The waterline integral in the Neumann-Kelvin theory of ship waves

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Highlight

A major feature of the classical Neumann-Kelvin (NK) theory of ship waves in calm water is that it involves an integral around the mean waterline of the ship hull surface. This waterline integral contains two terms. One of these two terms does not correspond to a consistent linear flow model, and can then be ignored. An elementary analysis shows that the remaining second term in the waterline integral in the *consistent* NK theory largely cancels out the hull-surface integral in both the low-Froude number limit and the short-wave limit, and therefore may not be ignored. These results likely explain the difficulties, widely reported in the literature, associated with numerical solutions of the NK theory, and corroborate the approach used in the Neumann-Michell theory, where the waterline integral in the consistent NK theory is combined with the hull surface integral via a mathematical transformation.

1. Introduction

The flow around a ship of length L that travels at a constant speed V along a straight path, in calm water of large depth and horizontal extent, is considered within the usual framework of linear potential flow theory. This theoretical framework is realistic and useful for most practical purposes as is well documented; e.g. [1-5]. The Froude number F is defined as $F \equiv V/\sqrt{gL}$ where g denotes the acceleration of gravity.

The flow due to the ship is observed in a system of orthogonal coordinates $\mathbf{X} \equiv (X, Y, Z)$ attached to the moving ship. The undisturbed free surface is chosen as the plane $Z = 0$ with the Z axis directed upward, and the X axis is taken along the ship path and directed toward the ship bow. The flow thus appears steady with flow velocity given by the sum of the apparent uniform current $(-V, 0, 0)$ that opposes the ship speed V and the (disturbance) flow velocity given by the gradient (Φ_X, Φ_Y, Φ_Z) of the flow potential $\Phi(\mathbf{X})$. The length L and the speed V of the ship are used to define the nondimensional coordinates $\mathbf{x} \equiv \mathbf{X}/L$, flow potential $\phi \equiv \Phi/(VL)$ and flow velocity $(\phi_x, \phi_y, \phi_z) \equiv (\Phi_X, \Phi_Y, \Phi_Z)/V$.

The mean wetted hull surface of the ship is denoted Σ. This surface intersects the undisturbed free-surface plane $z = 0$ along the mean waterline Γ. The unit vector $\mathbf{n} \equiv (n^x, n^y, n^z)$ is normal to the hull surface Σ and points into the water (outside the ship). Within the linear potential flow analysis considered here, the flow around Σ is expressed in terms of a Green function $G(\mathbf{x}, \boldsymbol{\xi})$ that satisfies the radiation condition and the Kelvin-Michell linearized boundary condition at the free surface $z = 0$, and represents the (nondimensional) velocity potential of the flow created at a flow-field point $\mathbf{x} \equiv (x, y, z)$ by a unit source located at a source point $\boldsymbol{\xi} \equiv (\xi, \eta, \zeta)$.

2. Classical (Brard-Guevel) and consistent Neumann-Kelvin theories

The Neumann-Kelvin theory proposed by Brard [6] and Guevel [7] expresses the flow potential $\phi \equiv \phi(\mathbf{x})$ at a flow field point $\mathbf{x}$ via the boundary integral representation

$$\phi = \phi^H + \psi^{BG} \quad \text{where} \quad \phi^H \equiv \int_\Sigma G\, n^x da \tag{1a}$$

$$\text{and} \quad \psi^{BG} \equiv F^2 \int_\Gamma \frac{(\phi\, G_\xi - G\, \phi_\xi)\, n^x}{\sqrt{(n^x)^2 + (n^y)^2}}\, d\ell - \int_\Sigma \phi\, \mathbf{n} \cdot \nabla G\, da \tag{1b}$$

Here, da and $d\ell$ denote the differential elements of area or length at a point $\boldsymbol{\xi}$ of the hull surface Σ or the mean waterline Γ. The components ϕ^H and ψ^{BG} in (1) correspond to the classical Hogner approximation given in [8] and the Brard-Guevel correction of ϕ^H.

An important feature of the classical NK flow representation (1) is that the Brard-Guevel potential ψ^{BG} contains an integral around the mean waterline Γ of the ship hull. The term $G\, \phi_\xi$ in the waterline integral (1b) is shown in [1] to correspond to an inconsistent linear flow model, and may then be ignored in a consistent linear flow model. The resulting consistent NK theory is associated with the modified

NK boundary integral flow representation

$$\phi = \phi^H + \psi \quad \text{where} \quad \psi \equiv F^2\!\int_\Gamma \frac{\phi\, G_\xi\, n^x\, d\ell}{\sqrt{(n^x)^2+(n^y)^2}} - \int_\Sigma \phi\,(G_\xi n^x + G_\eta n^y + G_\zeta n^z)\, da \tag{2}$$

and ϕ^H is given by (1a). The consistent NK flow representation (2) is now considered.

3. Wave potential in the consistent Neumann-Kelvin theory

The Green function G can be formally expressed as $G = L + W$ where L denotes a non-oscillatory local flow component, which can readily be evaluated via the simple global analytical approximation given in [9,10], and W represents the waves contained in G. Similarly, the flow potential $\phi \equiv \phi(\mathbf{x})$ at a flow-field point $\mathbf{x}$ can be expressed as

$$\phi = \phi_L + \phi_W \approx \phi_L^H + \phi_W^H + \psi_W \tag{3}$$

where ϕ_L^H and ϕ_W^H denote the local flow and wave components of the Hogner potential ϕ^H, and the local flow component ψ_L is ignored here, as in [1], because this component is negligible for typical nonlifting displacement ships. The wave potential ϕ_W is considered hereinafter.

This wave potential can be expressed as

$$\phi_W \equiv \phi_W^H + \psi_W \equiv \phi_W^H + \psi_W^x - \psi_W^{yz} \tag{4a}$$

The Hogner component ϕ_W^H is defined by (1a) where G is taken as W, and the components ψ_W^x and ψ_W^{yz} associated with the potential ψ given by (2) are defined as

$$\psi_W^x \equiv F^2\!\int_\Gamma \frac{\phi\, W_\xi\, n^x\, d\ell}{\sqrt{(n^x)^2+(n^y)^2}} - \int_\Sigma \phi\, W_\xi\, n^x\, da \quad \text{and} \quad \psi_W^{yz} \equiv \int_\Sigma \phi\,(W_\eta n^y + W_\zeta n^z)\, da \tag{4b}$$

The wave potential ψ_W^x combines the contributions of the waterline integral and part of the hull-surface integral in the consistent NK flow representation (2).

The wave component W in the Green function G is given in e.g. [1,9-11] as

$$W = \frac{H}{\pi F^2}\, \mathrm{Im}\!\int_{-q_\infty}^{q_\infty} \Lambda\, E\, \mathcal{E}\, dq \tag{5a}$$

where $H \equiv H(\xi - x)$ is the Heaviside unit-step function, Im means that the imaginary part is considered, and F is the Froude number. Moreover, the finite limits of integration $\pm q_\infty$ and the function Λ filter unrealistic short waves, and E and $\mathcal{E}$ are the elementary wave functions

$$E \equiv \mathrm{e}^{(1+q^2)z/F^2 + \mathrm{i}\, q_*(x+q\,y)/F^2} \quad \text{and} \quad \mathcal{E} \equiv \mathrm{e}^{(1+q^2)\,\zeta/F^2 - \mathrm{i}\, q_*(\xi+q\,\eta)/F^2} \quad \text{where} \quad q_* \equiv \sqrt{1+q^2} \tag{5b}$$

The wave potential ϕ_W defined by (4) is then given by the Fourier-Kochin representation

$$\phi_W = \frac{1}{\pi F^2}\, \mathrm{Im}\!\int_{-q_\infty}^{q_\infty} \Lambda\, A\, E\, dq \tag{6}$$

where the wave-amplitude function $A \equiv A(q;x)$ can be expressed as

$$A = A^H + \mathrm{i}\,(A^{yz} - A^x)\, q_*/F^2 \quad \text{where} \quad q_* \equiv \sqrt{1+q^2} \tag{7a}$$

Here, the components A^H, A^{yz} and A^x are defined as

$$A^H \equiv \int_\Sigma H\, n^x\, \mathcal{E}\, da \quad , \quad A^{yz} \equiv \int_\Sigma H\, \phi\, (q\, n^y + \mathrm{i}\, q_* n^z)\, \mathcal{E}\, da \tag{7b}$$

$$\text{and} \quad A^x \equiv F^2\!\int_\Gamma \frac{H\, \phi\, n^x\, \mathcal{E}\, d\ell}{\sqrt{(n^x)^2+(n^y)^2}} - \int_\Sigma H\, \phi\, n^x\, \mathcal{E}\, da \tag{7c}$$

The amplitude function A^H and the functions A^{yz} and A^x in (7) correspond to the Hogner wave potential ϕ_W^H and the NK wave potentials ϕ_W^{yz} and ϕ_W^x in (4). The amplitude function A^x is related to the waterline integral and part of the hull-surface integral in the consistent NK flow representation (2).

4. Waterline integral in the consistent Neumann-Kelvin theory

The importance of the integral around the waterline Γ in (7c) can readily be estimated for a ship hull surface Σ with a constant draft $d \equiv D/L$ and rectangular framelines, for which one has

$$A^x = \int_\Sigma H(A^\Gamma \phi^\Gamma - \phi)\, n^x\, \mathcal{E}\, da \quad \text{where} \quad 1 \le A^\Gamma \equiv (1+q^2)/(1-e^{-(1+q^2)\,d/F^2}) \le 1+q^2 \tag{8}$$

and ϕ^Γ denotes the value of the potential ϕ at the waterline Γ.

In the low Froude number limit $1 \ll d/F^2$ or the short-wave limit $1 \ll q$, (8) yields $A^\Gamma \approx 1+q^2$ and

$$A^x \approx q^2 \int_\Sigma H\, \phi\, n^x\, \mathcal{E}\, da \tag{9}$$

Thus, in the limit $1 \ll q$, the waterline integral dominates the hull-surface integral in (7c). Moreover, expression (7b) for A^{yz}, the identities $(n^x, n^y, n^z) = (-t^y, t^x, 0)$ and the approximation (9) yield

$$A^{yz} - A^x \approx q \int_\Sigma H\, \phi\, (t^x + q\, t^y)\, \mathcal{E}\, da \approx \frac{F^2 q}{1+q^2} \int_\Gamma H\, \phi^\Gamma (t^x + q\, t^y)\, \mathrm{e}^{-\,\mathrm{i}\, q_* (\xi + q\, \eta)/F^2} d\ell \tag{10}$$

The dominant contribution to the waterline integral (10) stems from the points of stationary phase of the trigonometric function, i.e. from the points where $d\xi/d\ell + q\, d\eta/d\ell \equiv t^x + q\, t^y$ vanishes. The dominant contributions to the functions A^{yz} and A^x therefore cancel out in the limit $1 \ll (1+q^2)\, d/F^2$.

The foregoing elementary analysis shows that the waterline integrals in (4b) and (2) may not be ignored. This analysis also corroborates the approach used in [1] where the integrals around the waterline Γ and over the hull surface Σ in (4b) are combined and expressed as a hull-surface integral via a mathematical transformation. Specifically, this transformation is based on Stokes' theorem and a vector wave function $\mathbf{W} \equiv (\,0\,, W_z^x\,, -W_y^x\,)$ associated with the scalar wave function W in the Green function $G \equiv W + L$ via the relation $\nabla \times \mathbf{W} = \nabla W$.

5. Neumann-Michell theory

Specifically, the wave amplitude function A in (6) is expressed as the hull-surface integral

$$A = \int_\Sigma H [\, n^x + (q\, \nu^y + \mathrm{i}\, q_* \nu^z)\, \phi_t + n^x (q\, \nu^z - \mathrm{i}\, q_* \nu^y)\, \phi_d\,]\, \mathcal{E}\, da \tag{11}$$

in the Neumann-Michell (NM) theory given in [1]. Here, ϕ_d and ϕ_t denote the derivatives of the potential ϕ along two unit vectors $\mathbf{d}$ and $\mathbf{t}$ that are tangent to the ship hull surface Σ and defined as

$$\mathbf{d} \equiv (0, -\nu^z, \nu^y) \text{ and } \mathbf{t} \equiv [\sqrt{(n^y)^2 + (n^z)^2}, -n^x \nu^y, -n^x \nu^z] \text{ where } (\nu^y, \nu^z) \equiv (n^y, n^z)/\sqrt{(n^y)^2 + (n^z)^2}$$

Expressions (7b) and (9) show that the integrands of the hull-surface integrals $A^{yz}\sqrt{1+q^2}$ and $A^x\sqrt{1+q^2}$ in the representation (7a) of ship waves within the consistent NK theory are $O(q^2)$ or $O(q^3)$ in the short-wave limit $1 \ll q$, whereas the integrand of the NM correction to the Hogner approximation in the hull-surface integral (11) is $O(q)$ in the limit $1 \ll q$. Major difficulties, reported in the literature, related to numerical solutions of the NK theory are then reduced within the NM theory. The NM representation (11) expresses the wave amplitude function A in (6) in terms of the (given) normal flow velocity $\phi_n \equiv n^x$, which is related to the Hogner potential ϕ^H in (2), and the tangential flow velocities ϕ_d and ϕ_t instead of the flow potential ϕ in the NK representation (2).

6. Validation, applications, extensions of the Neumann-Michell theory

Thus, the NM theory is a modification of the classical NK theory that, unlike the NK theory, does not involve a line integral around the mean ship waterline and corresponds to a consistent linear potential flow model. Validation studies of the NM theory are reported in [2-5] and several other studies, listed in [5]. An important feature of the theory is that it is a very practical. Specifically, the flow around a ship hull can be evaluated in about 1sec, using a common PC, in the NM theory. Moreover, the theory yields realistic flow predictions that are sufficiently accurate for most practical purposes, notably for early design and optimization, and compare favorably with far more complicated CFD methods;

[4,5]. The NM theory is then well suited for routine practical applications to ship design and hull-form optimization, and indeed has been widely used for optimization in numerous studies, listed in [5].

Three computer codes that solve the NK integro-differential equation to determine the flow potential ϕ at a ship hull surface Σ, and a fourth code that directly determines the tangential flow velocities ϕ_d and ϕ_t at Σ by solving a pair of coupled integral equations (readily obtained from the NK integro-differential equation for ϕ), exist so far. The surface Σ is discretized via flat triangles, and the iterative solution procedure given in [1,2] is used, in three of these four existing codes; The fourth code is based on a direct solution procedure and a NURB representation of Σ. The predictions given by these four numerical implementations of the NM theory are consistent, and compare favorably with the predictions given by panel methods that solve the Laplace equation (with linear or nonlinear free-surface boundary conditions) via distributions of Rankine sources on Σ and the free surface, as well as other alternative numerical methods that solve the Euler, RANS or URANS flow equations.

6. Concluding remarks about linearization

As is shown in [1], the narrow band of water bounded by the undisturbed free-surface plane $z = 0$ and the linear approximation $z = F^2\phi_x$ to the actual free surface yields a *linear* contribution to the hull-surface integral associated with the boundary condition at the ship hull surface, and this linear 'hull-boundary-condition' contribution must therefore be retained within the framework of a consistent linearization. Moreover, [1] shows that this linear contribution, ignored in the classical Brard-Guevel NK theory, precisely cancels out the term $G\phi_\xi$ in the waterline integral (1b). Thus, the common practice of enforcing the body-surface boundary condition up to the undisturbed free-surface plane $z = 0$ should not be automatically followed to formulate linear boundary-value problems for free-surface flows around ships and offshore floating structures.

It may be interesting to consider the formulation of consistent linear boundary-value problems for various free-surface flows around floating bodies. In particular, for the usual radiation and diffraction problems of a rigid floating structure (without mean forward speed) associated with time-harmonic motions of small amplitude ϵ, the contribution of the narrow band of water between the undisturbed free-surface plane $z = 0$ and the linear approximation to the actual free surface to the hull-surface integral associated with the boundary condition at the body surface is $O(\epsilon^2)$, i.e. is nonlinear. Evidently, this nonlinear contribution should be ignored in a consistent linear theory, as is indeed done in the classical linear theory of the radiation of water waves by floating structures.

References

[1] F. Noblesse, F. Huang, C. Yang. The Neumann-Michell theory of ship waves, J Eng. Math. 79:1 (2013) 51-71

[2] F. Huang, C. Yang, F. Noblesse. Numerical implementation and validation of the Neumann-Michell theory of ship waves. European J Mech. / B Fluids 42 (2013) 47-68

[3] C. Yang, F. Huang, F. Noblesse. Practical evaluation of the drag of a ship for design and optimization, J Hydro./B 25:5 (2013) 645-654

[4] C. Ma, C. Zhang, F. Huang, C. Yang, X. Gu, W. Li, F. Noblesse. Practical evaluation of sinkage and trim effects on the drag of a common generic freely floating monohull ship, Applied Ocean Research 65 (2017) 1-11

[5] C. Ma, Y. Zhu, J. He, C. Zhang, D. Wan, C. Yang, F. Noblesse. Nonlinear corrections of linear potential-flow theory of ship waves, European J Mech. / B Fluids 67 (2018) 1-14

[6] R. Brard. The representation of a given ship form by singularity distributions when the boundary condition on the free surface is linearized, J. Ship Research 16 (1972) 79-92

[7] P. Guevel, P. Vaussy, J.M. Kobus. The distribution of singularities kinematically equivalent to a moving hull in the presence of a free surface. Il Shipbuilding Progress 21 (1974) 311-324

[8] E. Hogner. Hydromech. Probl. d. Schiffsantriebs, Herausgeg.v.Kempf u. E. Foerster, Hamburg (1932) 99-114

[9] F. Noblesse, G. Delhommeau, F. Huang, C. Yang. Practical mathematical representation of the flow due to a distribution of sources on a steadily-advancing ship hull, J Eng. Math. 71 (2011) 367-392

[10] H. Wu, C. Zhang, C. Ma, F. Huang, C. Yang, F. Noblesse. Errors due to a practical Green function for steady ship waves, European J Mech. / B Fluids 55 (2016) 162-169

[11] F. Noblesse. Alternative integral representations for the Green function of the theory of ship wave resistance, J Eng. Math. 15 (1981) 241-265