Wave-making problem by a vertical cylinder: Neumann-Kelvin theory versus Neumann-Michell theory

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Highlights

1. The Green function is analytically integrated over a smooth surface and a closed waterline using the Fourier-Laguerre spectral method.
2. By measuring condition numbers of coefficient matrices, the Neumann-Kelvin theory is ill-posed, since the condition number is up to $O(10^6)$. In contrast, the Neumann-Michell theory is well-behaved with a condition number in the order of 10^2.
3. The complete form of the boundary integral equation for the Neumann-Michell theory accounting for the local component of the waterline integral is given, and the waterline integral and hull surface integral in the modified Neumann-Kelvin theory are partially cancelled out.

1 Statement of the problem

A Cartesian coordinate system $OXYZ$ is defined with the positive $X-$axis pointing to upstream and $Z-$axis orienting upward, and it travels at a constant speed $\mathcal{U}$ with the cylinder along the positive $X-$axis. In this frame of reference, the problem is defined as an incoming uniform flow with the velocity $\mathcal{U}$ in the direction of negative $X-$axis. The reference length L and gravitational acceleration g are used to define non-dimensional coordinates $\boldsymbol{x} = (x, y, z)$, velocity components $\boldsymbol{u} = (u, v, w)$ and velocity potential Φ with respect to L, $\sqrt{gL}$ and $\sqrt{gL^3}$, respectively. The Froude number is defined as $F = \mathcal{U}/\sqrt{gL}$. The total potential in the fluid domain is decomposed into $\Phi = F(-x + \phi)$. On the hull surface Σ^H, the body boundary condition is satisfied:

$$\phi_n = n_x \quad \text{on} \quad \Sigma^H \tag{1}$$

where the normal vector $\boldsymbol{n} = (n_x, n_y, n_z)$ is defined positively pointing into the fluid domain. On the mean free surface Σ^F, the Kelvin-Michell free-surface boundary condition is satisfied:

$$F^2\phi_{xx} + \phi_z = 0 \quad \text{at} \quad z = 0 \tag{2}$$

The free-surface Green function for the steadily translating problem satisfying the free-surface condition (2) is: $\mathcal{G} = \mathscr{R} + \mathscr{F}$ where $\mathscr{R}$ and $\mathscr{F}$ denote the Rankine term and free-surface term defined as [1]:

$$\mathscr{R}(\boldsymbol{x}, \boldsymbol{\xi}) = \frac{1}{4\pi}\left(-\frac{1}{r} + \frac{1}{r'}\right) \quad \text{and} \quad \mathscr{F}(\boldsymbol{x}, \boldsymbol{\xi}) = \frac{1}{4\pi^2}\Re\int_{-\pi}^{\pi}\int_0^{\infty}\frac{\kappa \mathrm{e}^{\kappa(z+\zeta) - \mathrm{i}\kappa[(x-\xi)\cos\theta + (y-\eta)\sin\theta]}}{F^2\kappa^2\cos^2\theta - \kappa}\mathrm{d}\kappa\mathrm{d}\theta \tag{3}$$

where $\boldsymbol{\xi} = (\xi, \eta, \zeta)$ and $\boldsymbol{x} = (x, y, z)$ denote the source point and flow-field point. Several alternative methods to efficiently and accurately evaluate $\mathscr{F}$ are reviewed in [2]. Here, we introduce a new method [3] which is suitable for the spatial integration. The free-surface term $\mathscr{F}$ is expanded into:

$$\mathscr{F} = \frac{1}{4\pi^2}\Re\sum_{\ell=-\infty}^{\infty}(-\mathrm{i})^{\ell}\mathrm{e}^{-\mathrm{i}\ell\varphi}\int_0^{\infty}\mathrm{e}^{\kappa(z+\zeta)}J_{\ell}(\kappa h)\, g_{\ell}(\kappa)\,\mathrm{d}\kappa \quad \text{with} \quad g_{\ell}(\kappa) = \int_{-\pi}^{\pi}\frac{\mathrm{e}^{\mathrm{i}\ell\theta}\mathrm{d}\theta}{F^2\kappa\cos^2\theta - 1} \tag{4}$$

By using Cauchy's theorem of residue, the integral with respect to θ can be analytically expressed [3]. The analytical expression is well-behaved and not oscillatory with κ. For a large κ, the asymptotic representation of $g_{\ell}(\kappa)$ is:

$$\widehat{g}_{\ell}(\kappa) = -\frac{2\ell(-\mathrm{i})^{\ell}\pi}{F^2\kappa}\left(1 - \frac{\ell^2 - 1}{6}\frac{1}{F^2\kappa}\right) + O\left(\kappa^{-3}\right) \tag{5}$$

2 Boundary integral equations for different flow models

2.1 Neumann-Kelvin theory

We apply Green's identity in the fluid domain bounded by the hull surface Σ^H free surface Σ^F and the surface at infinity Σ^{∞}. The integral over the surface Σ^{∞} is null due to the radiation condition. By applying the Kelvin-Michell free-surface condition, the free surface integral is reduced to a waterline integral using the Stokes' theorem. Then, we obtain the boundary integral equation for the Neumann-Kelvin (NK) theory [4, 5]:

$$\phi = \iint_{\Sigma^H}(\mathcal{G}n_{\xi} - \phi\mathcal{G}_n)\,\mathrm{d}S - F^2\oint_{\Gamma}(\mathcal{G}\phi_{\xi} - \phi\mathcal{G}_{\xi})\, n_{\xi}\Big/\sqrt{n_{\xi}^2 + n_{\eta}^2}\mathrm{d}L \tag{6}$$

where Γ stands for the waterline intersected by the cylindrical surface Σ^H and free surface Σ^F.

2.2 Modified Neumann-Kelvin theory

In the work by Noblesse et al [6], the component between the mean free surface and the actual free surface is accounted for in the source term, and it is written as:

$$\iint_{\Sigma_a^H} \mathcal{G} n_\xi \mathrm{d}S \approx \iint_{\Sigma^H} \mathcal{G} n_\xi \mathrm{d}S + F^2 \oint_\Gamma \mathcal{G} \phi_\xi n_\xi \Big/ \sqrt{n_\xi^2 + n_\eta^2} \mathrm{d}L \tag{7}$$

The waterline integral in (7) and the one in (6) can be cancelled out. Therefore, we obtain a new boundary integral equation with the waterline integral component associated with ϕ_ξ removed:

$$\phi = \iint_{\Sigma^H} \left(\mathcal{G} n_\xi - \phi \mathcal{G}_n\right) \mathrm{d}S + F^2 \oint_\Gamma \phi \mathcal{G}_\xi n_\xi \Big/ \sqrt{n_\xi^2 + n_\eta^2} \mathrm{d}L \tag{8}$$

This is refered to as the modified Neumann-Kelvin (mNK) theory. In contrast to the boundary integral equation for the NK theory given by (6), the waterline integral does not include the term associated with the spatial derivative of velocity potential, and is simplified to some extent.

2.3 Neumann-Michell theory

In the mNK theory, the term associated with ϕ_ξ in the waterline integral is removed, but it still requires the integration of the Green function along the waterline. Then, we follow the work by Noblesse et al [6] to further eliminate the waterline integral which is referred to as the Neumann-Michell (NM) theory. However, different from [6], the local component in the waterline integral is retained. We introduce a vector function $\boldsymbol{F}$ satisfying:

$$\nabla \times \boldsymbol{F} = \nabla \mathscr{F} \quad \text{with} \quad \boldsymbol{F} = \left(0, \mathscr{F}_\zeta^\xi, -\mathscr{F}_\eta^\xi\right) \tag{9}$$

where the subscript means differentiation while the superscript represents integration. If the vector function $\boldsymbol{F}$ and the scalar function $\mathscr{F}$ satisfy the relation (9), the integral of $\boldsymbol{n} \cdot [\nabla \times (\phi \boldsymbol{F})]$ over a closed surface Σ is null. Then, we obtain the boundary integral equation for the NM theory which is expressed as:

$$\phi(\boldsymbol{x}) = \iint_{\Sigma^H} \left[\mathcal{G} n_\xi - \phi \mathscr{R}_n + (\boldsymbol{n} \times \nabla \phi) \cdot \boldsymbol{F}\right] \mathrm{d}S - \oint_\Gamma \mathscr{R}_\zeta^\xi \phi n_\xi \Big/ \sqrt{n_\xi^2 + n_\eta^2} \mathrm{d}L \tag{10}$$

In the formulation (10), the waterline integral is only associated with the Rankine term $\mathscr{R}$, and the waterline integral of the free-surface term $\mathscr{F}$ is eliminated.

3 Fourier-Laguerre spectral method

To implement the boundary integral flow representations given in section 2, we study the wave-making problem by a long vertical cylinder with a radius $\mathcal{R}$. For the purpose of coping with the singular and highly-oscillatory behaviours of the Green function properly, we analytically integrate the Green function over a smooth boundary surface and a closed waterline. On the hull surface in the form of a circular cylinder Σ^H, the velocity potential and its normal derivative are expanded into a series of base function composed of the Laguerre function in the vertical direction and Fourier series in circumference [7]:

$$\phi(\varphi, \zeta) = \sum_{k=0}^{\infty} \sum_{l=-\infty}^{\infty} \phi_{kl} \mathcal{L}_k(-s\zeta) \mathrm{e}^{\mathrm{i}l\varphi} \quad \text{and} \quad \frac{\partial \phi}{\partial n}(\varphi, \zeta) = \psi(\varphi, \zeta) = \sum_{k=0}^{\infty} \sum_{l=-\infty}^{\infty} \psi_{kl} \mathcal{L}_k(-s\zeta) \mathrm{e}^{\mathrm{i}l\varphi} \tag{11}$$

via which the Green function is analytically integrated over a smooth surface and along a closed waterline. On the waterline, the velocity potential and radial velocity component are consistent with the distribution over the hull surface.

3.1 Fourier-Laguerre spectral method for the NK theory

To construct the boundary integral equation on the cylindrical surface, the collocation of the Galerkin type is applied through integrating a test function in the form of $\mathcal{L}_m(-sz) \mathrm{e}^{-\mathrm{i}n\gamma}$ on both sides of the boundary integral equation over the cylinder surface [7], and the resultant formulation is expressed as:

$$\pi \phi_{mn} + \sum_{k=0}^{\infty} \sum_{l=-\infty}^{\infty} \phi_{kl} \mathcal{H}_{mn,kl}^{HH} - \sum_{k=0}^{\infty} \sum_{l=-\infty}^{\infty} \phi_{kl} \mathcal{H}_{mn,kl}^{HW} = \sum_{k=0}^{\infty} \sum_{l=-\infty}^{\infty} \psi_{kl} \mathcal{G}_{mn,kl}^{HH} - \sum_{k=0}^{\infty} \sum_{l=-\infty}^{\infty} \psi_{kl} \mathcal{G}_{mn,kl}^{HW} \tag{12}$$

with

$$\mathcal{G}_{mn,kl}^{HH} = \int_{-\infty}^{0} \int_{-\pi}^{\pi} \iint_{\Sigma^{\mathrm{H}}} \mathcal{G}(\boldsymbol{x}, \boldsymbol{\xi}) \mathcal{L}_m(-sz) \mathcal{L}_k(-s\zeta) \mathrm{e}^{-\mathrm{i}n\gamma} \mathrm{e}^{\mathrm{i}l\varphi} \mathrm{d}S \mathrm{d}\gamma \mathrm{d}z \tag{13a}$$

$$\mathcal{H}_{mn,kl}^{HH} = \int_{-\infty}^{0} \int_{-\pi}^{\pi} \iint\!\!\!\!\!\!\bigcirc_{\Sigma^{\mathrm{H}}} \mathcal{G}_n(\boldsymbol{x}, \boldsymbol{\xi}) \mathcal{L}_m(-sz) \mathcal{L}_k(-s\zeta) \mathrm{e}^{-\mathrm{i}n\gamma} \mathrm{e}^{\mathrm{i}l\varphi} \mathrm{d}S \mathrm{d}\gamma \mathrm{d}z \tag{13b}$$

$$\mathcal{G}_{mn,kl}^{HW}=\sqrt{s}F^2\int_{-\infty}^{0}\int_{-\pi}^{\pi}\oint_{\Gamma}\mathscr{F}\left(\boldsymbol{x},\boldsymbol{\xi}\right)\mathcal{L}_m\left(-sz\right)\mathrm{e}^{-\mathrm{i}n\gamma}\mathrm{e}^{\mathrm{i}l\varphi}\cos^2\varphi\mathrm{d}s\mathrm{d}\gamma\mathrm{d}z \tag{13c}$$

$$\mathcal{H}_{mn,kl}^{HW}=\sqrt{s}F^2\int_{-\infty}^{0}\int_{-\pi}^{\pi}\oint_{\Gamma}\left[\frac{\mathrm{i}l}{\mathcal{R}}\mathscr{F}\left(\boldsymbol{x},\boldsymbol{\xi}\right)\sin\varphi+\mathscr{F}_{\xi}\left(\boldsymbol{x},\boldsymbol{\xi}\right)\right]\mathcal{L}_m\left(-sz\right)\mathrm{e}^{-\mathrm{i}n\gamma}\mathrm{e}^{\mathrm{i}l\varphi}\cos\varphi\mathrm{d}s\mathrm{d}\gamma\mathrm{d}z \tag{13d}$$

3.2 Fourier-Laguerre spectral method for the mNK theory

In the same manner, the boundary integral equation for the mNK theory constructed on the cylinder surface is represented as:

$$\pi\phi_{mn}+\sum_{k=0}^{\infty}\sum_{l=-\infty}^{\infty}\phi_{kl}\mathscr{H}_{mn,kl}^{HH}-\sum_{k=0}^{\infty}\sum_{l=-\infty}^{\infty}\phi_{kl}\mathscr{H}_{mn,kl}^{HW}=\sum_{k=0}^{\infty}\sum_{l=-\infty}^{\infty}\psi_{kl}\mathscr{G}_{mn,kl}^{HH} \tag{14}$$

where $\mathscr{H}_{mn,kl}^{HH}=\mathcal{H}_{mn,kl}^{HH}$, $\mathscr{G}_{mn,kl}^{HH}=\mathcal{G}_{mn,kl}^{HH}$ and $\mathscr{H}_{mn,kl}^{HW}$ is expressed as:

$$\mathscr{H}_{mn,kl}^{HW}=\sqrt{s}F^2\int_{-\infty}^{0}\int_{-\pi}^{\pi}\oint_{\Gamma}\mathscr{F}_{\xi}\left(\boldsymbol{x},\boldsymbol{\xi}\right)\mathcal{L}_m\left(-sz\right)\mathrm{e}^{-\mathrm{i}n\gamma}\mathrm{e}^{\mathrm{i}l\varphi}\cos\varphi\mathrm{d}s\mathrm{d}\gamma\mathrm{d}z \tag{15}$$

3.3 Fourier-Laguerre spectral method for the NM theory

Similarly, the boundary integral equation for the NM theory is expressed as:

$$\pi\phi_{mn}+\sum_{k=0}^{\infty}\sum_{l=-\infty}^{\infty}\phi_{kl}\mathbb{H}_{mn,kl}^{HH}-\sum_{k=0}^{\infty}\sum_{l=-\infty}^{\infty}\phi_{kl}\mathbb{H}_{mn,kl}^{HW}=\sum_{k=0}^{\infty}\sum_{l=-\infty}^{\infty}\psi_{kl}\mathbb{G}_{mn,kl}^{HH} \tag{16}$$

where $\mathbb{G}_{mn,kl}^{HH}=\mathcal{G}_{mn,kl}^{HH}$. In addition, $\mathbb{H}_{mn,kl}^{HH}$ and $\mathbb{H}_{mn,kl}^{HW}$ are expressed as:

$$\begin{aligned}\mathbb{H}_{mn,kl}^{HH}=&+\int_{-\infty}^{0}\int_{-\pi}^{\pi}\iint_{\Sigma^{\mathrm{H}}}\frac{\partial\mathscr{R}\left(\boldsymbol{x},\boldsymbol{\xi}\right)}{\partial n_{\boldsymbol{\xi}}}\mathcal{L}_m\left(-sz\right)\mathcal{L}_k\left(-s\zeta\right)\mathrm{e}^{-\mathrm{i}n\gamma}\mathrm{e}^{\mathrm{i}l\varphi}\mathrm{d}S\mathrm{d}\gamma\mathrm{d}z\\&-s\int_{-\infty}^{0}\int_{-\pi}^{\pi}\iint_{\Sigma^{\mathrm{H}}}\mathscr{F}_{\zeta}^{\xi}\left(\boldsymbol{x},\boldsymbol{\xi}\right)\mathcal{L}_m\left(-sz\right)\mathcal{L}_k'\left(-s\zeta\right)\mathrm{e}^{-\mathrm{i}n\gamma}\mathrm{e}^{\mathrm{i}l\varphi}\cos\varphi\mathrm{d}S\mathrm{d}\gamma\mathrm{d}z\\&+\frac{\mathrm{i}l}{\mathcal{R}}\int_{-\infty}^{0}\int_{-\pi}^{\pi}\iint_{\Sigma^{\mathrm{H}}}\mathscr{F}_{\eta}^{\xi}\left(\boldsymbol{x},\boldsymbol{\xi}\right)\mathcal{L}_m\left(-sz\right)\mathcal{L}_k\left(-s\zeta\right)\mathrm{e}^{-\mathrm{i}n\gamma}\mathrm{e}^{\mathrm{i}l\varphi}\mathrm{d}S\mathrm{d}\gamma\mathrm{d}z\end{aligned} \tag{17a}$$

$$\mathbb{H}_{mn,kl}^{HW}=-\sqrt{s}\int_{-\infty}^{0}\int_{-\pi}^{\pi}\oint_{\Gamma}\mathscr{R}_{\zeta}^{\xi}\left(\boldsymbol{x},\boldsymbol{\xi}\right)\mathcal{L}_m\left(-sz\right)\mathrm{e}^{-\mathrm{i}n\gamma}\mathrm{e}^{\mathrm{i}l\varphi}\cos\varphi\mathrm{d}s\mathrm{d}\gamma\mathrm{d}z \tag{17b}$$

3.4 Matrix-vector form

Subsections 3.1, 3.2 and 3.3 set forth the boundary integral equations based on the Fourier-Laguerre series for the NK, mNK and NM flow models. Expressions (12), (14) and (16) can be concisely expressed as

$$[\boldsymbol{H}]\cdot\{\boldsymbol{\Phi}\}=[\boldsymbol{G}]\cdot\{\boldsymbol{\Psi}\}=\{\boldsymbol{B}\} \tag{18}$$

Here, the vector $\{\boldsymbol{B}\}$ on the right-hand side of (18) is known according to the body boundary condition (1). Therefore, the implementation of the Fourier-Laguerre series yields a linear equation system with Fourier-Laguerre coefficients ϕ_{kl} as unknowns.

4 Discussions

4.1 Cancellation between hull surface integral and waterline integral

In subsection 2.3, the waterline integral is eliminated indicating that the waterline integral and hull surface integral in the mNK theory are partially cancelled out. By using asymptotic expansions for $g_\ell(\kappa)$ and $J_l(\kappa\mathcal{R})$, the integrand of $\mathscr{H}_{mn,kl}^{HH}$ for a large argument κ can be expressed as:

$$\widehat{\mathscr{H}}_{mn,kl}^{\mathrm{HH}}=-\left(l-n\right)\frac{2\pi s\mathcal{R}}{F^2\kappa^2}\sqrt{\frac{2}{\pi\kappa\mathcal{R}}}J_n\left(\kappa\mathcal{R}\right)\cos\left(\kappa\mathcal{R}-\frac{l-1}{2}\pi-\frac{\pi}{4}\right)+O\left(\kappa^{-4}\right) \tag{19}$$

and then, the asymptotic expression of the integrand of $\mathscr{H}_{mn,kl}^{HW}$ is expressed as:

$$\begin{aligned}\widehat{\mathscr{H}}_{mn,kl}^{\mathrm{H\Gamma}}=&-\left(l-n\right)\frac{2\pi s\mathcal{R}}{F^2\kappa^2}\sqrt{\frac{2}{\pi\kappa\mathcal{R}}}J_n\left(\kappa\mathcal{R}\right)\cos\left(\kappa\mathcal{R}-\frac{l-1}{2}\pi-\frac{\pi}{4}\right)\\&-\frac{2\pi ls}{\kappa^2}\sqrt{\frac{2}{\pi\kappa\mathcal{R}}}J_n\left(\kappa\mathcal{R}\right)\sin\left(\kappa\mathcal{R}-\frac{l-1}{2}\pi-\frac{\pi}{4}\right)+O\left(\kappa^{-4}\right)\end{aligned} \tag{20}$$

Representation $\widehat{\mathscr{H}}_{mn,kl}^{HH}$ is identical to the first expression in $\widehat{\mathscr{H}}_{mn,kl}^{HW}$ which means that the hull surface integral and waterline integral in the mNK theory are partially cancelled out. As $F\ll 1$, the first expression in (20) plays a dominant role and the cancellation of the leading terms is more eminent, which is consistent with [8].

4.2 Condition numbers of coefficient matrices

The condition number of coefficient matrices is now considered. Due to the fact that the mNK theory is mathematically equivalent to the NM theory, there are essentially two flow models. Figure 1 depicts condition numbers of coefficient matrices [**G**] and [**H**] for NK and NM theories as a function of F. When $F \leq 0.2$, condition numbers for NK and NM theories are almost at the same order. When $F > 0.3$, the difference between NK and NM theories is obvious. The condition numbers for the NK theory are very large, and matrices [**G**] and [**H**] are in orders of 10^5 and 10^4, respectively, indicating that the NK theory yields singular coefficient matrices. In contrast, condition numbers of matrices [**G**] and [**H**] for the NM theory are between 10^2 and 10^3. In addition, they keep stable with F. Therefore, the NM theory is comparatively well-behaved, it is adopted to study the wave-making problem by a vertical cylinder.

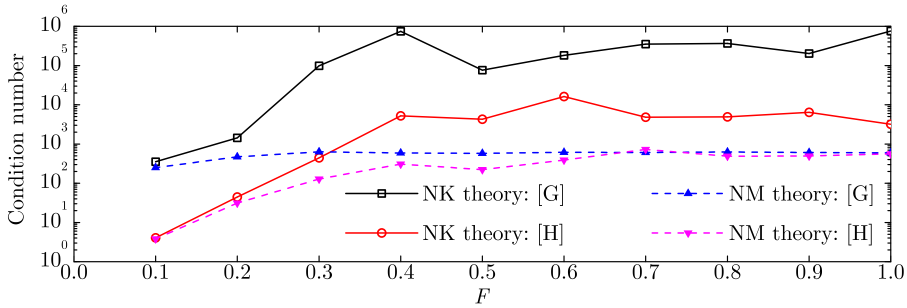

Figure 1: Condition numbers of coefficient matrices [**G**] and [**H**] for NK theory and NM theories.

4.3 Wave-resistance of a translating cylinder

The wave-making by a translating vertical cylinder is now considered. According to the body boundary condition $\psi = n_\xi = \cos\varphi$, the element of vector $\{\boldsymbol{B}\}$ for the mNK theory or NM theory is expressed as:

$$\mathscr{B}_{mn} = \mathcal{R} \int_{-\pi}^{\pi} \int_{-\infty}^{0} \int_{-\pi}^{\pi} \int_{-\infty}^{0} \mathcal{G}\left(\boldsymbol{x}, \boldsymbol{\xi}\right) \mathcal{L}_m\left(-sz\right) \mathrm{e}^{-\mathrm{i}n\gamma} \cos\varphi \mathrm{d}\zeta \mathrm{d}\varphi \mathrm{d}z \mathrm{d}\gamma \tag{21}$$

Solving the linear equation system given by (18) yields the Fourier-Laguerre coefficients ϕ_{kl}. We can get the velocity potential distribution over the hull surface using (11), and then the wave resistance experienced by the vertical cylinder based on Bernoulli's equation is given by:

$$d = 2\pi \sum_k \frac{(-1)^k}{\sqrt{s}} \left(\phi_{k,-2} + \phi_{k,2}\right) + \frac{\pi\mathcal{R}}{2} \sum_m \sum_n \sum_k \sum_l \delta_{n+l,\pm1} \phi_{mn} \phi_{kl} \left\{ \delta_{m,k} \left(\frac{nl}{\mathcal{R}^2} + \frac{s^2}{4} \right) - s^2 \left[\frac{1}{2} + \min\left(m, k\right) \right] \right\} \tag{22}$$

Application of the Fourier-Laguerre spectral method yields a very simple expression for the wave resistance given by (22). Results of wave resistance as well as generated wave patterns will be presented at the workshop.

References

[1] JV Wehausen, EV Laitone, Surface waves, *Hanbuch der Physik*, vol. 9, 1960.

[2] F Noblesse, Alternative integral representations for the Green function of the theory of ship wave resistance, *J. Engng Maths*, 15: 241–265, 1981.

[3] I Ten, H Liang, XB Chen, New formulations of the ship-motion Green function, *J. Engng Maths*, 2017.

[4] R Brard, The representation of a given ship form by singularity distributions when the boundary condition on the free surface is linearized, *J. Ship Res.*, 16: 79–92, 1972.

[5] P Gueval, P Vaussy, JM Kobus, The distribution of singularities kinematically equivalent to a moving hull in the presence of a free surface, *Int. Shipbuilding Progr.*, 21: 311–324, 1974.

[6] F Noblesse, F Huang, C Yang, The Neumann–Michell theory of ship waves, *J. Engng Maths*, 79: 51–71, 2013.

[7] H Liang, XB Chen, A new multi-domain method based on an analytical control surface for linear and second-order mean drift wave loads on floating bodies, *J. Comput. Phys.*, 347: 506–532, 2017.

[8] J He, F Noblesse, CJ Yang, W Li, G Delhommeau, The waterline integral in the Neumann–Kelvin theory of ship waves, *Proc. of 33rd Intl Workshop on Water Waves and Floating Bodies, Brest, France*, 2018.