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Reconstruction of an extreme wave profile with analytical methods

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Highlights:

- Simulation of nearly breaking focused wave group with fully nonlinear potential flow solver (HOS-NWT).
- Harmonic analysis of the focused wave group using the four-wave decomposition method.
- Extraction of the evolved free-wave spectrum from the fully nonlinear computations.
- Use of analytical methods to reconstruct the wave profile by adding nonlinear bound harmonics.
- Comparison of exact second order theory, approximate fifth order expansion and Creamer transform against a fully nonlinear wave profile.

1. Introduction

Traditionally, the design of marine structures was performed using regular waves, with characteristics chosen to represent a wave with a period and height selected from statistical distributions of exceedance probabilities, and then using analytical models or empirical formulas to estimate the kinematics. Over time, powerful numerical models have emerged, such as Computational Fluid Dynamics solvers, which are capable of computing the fully nonlinear solution of wave-structure interaction. Nevertheless, the selection of the design wave which serves as input to the numerical models is of primary importance for the reliability of the results. It was shown that the average shape of the largest waves in the ocean can be described by the scaled autocorrelation function of the underlying free-wave spectrum, which introduced the concept of the NewWave theory [1] for the design of offshore and even coastal structures. In practice, according to NewWave theory, the largest waves based on an underlying spectrum have the shape of a wave group, which components are in phase at a specific location and time, forming a focused wave.

Focused waves are widely used in physical and numerical modelling because they offer a deterministic alternative to long random simulations, which require large resources, and minimize possible spurious effects of wave reflections. Despite the success and the popularity of the NewWave theory, there are some limitations of the method which may hinder the realistic replication of extreme ocean waves: i) NewWave is based on linear theory, while, especially steep waves in the ocean exhibit high degree of nonlinearity, and ii) during the linear dispersive focusing of a NewWave-type wave group, the underlying free-wave spectrum remains constant. Experiments and numerical simulations [2] have proven that the propagation of steep focused waves is associated with both emergence of high order bound nonlinearities that make the wave profile steeper and higher and with resonant nonlinearities that change the dispersive properties, i.e., amplitudes and phases, of the free-wave components of the wave group. Although these effects can be implicitly reproduced by fully nonlinear models, they cause undesirable effects that deteriorate the quality of focusing. These can be controlled to a good extent with focusing methodologies that iteratively correct the input signal for the wave paddles. However, the evolution of the free-wave regime may have strong impacts to analytical models for estimating the bound nonlinearities and are commonly used in engineering practice. Johannessen & Swan (2003) [2] demonstrated that using the locally broadened free-wave spectrum improves considerably the second order theory prediction of the bound waves and the analytically calculated result approaches the fully nonlinear solution. For directional waves close to their breaking limit, they found that second order theory is adequate to replicate the crest elevation, while long-crested waves included higher than second order bound waves. The free + bound wave structure can also describe realistic large ocean waves [3].

In this study, the scope is to explore a "high order" unidirectional NewWave-type wave group calculated using: the linear NewWave with incorporated bound waves according to second order theory [4]; approximate fifth order expansion [5]; and Creamer transform [6]. The results are compared with fully nonlinear simulations of the high order spectral numerical wave tank (HOS-NWT) model [7], using similar testing conditions and techniques for accurate focusing of the waves as used by Vyzikas *et al.* (2018) [8]. The wave conditions correspond to a broadband Gaussian amplitude spectrum of peak frequency of f_p=0.64 Hz, which is discretised by 320 components ranging between 0-2.5 Hz, propagating in intermediate water depth of d=1 m. The original (theoretical) spectrum and the locally broadened (evolved) spectrum at the focusing location are examined, following the principle of [2] in order to calculate, with greater accuracy, the bound waves, as it is known that the latter are explicitly defined by the free waves.

2. Fully nonlinear simulations

In a recent study, unidirectional focused wave groups close to their breaking limit were simulated using the two-phase Reynolds Averaged Navier Stokes (RANS) model in OpenFOAM and the results showed excellent agreement with experiments [8]. A key element for this is the employment of an accurate focusing methodology. The methodology used is similar to that proposed by Stagonas *et al.* (2014) [9], which iteratively corrects the amplitudes and phases of the wave components of the extracted linearized harmonics that comprise the free-wave spectrum. However, instead of correcting both the phases and the amplitudes at the same location, the latter are matched to the target spectrum near the wave paddle. This technique allows for tracking the natural evolution of the amplitude spectrum, due to near-resonant third order nonlinearities, towards the phase focusing location where the extreme wave is formed.

Aiming at decreasing the computational cost of the OpenFOAM simulation, the efficient HOS-NWT [7] is used here, since the waves are not breaking. HOS is a pseudo-spectral method based on potential flow theory, which evaluates the spatial derivatives of the flow variables in the Fourier space. The NWT is a numerical mirror of the physical experiment and has a linear piston-type wave generation and an absorption zone designed to guarantee that there are no reflections at the time window of interest that contains the focused wave group. The wave group propagates for approximately $3.5L_p$, where L_p=3.59 m is the wavelength corresponding to the component of f_p, from the amplitude correction to the phase focal point (1.63 m and 14.1 m downstream of the wave paddle, respectively). After careful convergence analysis, the order of HOS is selected to be 5, corresponding to a fully nonlinear simulation; the total length of the NWT (50 m) is discretized by 500 modes; and the wave maker has 40 modes. Full dealiasing is performed. The results of HOS-NWT are in good agreement with those of OpenFOAM and can replicate very reliably the evolution of the free-wave spectrum. Detailed validation of the HOS-NWT will follow in future work.

The free-wave spectrum is extracted by the fully nonlinear measurements using the four-wave decomposition method which requires the independent simulation of four wave groups with components of phase differences 0, $\pi/2$, π and $3\pi/2$ rad. Appropriate algebraic combination of the simulated timeseries returns the linear + fifth order, second order sum, second order difference + fourth order and third order harmonics. The advantage compared to two-wave decomposition is that it can isolate the linear harmonics with trivial frequency filtering of the fifth order harmonics, allowing for application on realistic broadbanded spectra, such as the one in the present study [8].

An example of the spectral analysis of the harmonics for the nearly breaking wave group is presented in Figure 1a after the correction with the focusing methodology. The corresponding timeseries of the harmonics are presented in Figure 1b. On one hand, it can be observed that the amplitude spectrum has been altered from the original as the group approached the phase focal location, with the spectral peak been downshifted and considerable energy been transferred to high frequencies between 0.9-1.5 Hz. On the other hand, the analysis of the timeseries shows that bound waves at least up to fourth order are formed and the linear harmonic can only account for 75% of the crest elevation.

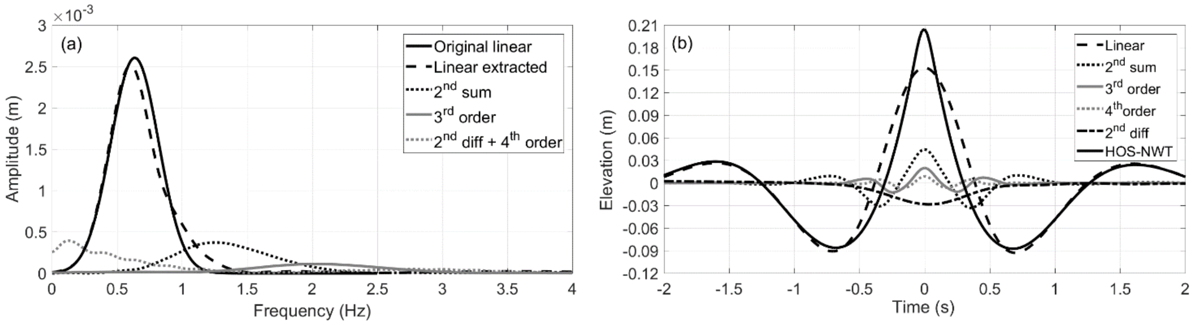

Figure 1: Decomposition of the wave group into harmonics at the focal location: (a) Spectral analysis and comparison with the original spectrum; (b) Timeseries and comparison with the fully nonlinear simulation.

3. Analytical methods

The best-established approach to include bound waves is the exact second order solution of Dalzell [4], which calculates the second sum and difference harmonics as a combination of any possible wavenumbers of the linear spectrum, without any presumption of the spectral bandwidth. As mentioned, second order bound nonlinearities in finite water depth can adequately describe the wave profile of directional wave groups, but unidirectional groups have a stronger bound wave structure [2].

The analytically calculated second sum and second difference harmonics are compared with the extracted harmonics in Figure 2b and 2c, respectively. For comparison, the original and evolved linear harmonics are presented in Figure 2a, where it is seen that the evolved harmonics have shallower troughs and wider and lower lateral crests than the original, but practically the same crest elevation. As the calculation of the second order harmonics is based solely on the linear harmonics, it can be seen that the overall agreement between the analytical solution and the extracted harmonics is good, but it improves considerably when the evolved free-wave spectrum is used. The main and the lateral crests of the second sum harmonics are reproduced with good accuracy, but the depth of the adjacent troughs is underestimated. For the second difference harmonics, the original spectrum results in a considerably wider and shallower trough, while the evolved spectrum gives a better estimate, however, still with a shallow trough.

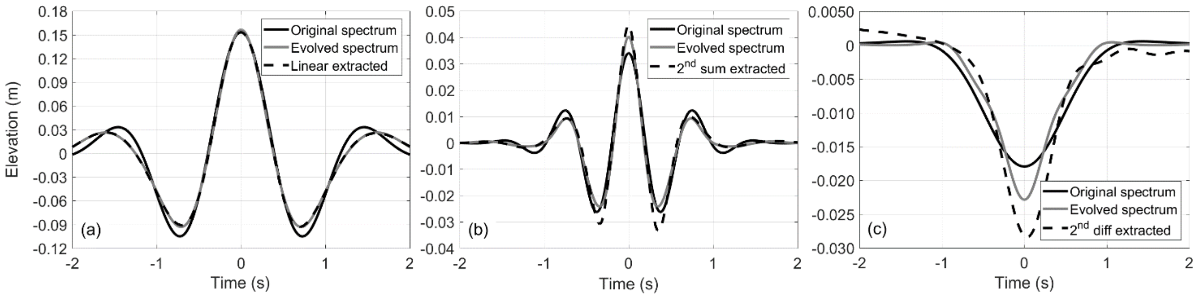

Figure 2: Comparison of the timeseries of the extracted harmonics at the focal location against the analytically calculated harmonics based on the original and evolved free-wave spectra: (a) Linear; (b) Second sum; (c) Second difference harmonics.

Next, the approximate solution of the fifth order expansion is examined [5], which to the best of the authors' knowledge has only be applied in the original publication for the reproduction of the New Year Draupner Wave, where the free-wave spectrum had to be crudely estimated from the field measurements. Since the free-wave spectrum is known here, the performance of the approximation can be more accurately tested. The fifth order expansion is based on the Stokes expansion of a slowly modulated wave train with modified coefficients (S_{ij}). The indices i and j refer to the Stokes amplitude order and harmonic of frequency, respectively. The expression is shown in Equation 1, where the terms are grouped according to ascending frequency order. The D_{ij} coefficients are calculated using the linear signal and its Hilbert transform, as shown in the appendix of [5].

$$n(t) = \left[S_{11}D_{11} + \frac{S_{31}}{d^2}D_{31} + \frac{S_{51}}{d^4}D_{51}\right] + \left[\frac{S_{22}}{d}D_{22} + \frac{S_{42}}{d^3}D_{42}\right] + \left[\frac{S_{33}}{d^2}D_{33} + \frac{S_{53}}{d^4}D_{53}\right] + \left[\frac{S_{44}}{d^3}D_{44}\right] + \left[\frac{S_{55}}{d^4}D_{55}\right] \quad (1)$$

To apply the expansion, a representative wavenumber for the wave group should be selected, which is used to calculate Fenton's B_{ij} coefficients included in the S_{ij} coefficients. After a sensitivity analysis, it was found that the wavenumber of the f_p component (k_p) is a good approximation, which here is calculated separately for the original and evolved free-wave spectra. It is noted that the expansion calculates only the high order harmonics and the set-down should be calculated separately. In the original publication, the low-frequency harmonics are estimated by the self-interactions only. Tests showed that for the present case this results in negligible set-down, which artificially increases the crest elevation. Thus, it is preferred to compute the exact set-down according to second order theory [4]. Comparisons for this method are given only for the total free surface profile (Figure 3) and not for the individual harmonics.

The last method for introducing nonlinearity to a linear wave profile refers to the Creamer transform [6], which is based on the Hamiltonian representation of weakly nonlinear water waves. By employing a canonical transformation of the flow variables based on the Lie transform, the Creamer transform cancels the lowest order of nonlinearity, which results in the complete elimination at Hamiltonian third order expansion. In the unidirectional infinite depth case, the Creamer transform is expressed as a simple integral of the spatial profile of the free surface (Equation 2), which essentially adds bound waves of infinite order to a linear surface profile at a given time instance. Since the Creamer transform is formulated in space, the timeseries can be produced at the location of interest (x_f), here the focal point, by time propagating the wave group at some discrete times before and after the focal time using the linear dispersion relation and keeping the values of the surface elevation only at x_f. The time range should be sufficiently short to assume that the underlying free-wave spectrum has remained practically the same. For the present study, this is ±2 s from the focal time.

$$n_{NL}(k) = \frac{1}{|k|}\int e^{-ikx}\left(e^{ikn_H(x)} - 1\right)dx \quad (2)$$

where $n_{NL}(k)$ is the nonlinear amplitude wavenumber spectrum and $n_H(x)$ the Hilbert transform of the spatial profile.

4. Results and discussion

The reconstructed profiles of the free surface elevation using the three analytical methods of the previous section are presented in Figure 3a and 3b using the original and the evolved free-wave spectra, respectively, and they are compared with the fully nonlinear solution of HOS-NWT. By comparing the two subplots, it can be readily seen that the use of the evolved free-wave spectrum gives a significantly improved prediction for the lateral crests and troughs, while more subtle differences are observed for the main crest. More specifically, the use of the evolved free-wave spectrum results in shallower troughs and flatter lateral crests compared to the original spectrum. The main crest remains almost identical. These trends were already identified for the linear harmonics in Figure 2a, which confirms in an indirect way that the bound nonlinear harmonics are controlled by the underlying linear harmonics. Thus, the correct estimation of the linear harmonics is vital for the realistic representation of NewWave-type wave groups.

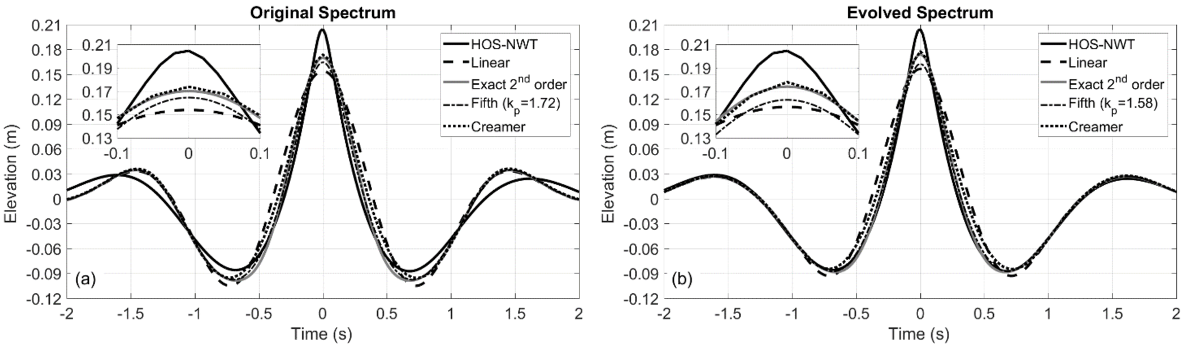

Figure 3: Comparison of the predicted nonlinear NewWave-type profiles of the different analytical methods against the fully nonlinear profile of HOS-NWT using the (a) original and (b) the evolved free-wave spectra.

The inter-comparison of the analytical methods is consistent for the two examined free-wave spectra. Examining the main crest shows that all the methods predict a steeper and narrower profile compared to linear theory and increase the linear theory prediction of the crest elevation on average by approximately 12%. The nonlinear crest is predicted best by second order theory and the Creamer transform that have almost identical results. The use of the evolved spectrum gives a moderate increase of the crest elevation for both methods. The fifth order expansion predicts a lower crest elevation, which is marginally decreased when the evolved linear spectrum is used. It is noted that for the second difference harmonic, the exact solution was employed, otherwise this method would artificially predict a crest elevation similar to that of the fully nonlinear solution, but with wider main crest and elevated lateral troughs. Furthermore, when the fifth order expansion was used for the Draupner wave [5], it was tuned to match exactly its crest height, which explains the good agreement obtained in the original study and the less impressive performance that it exhibits here. More in-detail comparison of the two best-performing, i.e., second order theory and Creamer transform, reveals that the latter predicts a wider main crest and consequently, the former gives overall more reliable results. However, this may be an effect of the infinite water depth assumption in the form of the Creamer transform used here (Equation 2). Future work will expand the present study in deep water depth for greater consistency in comparing the methods.

To conclude, the present study examined the performance of analytical methods for adding nonlinear bound waves in a linear wave profile aiming at improving the linear NewWave profile. The results demonstrated that all the methods provide a more realistic NewWave profile, which is more appropriate for engineering purposes as a design wave for marine structures. Employing the locally broadened free-wave spectrum gives a considerable improvement of the overall wave profile, but an almost negligible increase of the crest elevation. This finding is contrasting previous studies [2] that reported appreciable increase of the linear and second order theory prediction for nearly breaking unidirectional wave groups when the evolved free-wave spectrum was used. This contradiction can be attributed to the different underlying original spectra and partially to the less precise two-wave harmonic decomposition used in [2].

It is the subject of future work to examine the kinematics of the different methods in correspondence to the free-wave spectrum and to investigate whether more conservative structural loads are finally calculated.

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