Linearized seakeeping using CFD

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Introduction

The current trends in the seakeeping simulations seem to be more and more oriented toward the use of the RANS based CFD methods instead of the more classical potential flow methods. There are several reasons for that among which the inclusion of the nonlinear effects and the effects of the forward speed are probably the most important ones. Indeed, due to the very complex behavior of the free surface for large amplitude waves the potential flow methods have enormeous difficulties to model the associated nonlinear effects, especially when the wave breaking occurs. In addition, the problem of seakeeping with the forward speed also introduce the huge difficulties so that the fully consistent potential flow solution, even linear, based on the Boundary Integral Equation (BIE) technique is still missing. On the other hand, it must be recognized that the CFD methods became nowadays very efficient allowing for quite accurate evaluation of the ship seakeeping characteristics for very general operating conditions. One of the main drawbacks of the CFD are very large CPU time requirements which typically exceeds those of potential flow by an order of magnitude. One of the ways to reduce the overall CPU time is to linearize the seakeeping problem where by linearization we understand the linearization of the boundary conditions only. This means that the other terms in the NS equations remain nonlinear. Lot of work has been done in the past, regarding the linearization of the potential flow formulation [2, 5, 1, 3] but there are not so many publications on the linearized Navier Stokes formulation [7].

Potential flow formulation

Before presenting the linearized problem for seakeeping within the CFD approach, it is useful to recall the basic principles of the linearization within the potential flow approach. We start by defining the different coordinate systems in Figure 1 [$O(\boldsymbol{X}) = O(X, Y, Z)$ - earth fixed, $o(\boldsymbol{x}) = o(x, y, z)$ - steady translating and $o'(\boldsymbol{x}') = o'(x', y', z')$ - ship fixed]. The problem of body advancing with constant forward speed U

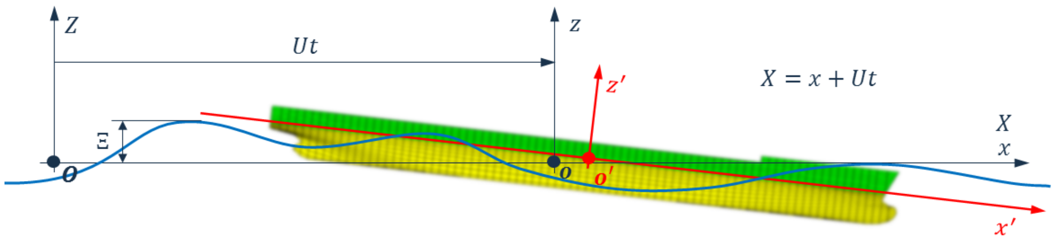

Figure 1: Coordinate systems.

in X direction is equivalent to the problem of the uniform flow passing the ship. Since this approach is preffered for the use within the CFD solution methodology we describe the fully nonlinear potential flow problem using this terminology. In this case the coordinate systems $O(\boldsymbol{X})$ and $o(\boldsymbol{x})$ coincide and both are earth fixed, with $Z = z = 0$ denoting the free surface position. The total potential Φ is represented as the sum of the uniform flow potential $-Ux$ and its perturbation denoted ϕ:

$$\Phi(\boldsymbol{x}, t) = \phi(\boldsymbol{x}, t) - Ux \tag{1}$$

The velocity potential $\Phi(\boldsymbol{x}, t)$ and the free surface position $\Xi(x, y, t)$ define the pair of the unknown quantities or state variables. At the free surface, they are related to each other through the kinematic boundary condition (KBC) and the dynamic boundary condition (DBC). The kinematic boundary condition states that the free surface is the material surface and it remains such for all times. This leads to the following mathematical description:

$$\Big(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x} + \nabla\phi\nabla\Big)\Xi = \frac{\partial \phi}{\partial Z} \tag{2}$$

The dynamic free surface condition states that the pressure in the fluid and the pressure at the free surface are equal to each other. The Bernoulli equation is used to calculate the pressure and we can write:

$$p = -\varrho\Big[\frac{\partial \Phi}{\partial t} + \frac{1}{2}(\nabla\Phi)^2 + gz - \frac{1}{2}U^2\Big] \tag{3}$$

With this in mind, the dynamic free surface boundary condition for the potential ϕ becomes:

$$\Big(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\Big)\phi + \frac{1}{2}(\nabla\phi)^2 = -g\Xi \tag{4}$$

Within the potential flow formulation it is possible to combine the kinematic and the dynamic boundary condition into one single condition. The easiest way to do this is to take the total time derivative of the dynamic boundary condition. The following combined boundary condition is obtained (e.g. [2]):

$$\frac{\partial^2\phi}{\partial t^2} - 2U\frac{\partial^2\phi}{\partial x\partial t} + U^2\frac{\partial^2\phi}{\partial x^2} + 2\nabla\phi\nabla\frac{\partial\phi}{\partial t} - 2U\nabla\phi\nabla\frac{\partial\phi}{\partial x} + \frac{1}{2}\nabla\phi\nabla(\nabla\phi)^2 + g\frac{\partial\phi}{\partial z} = 0 \tag{5}$$

The above formulated potential flow problem is fully nonlinear and does not make any assumptions about the particular nature of the flow. The problem is extremely complex and no consistent general numerical solution exists due to the presence of highly nonlinear terms. That is why the different linearization procedures were proposed in the past. The main goals of the linearization are to apply the boundary conditions at the fixed boundaries and, at the same time, to make those conditions linearly dependent on the unknown quantities. The linearization is usually performed in two steps. The first step consists in assuming that the different flow quantities at the instantaneous boundary can be expressed as a small perturbaton of the same quantities at their mean position. In that respect, both the kinematic and the dynamic free surface conditions are expanded in Taylor series around the initial calm free surface $z = 0$. The following expressions are obtained up to the order $O(\Xi)$:

$$\begin{aligned} &KBC \qquad \Big(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x} + \nabla\phi\nabla\Big)\Xi = \frac{\partial\phi}{\partial z} + \Xi\frac{\partial^2\phi}{\partial z^2} + O(\Xi^2) \\ &DBC \qquad \Big(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\Big)\phi + \frac{1}{2}(\nabla\phi)^2 + \Xi\frac{\partial}{\partial z}\Big[\Big(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\Big)\phi + \frac{1}{2}(\nabla\phi)^2\Big] = -g\Xi + O(\Xi^2) \end{aligned} \tag{6}$$

where all quantities are to be evaluated at $z = 0$.
The second step in the linearization procedure, consists in developing the total solution into the dominant part and the small perturbation around it. In that respect we formally decompose the state variables ϕ and Ξ as follows:

$$\phi(\boldsymbol{x}, t) = \phi_0(\boldsymbol{x}) + \phi_1(\boldsymbol{x}, t) \qquad , \qquad \Xi(x, y, t) = \Xi_0(x, y) + \Xi_1(x, y, t) \tag{7}$$

where ϕ_0 denotes the basis flow, assumed to be time independent, and ϕ_1 the perturbation flow.
For the time being we do not define the exact order of the different quantities and we just mention that the basis flow state variables (ϕ_0, Ξ_0) are one order of magnitude larger than the perturbed ones (ϕ_1, Ξ_1). However, the free surface elevation Ξ_0 is still assumed to be small enough in order for Taylor series expansion (6) to be valid. Formally we state that the basis flow quantities are of the order $O(1)$ and the perturbed quantities are of the order $O(\varepsilon)$, without explicitely defining what is ε. It is important to understand that, for the general case, this decomposition is not unique.
After introducing the decomposition (7) into (6), and neglecting the terms of the higher order, we obtain the expressions (8) and (9) forkinematic and dynamic boundary conditions at first two orders. It is important to note that, up to this point, the linearization procedure does not make any distinction in between the steady and the unsteady potential flow at order $O(\varepsilon)$ and all the conditions apply both to the steady and the unsteady flow. In particular, when applied to the steady wave resistance problem, the above free surface condition belongs to the so called Dawson type of approaches.

$$O(1) \qquad (-U\frac{\partial}{\partial x}+\nabla\phi_0\nabla)\Xi_0=\frac{\partial\phi_0}{\partial z}+\Xi_0\frac{\partial^2\phi_0}{\partial z^2}$$

$$-U\frac{\partial\phi_0}{\partial x}+\frac{1}{2}(\nabla\phi_0)^2+\Xi_0\frac{\partial}{\partial z}\Big[-U\frac{\partial\phi_0}{\partial x}+\frac{1}{2}(\nabla\phi_0)^2\Big]=-g\Xi_0 \tag{8}$$

$$O(\varepsilon) \qquad (\frac{\partial}{\partial t}-U\frac{\partial}{\partial x}+\nabla\phi_0\nabla)\Xi_1+\nabla\phi_1\nabla\Xi_0=\frac{\partial\phi_1}{\partial z}+\Xi_1\frac{\partial^2\phi_0}{\partial z^2}+\Xi_0\frac{\partial^2\phi_1}{\partial z^2}$$

$$(\frac{\partial}{\partial t}-U\frac{\partial}{\partial x}+\nabla\phi_0\nabla)\phi_1+\Xi_0\frac{\partial}{\partial z}\Big[(\frac{\partial}{\partial t}-U\frac{\partial}{\partial x}+\nabla\phi_0\nabla)\phi_1\Big] \tag{9}$$

$$+\Xi_1\frac{\partial}{\partial z}\Big[-U\frac{\partial\phi_0}{\partial x}+\frac{1}{2}(\nabla\phi_0)^2\Big]=-g\Xi_1$$

Only in the case of the zero forward speed the linearization is trivial because the basis flow is naturally equal to zero so that we can simply ignore all the quadratic terms in the above free surface conditions (8, 9) and obtain:

$$\frac{\partial\Xi_1}{\partial t}=\frac{\partial\phi_1}{\partial z} \quad , \quad \frac{\partial\phi_1}{\partial t}=-g\Xi_1 \quad , \quad \frac{\partial^2\phi_1}{\partial t^2}+g\frac{\partial\phi_1}{\partial z}=0 \tag{10}$$

which are the well known kinematic, dynamic and combined free surface conditions for zero speed case. In the case of non zero forward speed different choices for the basis flow have been proposed in the litterature going from the simplest uniform flow approximation ($\phi_0=0$), passing thorugh the double body flow linearization ($\partial\phi_0/\partial z=0$, at $z=0$) and finally using the fully nonlinear steady flow [1].

Navier Stokes formulation

First of all, let us again mention that, in the present context, the linearization of the Navier Stokes equations means the linearization with respect to the boundary conditions only and other nonlinear terms in the Navier Stokes equations remain. Within the Navier Stokes formulation the state variables are the pressure in the fluid p and the flow velocity $\boldsymbol{v}=(u,v,w)$. We will not enter here into the details of the Navier Stokes equations and just make the direct analogy in between the different expressions which were formulated for the potential flow problem. Similar to potential flow formulation, and for the time being, we will concentrate on the linearization around the mean free surface position $z=0$ i.e. the solution formally valid for small free surface disturbances. Whatever the formulation i.e. potential flow or Navier Stokes, the free surface boundary condition will always consist of the similar kinematic (KBC) and the dynamic (DBC) boundary conditions. Assuming the shear stress at the free surface to be negligible, we can rewrite the conditions (6) in the form:

$$KBC \qquad \Big(\frac{\partial}{\partial t}-U\frac{\partial}{\partial x}+\boldsymbol{v}\nabla\Big)\Xi=w+\Xi\frac{\partial w}{\partial z}$$

$$DBC \qquad p_d+\Xi\frac{\partial p_d}{\partial z}=-g\Xi \tag{11}$$

The conditions have to be applied at $z=0$, and the notation p_d is used to denote the dynamic part of the pressure:

$$p_d(\boldsymbol{x})=p(\boldsymbol{x})-gz \tag{12}$$

It is very important to understand that, even if the above conditions have to be applied at $z=0$, they remain nonlinear. It is also important to note that, if we could solve the seakeeping problem with the above conditions, there will be no need for further linearization and the solution would be correct up to the order $O(\Xi)$, which was our final goal here. However, within the potential flow theory, solving the above formulated problem is still not very convenient so that the additional effort has been made in order to further linearize the free surface conditions. As we have seen, this additional linearization process, passes through the separation of the total flow into the basis flow and its first order correction leading to the quite complex final expressions. Within the Navier Stokes formulation it looks like solving the problem directly with the above conditions (11) is much more convenient. Indeed, as already mentionned there exist other nonlinear terms in the Navier Stokes equations which should be treated in nonlinear sense anyway. This means that the boundary conditions at the free surface for the linearized Naviers Stokes problem is defined by (11) and the rest of the procedure for solving the Navier Stokes equations remains the same. The method described above was implemented within the OpenFOAM framework [6] and herebelow we present few preliminary validation results.

Preliminary results and discussions

The well known test cases of MOERI (container ship KCS and tanker KVLCC2) were chosen for comparisons. In left part of Figure 2, the results for steady wave resistance of KCS are shown for Froude

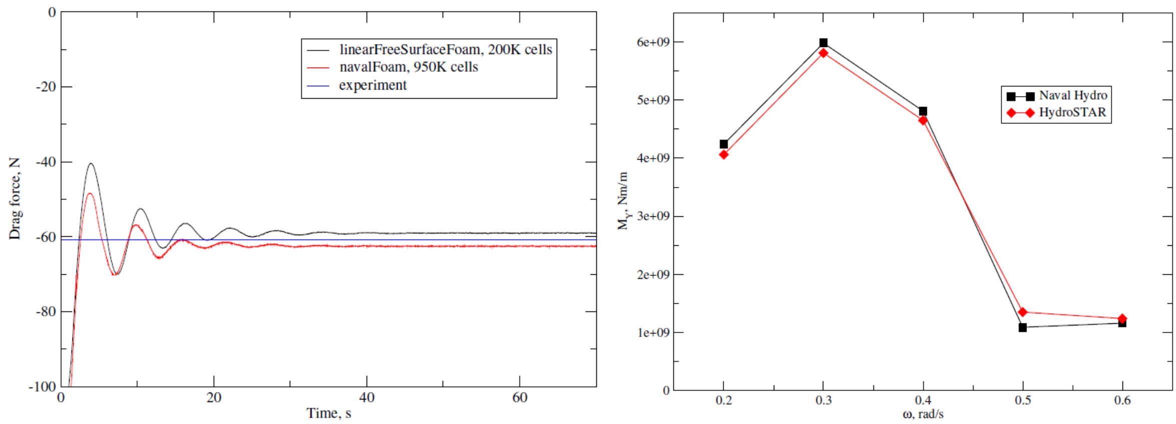

Figure 2: Steady wave resistance (left) and the pitch excitation moment (right) for KVLCC ship.

number equal to 0.26. We can observe very good agreement in between the fully nonlinear, linear and the experimental results. In the right part of the same figure, the results for the unsteady pitch excitation are presented for KVLCC2. The CFD results are compared to potential flow results obtained by HYDROSTAR. Once again, very good agreement in between two clases of results is observed. All this shows that the linearized CFD model represents the fast and the efficient tool for seakeeping simulations and can be used safely for the moderate sea conditions conditions.

We mention here that there exist some differences in between the formulation employed in [7] and the present formulation. Indeed, in [7] the following kinematic and dynamic bundary conditions were used:

$$\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)\Xi = w \qquad , \qquad p_d = -g\Xi \tag{13}$$

Comparisons in between the two formulations will be presented and discussed at the Workshop.

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