiary interactions in a random sea, we constructed similar NewWave profiles to those shown in Fig. 3 from the irregular wave tests, as in Zhao et al. (2018). By creating shorter time series with each maximum point located at relative time $t = 0$ and then averaging across the short records, we produce the so-called NewWave shape. Based on the constructed four-phase signals in combination, we extracted the first four harmonics, the first two of which are shown in Fig. 4 where (a) is obtained by searching for the peak responses and (b) the peak incident (undisturbed) waves. We did not observe visually any third harmonics, and the second harmonic (φ^{2+}) in the response signal is very small as shown in Fig. 4 (a). This is consistent with the tertiary interaction theory of Molin et al. (2005) based on Longuet-Higgins and Phillips (1962). This does not result in obvious high-frequency phenomena but in modifications to apparently linear (1st-harmonic) quantities. The most important information shown in Fig. 4 is that in comparison with Fig. 3 an obvious net time delay of 0.44 s is identified between the peak of the envelope of the incident waves (black) which give rise to the large responses and that of the responses (red), presumably due to the reduction in phase speed from the tertiary interactions.

4.2 Reciprocity

If the coupling between the input and output signals is linear (no higher harmonic quantities), a simple reciprocity result should hold. From the results shown in Fig. 4, the incident wave time history ($\eta|\varphi^{NW}$) given the largest responses (NewResponse) and the response time history ($\varphi|\eta^{NW}$) given the largest incident waves (NewWave) are a mirror image pair in time centred around the zero conditioning instant, with a simple scaling factor. Fig. 5 shows the reciprocity of the two signals. The good agreement suggests that the coupling between the incident waves and the responses is linear (first-harmonic quantities) in the local field. This linearity is consistent with the Molin lensing of each frequency component being due to tertiary interactions with the reflected field of every other frequency component, occurring over a large but finite area on the weather side of the model.

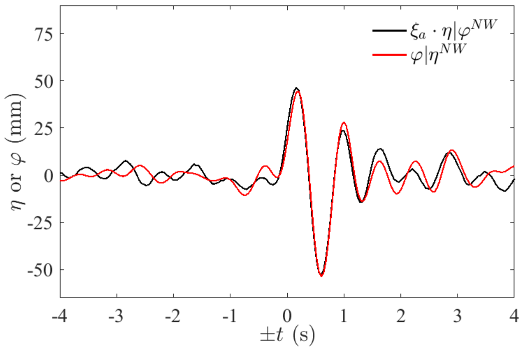

Fig. 5 Reciprocity of the undisturbed wave η (time reversed about $t = 0$ and thus "$\pm t$" used here) and wave surface elevations in front of the models (φ)

5. Concluding remarks

Following the pioneering work by Molin et al. (2003), we provide experimental evidence for large wave interactions on the weather side of a fixed body in random waves. Using a NewWave-type analysis, we have defined the time delay for large events and the amplitude magnification in a random sea. The most striking effect is that the wave surface elevation in front of the fixed body can reach up to 4x the amplitude of the incident waves at high frequencies in a broad-banded sea state. We see no significant tank resonant modes or standing waves between wave paddles and the fixed model (further evidence will be presented at the workshop). We believe that we have provided sufficient evidence that the significant wave amplitude magnification observed in our irregular wave test is a result of tertiary wave interactions, the same mechanism as in Molin et al. (2003, 2005, 2010) for regular waves. Such investigation of tertiary interactions in irregular waves has not previously been reported. This effect may be of practical importance, e.g. survival of disabled floating structures in beam seas, air gap of multi-column offshore structures, breakwaters, etc.

References

Bingham, H.B., Fuhrman, D.R., Jamois, E., Kimmoun, O., 2004. Nonlinear wave interaction with bottom-mounted structures by a high-order Boussinesq method. *Proc. 19th Int. Workshop on Water Waves and Floating Bodies*. Cortona, Italy, 2004.

Jonathan, P., Taylor, P.H., 1997. On irregular, nonlinear waves in a spread sea. *J. Offshore Mech. Arct. Eng.*, 119, 37-41.

Longuet-Higgins, M.S., Phillips, O.M., 1962. Phase velocity effects in tertiary wave interactions. *Journal of Fluid Mechanics*, 12, 333-336.

Molin, B., Remy, F., Kimmoun, O., Ferrant, P., 2003. Third-order interactions and wave run-up. *Proc. 18th Int. Workshop on Water Waves and Floating Bodies*. Le Croisic, France, 2003.

Molin, B., Remy, F., Kimmoun, O., Jamois, E., 2005. The role of tertiary wave interactions in wave-body problems. *Journal of Fluid Mechanics*, 528, 323-354.

Molin, B., Kimmoun, O., Liu, Y., Remy, F., Bingham, H.B., 2010. Experimental and numerical study of the wave run-up along a vertical plate. *Journal of Fluid Mechanics*, 654, 363-386.

Sun, L., Eatock Taylor, R., Taylor, P.H., 2015. Wave driven free surface motion in the gap between a tanker and an FLNG barge. *Applied Ocean Research*, 51, 331-349

Zhao, W., Wolgamot, H. A., Taylor, P. H., Eatock Taylor, R., 2017. Gap resonance and higher harmonics driven by focused transient wave groups. *Journal of Fluid Mechanics*, 812, 905-939.

Zhao, W., Taylor, P.H., Wolgamot, H.A, Eatock Taylor, R., 2018. Linear viscous damping in random wave excited gap resonance at laboratory scale – NewWave analysis and reciprocity. *Journal of Fluids and Structures*, under review.