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Challenges in developing a SWENSE two-phase CFD solver for complex wave conditions

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Introduction

Simulating wave-structure interaction with CFD tools is often computational resource demanding: large computational domains are used to prevent waves from being reflected at the boundary and fine meshes are always needed to avoid excessive numerical attenuation of incoming waves.

To reduce the computational cost, the Spectral Wave Explicit Navier-Stokes Equations (SWENSE) method was proposed to couple the solutions from potential theory and Navier-Stokes (NS) equations[1]. In this method, the incident wave solution is evaluated by the wave models based on potential theory in the entire computational domain, leaving only the perturbation caused by the structure and the influence of viscosity to be solved with CFD. In its original single-phase formalism, it has been proved that coarse meshes can be used in the farfield for incident wave propagation while keeping a good accuracy[2].

The extension of the SWENSE method to two-phase solvers is not straightforward. Few attempts appeared recently[3, 4]. This paper presents this extension in the frame of Volume of Fluid (VOF) solvers. The two main challenges are: (a) developing numerically stable two-phase SWENSE equations; (b) mapping accurately the solution of potential wave models to the CFD calculation grids. The latter is not only the bottleneck of the development of the two-phase SWENSE method but also a common difficulty in coupling potential wave models and two-phase CFD solvers. The authors propose a solution to overcome these difficulties and develop a two-phase SWENSE solver with the open source CFD package OpenFOAM. Several test cases are proposed for validating the model, including 2D regular and irregular waves propagation and the computation of wave forces on a 3D cylinder.

Challenge 1: SWENSE equations for two-phase VOF solvers

The SWENSE method decomposes a wave structure-interaction problem χ into an incident part χ_I and a complementary part χ_C as shown in Eqn.(1). The incident part takes into account the propagation of the incident waves, whose solution is given by a dedicated wave model; the complementary part serves as a correction due to the disturbances caused by the viscosity and the presence of structures.

$$\chi = \chi_I + \chi_C \tag{1}$$

Complying with the idea of the SWENSE method for single-phase solver[1], the two-phase SWENSE governing equations should be obtained by subtracting the Euler equations from two-phase Navier-Stokes equations. The NS momentum equation for two-phase incompressible fluid, written with VOF is as follows, where w and a stand for the properties in the water and in the air respectively and the α field is the VOF field.

$$\rho = \alpha\rho^a + (1-\alpha)\rho^w \tag{2}$$

$$\frac{\partial \alpha}{\partial t} + \mathbf{u}.\nabla\alpha = 0 \tag{3}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u} = -\frac{\nabla p}{\rho} + \mathbf{g} + \nu\nabla^2\mathbf{u} \tag{4}$$

The perfect fluid Euler momentum equation behind potential wave models reads:

$$\rho_I = \rho^w \tag{5}$$

$$\frac{\partial \mathbf{u}_I}{\partial t} + \mathbf{u}_I.\nabla\mathbf{u}_I = -\frac{\nabla p_I}{\rho_I} + \mathbf{g} \tag{6}$$

To demonstrate the challenge, we subtract directly the two equations as in the single-phase SWENSE method[1]. The hypothesis done is that the solution from wave models can be extended in the air. The momentum equation obtained, written with the notation of $\mathbf{u}_C = \mathbf{u} - \mathbf{u}_I$ and $p_C = p - p_I$, reads:

$$\frac{\partial \mathbf{u}_C}{\partial t} + \mathbf{u}_C.\nabla\mathbf{u}_C + \mathbf{u}_C.\nabla\mathbf{u}_I + \mathbf{u}_I.\nabla\mathbf{u}_C = -\frac{\nabla p_C}{\rho} + \underline{\underline{\frac{\nabla p_I}{\rho_I} - \frac{\nabla p_I}{\rho}}} + \nu\nabla^2\mathbf{u}_C \tag{7}$$

The underlined terms are canceled out in the single-phase SWENSE momentum equation since $\rho = \rho_I$. However, these terms remain in two-phase flows, having non-zero values in the air phase. They behave as source terms and affect the numerical stability. To overcome this problem, we propose a mathematical reformulation for Eqn.(6). We introduce a modified incident pressure field p_I^* defined by:

$$p_I^* = \rho\frac{p_I}{\rho_I} \tag{8}$$

Eqn.(6) written in its modified version using p_I^* reads:

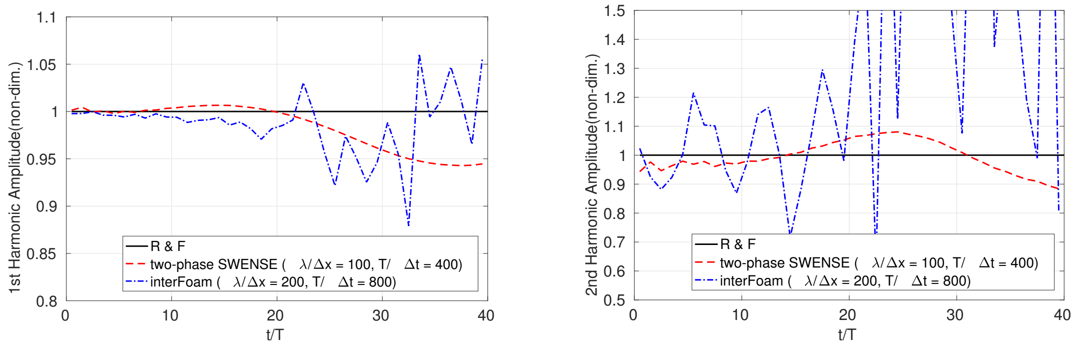
$$\frac{\partial \mathbf{u}_I}{\partial t} + \mathbf{u}_I.\nabla\mathbf{u}_I = -\frac{\nabla p_I^*}{\rho} + \frac{p_I}{\rho_I}\frac{\nabla\rho}{\rho} + \mathbf{g} \tag{9}$$

Subtracting Eqn.(9) from the two-phase Navier-Stokes equation Eqn.(4), and using the notion of $p_C = p - p_I^*$, the momentum equation written with the complementary variables is:

$$\frac{\partial \mathbf{u}_C}{\partial t} + \mathbf{u}_C.\nabla\mathbf{u}_C + \mathbf{u}_C.\nabla\mathbf{u}_I + \mathbf{u}_I.\nabla\mathbf{u}_C = -\frac{\nabla p_C}{\rho} - \underline{\underline{\frac{p_I}{\rho_I}\frac{\nabla\rho}{\rho}}} + \nu\nabla^2\mathbf{u}_C \tag{10}$$

The underlined term in Eqn.(10) is the only difference when comparing with the single-phase SWENSE momentum equation[1]. This term is similar to the dynamic pressure jump at the free surface in the conventional two-phase Navier-Stokes equation. The property of that dynamic pressure jump is that it equals to zero when the free surface matches the hydrostatic equilibrium position so that this hydrostatic equilibrium is maintained. In the case of the SWENSE method, the underlined term equals to zero when the free surface in the CFD matches the one from the wave models so that the incident wave solution is maintained.

This formulation is tested with two cases. The first one concerns the propagation of non-linear regular waves in a periodic 2D wave tank. The second deals with the calculation of the wave forces on a cylinder in regular waves. The details of the two test cases can be found in a separated paper[5]. For the first test case, Fig. 1 shows the first and second harmonic amplitudes of the free surface elevation. Compared with OpenFOAM native two-phase solver *interFoam*, an improvement on both the accuracy and the stability is confirmed.

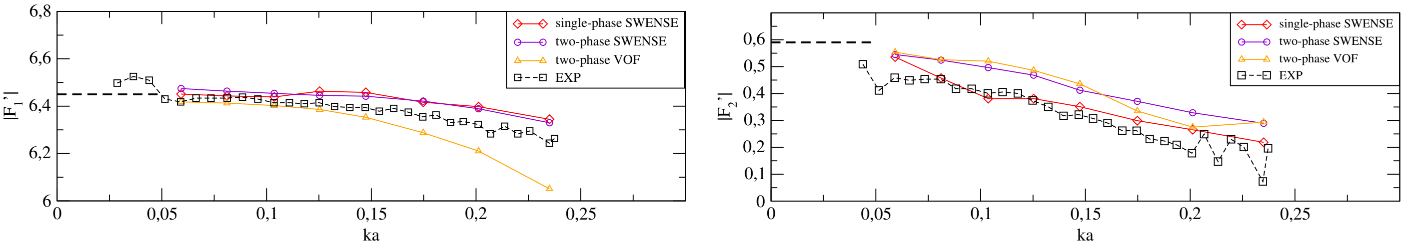


(a) First harmonic amplitude

(b) Second harmonic amplitude

Figure 1: Harmonic analysis of wave elevation for the regular wave propagation test case

For the second test case, Fig.2 shows the first and second harmonic amplitudes of the in-line wave force with different incident wave steepnesses. The results of the two-phase SWENSE solver have a good agreement with the single-phase SWENSE solver[6], and a smaller discrepancy from the experimental data for the first harmonic amplitude compared with a conventional two-phase VOF solver.



(a) First harmonic amplitude

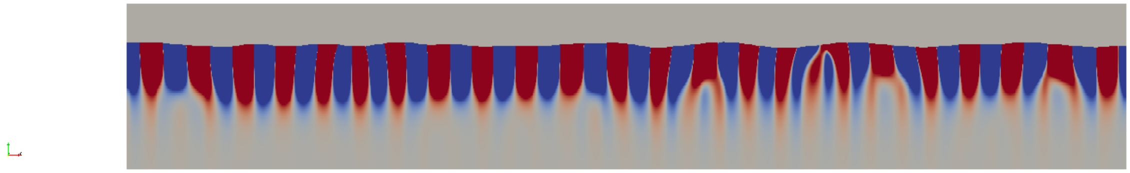
(b) Second harmonic amplitude

Figure 2: Harmonic analysis of wave force on a vertical cylinder

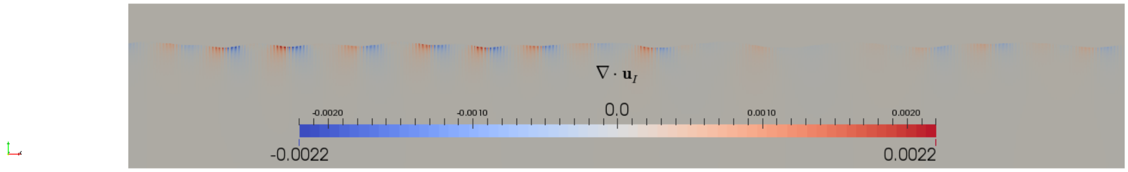
Challenge 2: Mapping solutions from wave models to CFD grids

The SWENSE method adopts spectral methods to efficiently model incident waves: the solution method of Rienecker-Fenton[7] is used for the regular waves and the High Order Spectral (HOS) method[8] is used for the irregular waves. The mapping of the solution from HOS solvers to the CFD grids is done with an open source wrapper program Grid2Grid[9]. However, an accurate mapping is a challenge. To model irregular waves, HOS solvers typically use less than 10 points per peak wavelength. Excessive points do not help capturing the physics of the waves and may generate stability problems due to the higher modes. However, the necessary cell number per wavelength is usually about 100 for two-phase VOF based solvers. Mapping the solution from wave models to the CFD cells is needed and done through interpolating from the neighbor HOS points. In a previous study[10], various interpolation techniques were compared from the simplest linear to the most sophisticated Lagrange method, but the interpolation errors are all too large to be accepted by the VOF based two-phase SWENSE method[4].

Recently, the authors come up with a solution for this problem. Instead of increasing the number of the HOS calculation points, ghost HOS points are added at the post-processing stage to provide more informations between two HOS calculation points and thus shorten the distance of the interpolation. This method avoids the time consuming point-by-point evaluation of the wave solution using the HOS modes; the value on each HOS ghost point is obtained directly through Fast Fourier Transforms (FFT). Fig.3 demonstrates a reduction of the interpolation errors on the divergence of velocity for an irregular wave case with $H_s = 0.028m$ and $T_p = 0.701s$. The waves are modeled by the HOS solver with 64 points in a computational domain with 10 peak wave lengths. By adding ghost points, the maximum value of the divergence decreased from 0.86 to 0.0022, corresponding to approximately 0.25% of the interpolation error using the original method.



(a) 64 HOS points $max(\nabla.\mathbf{u}_I) = 0.86$



(b) 64 HOS points + 448 ghost points $max(\nabla.\mathbf{u}_I) = 0.0022$

Figure 3: Interpolation errors for irregular waves with $H_s = 0.028m$ and $T_p = 0.701s$

Thanks to this improvement, the two-phase SWENSE solver is able to simulate irregular waves. Fig.4 shows an example of irregular waves propagating in a 2D wave tank simulated by the two-phase SWENSE method. The result of the two-phase VOF based SWENSE method and the HOS wave solution are in good agreement on the free surface elevation. More work including the simulation of structures in irregular waves is in progress and will be presented at the workshop.

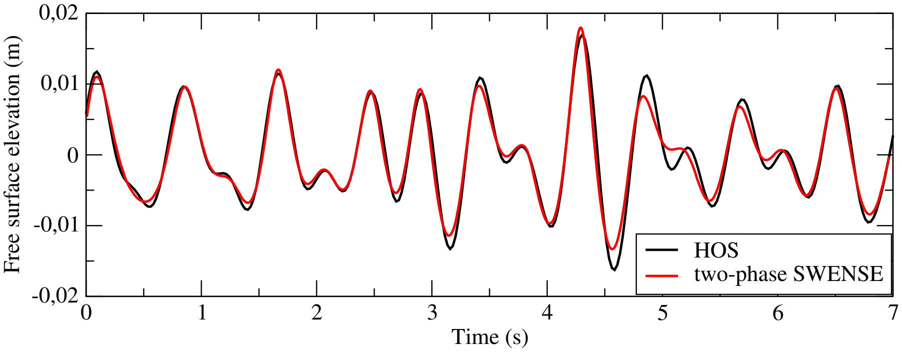


Figure 4: Time history of free surface elevation for irregular waves with $H_s = 0.028m$ and $T_p = 0.701s$

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