Water impact near the edge of a floating ice sheet

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The two-dimensional water impact of a symmetric rigid body in a close proximity to the edge of an ice field is investigated. Initially, $t = 0$, the water is at rest and occupies the lower half-plane $y < 0$, see figure 1. The rigid body, $y = f(x)$, touches the water surface, $x < d$, $y = 0$, at a single point taken as the origin of the Cartesian coordinate system, $f(0) = 0$, $f(-x) = f(x)$, where $|x| < L$. The ice sheet, $x > d$, is modeled here by a semi-infinite articulated plate of constant thickness h_i. The plate is stationary where $x > d + l$. The part of the plate near the edge, where $d < x < d + l$, may rotate around $x = d + l$ due to the hydrodynamic loads caused by the impact. This configuration models deflection of the ice plate near its edge. The body enters water with an initial velocity V_0. The pressure distribution along the wetted part of the body surface is not symmetric due to the presence of the ice plate. Only vertical motion of the body is considered in this study. The position of the body is described by the equation $y = f(x) - h(t)$, $h(0) = 0$, $h'(0) = V_0$, where $h(t)$ is the body displacement which should be determined as part of the solution. The body velocity, $h'(t)$, decays in time due to the hydrodynamic force acting on the wetted surface of the body. The wetted area of the body increases in time and should be determined together with the flow, motion of the body, and the deflection of the ice sheet. The initial stage of the impact is considered, when the body displacement, $h(t)$, is of the order of the vertical dimension of the body, H. In this study, we assume that $H/L \ll 1$. Therefore, the vertical displacements of the water boundary are small and can be neglected compared with the horizontal characteristic length L during the initial stage. Then the boundary conditions can be approximately linearized and imposed at the initial level of the water boundary. Gravity, surface tension and the fluid viscosity are not taken into account. The resulting model is known as the Wagner model of water impact [1]. The novelty of the present study compared with the standard Wagner theory of water impact is due to the presence of another solid boundary of the flow region, the motion of which is caused by the pressures generated by the impact. The problem is coupled. In addition, the body may come in contact with the edge of the ice sheet, if $L > d$, and penetrate water pushing the ice edge downwards. Such a problem was not considered within the Wagner model.

The present study is motivated by slamming of a ship moving along the edge of an ice field or in broken ice [2], and by the effect of ice conditions on a conventional free-fall lifeboat [3].

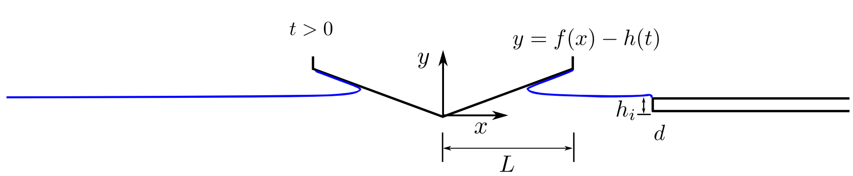

Fig. 1 Configuration of the problem

Ice conditions on the motion of a free-falling body

For safe operation of a conventional free-fall lifeboat in the presence of ice floes, the water in the place of the lifeboat entry should be free of floes. The presence of the floes near the impact place is expected to increase the lifeboat deceleration. We shall estimate the minimum distance from the lifeboat to the nearest floe, which does not affect significantly the body motion. To this aim, the two-dimensional water entry problem with semi-infinite rigid plate is considered, $l = 0$ and $d > L$.

Within the Wagner theory of water impact, the hydrodynamic force acting on a symmetric body is

$$F(t) = \frac{\mathrm{d}}{\mathrm{d}t}\left(m_a(t)h'(t)\right), \qquad m_a(t) = -\rho\int_{a(t)}^{b(t)} \varphi(x, 0, a, b)\mathrm{d}x,$$

where $m_a(t)$ is the added mass of an equivalent flat disc and $h'(t)$ is the vertical velocity of the body, $m_a(0) = 0$, $h'(0) = V_0$, ρ is the water density, and $\varphi(x, y, a, b)$ is the velocity potential of the flow caused by the flat plate, $a < x < b$, impacting water surface at unit speed. The equation of the body motion, $-Mh'' = F(t)$, can be integrated in time using the initial conditions with the result

$$h'(t) = \frac{MV_0}{M + m_a(t)},$$

where M is the mass of the body per unit length. the body deceleration in the two-dimensional problem is given by

$$-h''(t) = \frac{MV_0}{(M + m_a(t))^2}\frac{\mathrm{d}m_a}{\mathrm{d}t}.$$

For a parabolic contour, $y = x^2/(2R) - h(t)$, entering water without floating ice we have $-a(t) = b(t) = 2\sqrt{Rh(t)}$, $m_{as}(t) = (\pi/2)\rho b^2(t) = 2\pi\rho Rh(t)$. Then the maximum deceleration is achieved at the impact instant, $t = 0$, and it is equal to $2\pi\rho RV_0^2/M$. The added mass $m_{as}(t)$ can be also used in the presence of ice floes if the floes are far enough from the impact region.

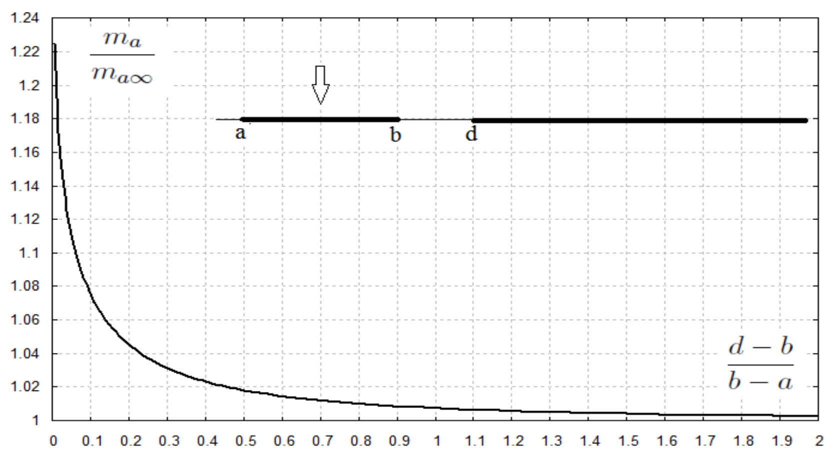

Fig. 2 M_a as a function of the relative gap length between the impacting plate (a, b) and the semi-infinite ice sheet (d, ∞).

The added mass when the wetted part of the body is close to the ice plate can be presented as

$$m_a = m_{a\infty}M_a(\omega), \qquad \omega = \frac{d-b}{b-a}, \qquad m_{a\infty} = \frac{\pi}{2}\rho(b-a)^2,$$

where $m_{a\infty}$ is the added mass of the "asymmetric" plate without ice floes, ω changes from ∞ at $t = 0$, when $b = a = 0$, to zero when the right contact point $x = b$ arrives at the edge of the ice plate, $x = d$. The function $M_a(\omega)$ is depicted in figure 2.

It is seen that the presence of the ice field can be neglected with the accuracy better that 1% when the gap between the plates is wider than the length of the impacting plate. The calculations are performed by using the theory of analytic functions and the Hilbert formulae. The calculations for $\omega = 0$ are performed separately without the gap between the plates with the result $M_a(0) = 16/\pi^2 \approx 1.6211$. This value can be achieved also numerically in the limit of a small gap between the plates. However, the Wagner theory with linearized free surface boundary conditions is unlikely to be valid in such a limit. Note that the added mass is independent of the shape of the body.

Wetted part of the body entering water near the ice edge

The wetted part of the body was calculated for a parabolic contour, $y = x^2/(2R) - h(t)$, by using the displacement potential and the conditions that the liquid displacements are finite at the contact points $x = a(t)$ and $x = b(t)$. The non-dimensional coordinates of the contact points, a/d and b/d, where d is the distance of the impact point from the edge of the semi-infinite rigid plate, are shown in figure 3 as the functions of the non-dimensional penetration depth $\hat{h} = 4Rh/d^2$ by solid lines. The asymmetry of the contact line is well visible even for relatively large gaps between the body and the ice sheet. The corresponding coordinates without the presence of the ice, $x_W = \pm\sqrt{\hat{h}}$, are shown by the dashed lines. The dotted lines depict the coordinates of the contact points without account of the free surface elevation.

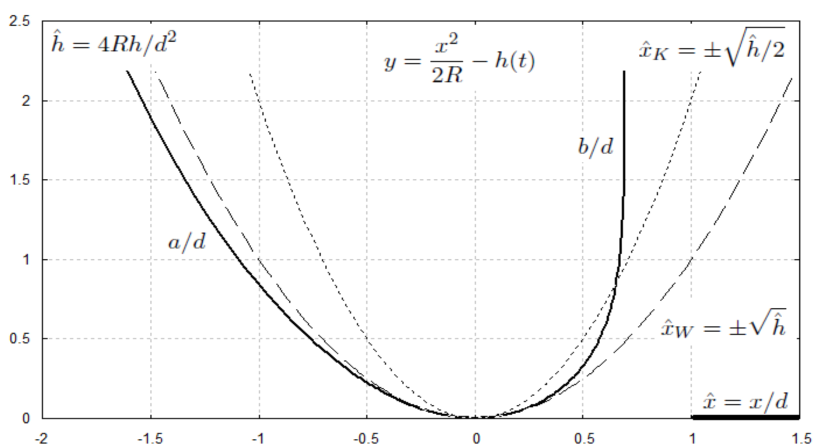

Fig. 3 The coordinates of the contact points calculated by different models.

We may conclude that the vertical motion of a body entering water near an ice field is not affected significantly by the presence of the ice. However, the flow between the entering body and the ice sheet is strongly dependent on the ice presence, which may result in highly asymmetric pressure distribution in the wetted area of the body and significant hydrodynamic torque caused by the ice presence.

CFD results for water impact near ice edge

The problem of a wedge impacting near a semi-infinite ice sheet is also solved numerically with a VOF-based Navier-Stokes solver from the OpenFOAM library. The effect of the ice sheet is studied by computing the flow for different values of gap between the ice and body ω. The deadrise angle is β. The configuration of the problem can be seen in figure 1. The influence of the ice on the body can be quantified with the ratio of the maximum force during the time series of the case with ice to that without ice:

$$M_f(\omega) = \frac{F_{\max}}{F^*_{\max}}, \qquad \omega = \frac{d-b}{b-a},$$

where $F^*_{\max}$ is the maximum force from the case without ice. The function M_f is plotted in figure 4. As can be seen, the influence of the ice sheet decreases rapidly as relative gap size increases. The

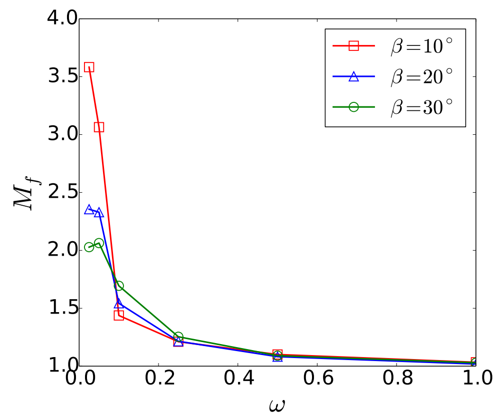

Fig. 4 M_f as a function of the relative gap length ω between the impacting body and the semi-infinite ice sheet.

maximum force ratio depends on the deadrise angle. For very small gap, $\omega < 0.07$, the smaller deadrise produces a larger maximum force. For larger gap, $\omega > 0.07$, the larger deadrise angle wedge has a larger maximum force.

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