Hybrid modeling of wave structure interaction with overlapping viscous-inviscid domains

Jeffrey C. Harris[1], Christopher M. O'Reilly[2,3], Amin Mivehchi[2], Konstantin Kuznetsov[1], Christian F. Janssen[4], Stephan T. Grilli[2], Jason M. Dahl[2]

(1) LHSV, Ecole des Ponts, CEREMA, EDF R&D, Université Paris-Est, Chatou, France
(2) Dept. of Ocean Engineering, University of Rhode Island, Narragansett, RI 02882, USA
(3) Navatek Ltd., South Kingstown, RI 02879, USA
(4) Fluid Dynamics and Ship Theory Inst., Hamburg University of Technology (TUHH), Germany

Email address of presenting author: `jeffrey.harris@enpc.fr`

Highlights

- Modeling of wave-structure interaction, particularly for studying forces on ships, is presented using a hybrid viscid-inviscid approach and overlapping domains
- Fully nonlinear potential flow provides the inviscid solution, solved with a boundary element approach, using cubic B-spline elements, and accelerated with the parallel fast multipole method
- The local viscous solution around the object is solved with a Navier-Stokes code; here this is demonstrated with an efficient Lattice Boltzmann approach, written strictly in terms of the viscous flow.

Introduction

Wave-structure interaction continues to be a major aspect of offshore engineering; here we report on recent progress of a three-dimensional hybrid model for naval hydrodynamics problems based on a perturbation method, in which both velocity and pressure are expressed as the sum of an inviscid flow with a viscous perturbation, using overlapping domains. The far-field solution is provided using an inviscid solver based on the boundary element method (BEM), solving for fully nonlinear potential flow theory. In the near-field, for a smaller domain near the body, viscous flow is solved with a Navier-Stokes (NS) model based; here this is done with a Lattice Bolztmann Method (LBM), capturing turbulence with a large-eddy simulation (LES) approach and turbulent wall model. We apply these methods to two typical problems: interaction of waves with monopiles, and modeling of the wake of a ship.

Nonlinear wave modeling with BEM has been used ever since the seminal work of Longuet-Higgins and Cokelet [13], but despite improvements in speed and accuracy, one still is limited to potential flow. Model coupling, however, would allow for connecting this fully nonlinear potential flow model to some CFD approach, such that potential flow is considered where it is physically relevant and faster or more accurate, and a NS model is used close to a body where viscosity and possibly wave breaking need to be considered. This logic has been the rational for initial developments for model coupling in the field (see, e.g., Grilli [5] for a review). Such models have already been applied to surfzone dynamics problems (e.g., Lachaume et al. [12], Biausser et al. [1]), wave structure interaction problems (e.g., Corte and Grilli [2]), and to model wave-induced flows and resulting sediment suspension over objects on the seabed (e.g., Gilbert et al. [3]).

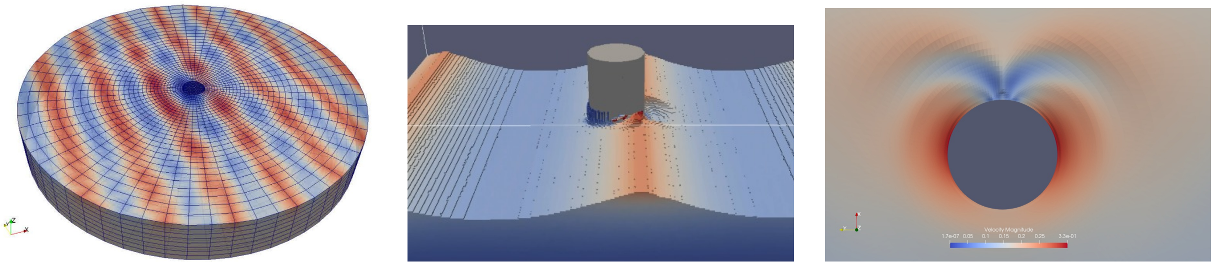
Theory

Coupled equations There are many approaches to coupling viscous and inviscid domains. One approach to model coupling is a decomposition of the total viscous flow (velocity u_i and pressure p) into the sum of the latter inviscid free-stream flow (u_i^I; p_I, which satisfy the Euler equations) and a defect or perturbation flow (u_i^P; p_P) [8]: $u_i = u_i^I + u_i^P$ and $p = p_I + p_P$. Substituting these values into the NS equations, and subtracting the Euler equations, we derive the governing equations for the "perturbation" fields as:

$$\frac{\partial u_i^P}{\partial x_i} = 0 \quad ; \qquad \frac{\partial u_i^P}{\partial t} + \frac{\partial}{\partial x_j}\left(u_i u_j - u_i^I u_j^I + \frac{p_P}{\rho}\delta_{ij} - \nu \frac{\partial u_i}{\partial x_j}\right) = 0. \tag{1}$$

Here the perturbation is potentially large, and defined in a region encompassing the boundary layer and wake of the structure of interest.

Fully nonlinear potential flow Our current approach is based on a higher-order approach similar to that of Grilli et al. [6], which has been used to model numerous applications, from landslide-generated tsunamis, rogue waves, waves generated by a surface effect ship, and the initiation of wave breaking. In the most recent incarnation of this type of model, we make use of cubic B-splines to represent the boundary geometry and field variables. For an incompressible



(a) (b) (c)

Figure 1: Different simulation results for nonlinear waves interacting with a cylinder, either: (a) BEM results for large monopile, $kR \approx 1.4$; (b) LBM result for coupled simulation with local wave breaking; and (c) BEM results for thin monopile just before wave breaking, $kR = 0.245$.

inviscid fluid with irrational motion, mass conservation is equivalent to the Laplace equation, $\nabla^2\phi = 0$, for the velocity potential, ϕ, such that the inviscid velocity $\mathbf{u}^I = \nabla\phi$. The Laplace equation is solved as a boundary integral expressed at each collocation point $\mathbf{x}_l$ ($l = 1, \ldots, N$),

$$\alpha(\mathbf{x}_l)\phi(\mathbf{x}_l) = \int \left[\frac{\partial\phi}{\partial n}(\mathbf{x})G(\mathbf{x}, \mathbf{x}_l) - \phi(\mathbf{x})\frac{\partial G}{\partial n}(\mathbf{x}, \mathbf{x}_l) \right] d\Gamma \tag{2}$$

where G is the free space Green's function based on the distance to point l, $\mathbf{r}_l = |\mathbf{x}_l - \mathbf{x}|$, α is the interior solid angle made by the boundary at a collocation point l (e.g., for a smooth boundary this would be 2π), and $\mathbf{n}$ points in the direction of the local outwards normal vector to the boundary. As this equation, written for each gridpoint, results in a dense $N \times N$ linear system of equations, the fast multipole method [4] can be used to accelerate this procedure.

Navier-Stokes model With the LBM, the macroscopic NS equations are modeled by solving mesoscopic equations on a lattice (i.e., grid), through distributions functions (DF), e.g., $f(\mathbf{x}, t, \xi)$, which represent the probability to find a particle at location $\mathbf{x}$, at time t with velocity ξ. All field variables (e.g., velocity, pressure), can be related to moments of these DFs. The time-evolution to be solved takes the form of a Boltzmann advection-collision equation,

$$\frac{Df_\alpha}{Dt} = \frac{\partial f_\alpha(\mathbf{x}, t)}{\partial t} + \mathbf{e}_\alpha \cdot \frac{\partial f_\alpha(\mathbf{x}, t)}{\partial \mathbf{x}} = \Omega_\alpha + B'_\alpha \tag{3}$$

where $\mathbf{e}_\alpha$ represents the discrete particle velocities, Ω_α is a collision operator which represents particle interactions, and B'_α represents volume forces such as gravity. As the LBM is weakly compressible, all operations are local, and it is efficiently parallelized on GPUs.

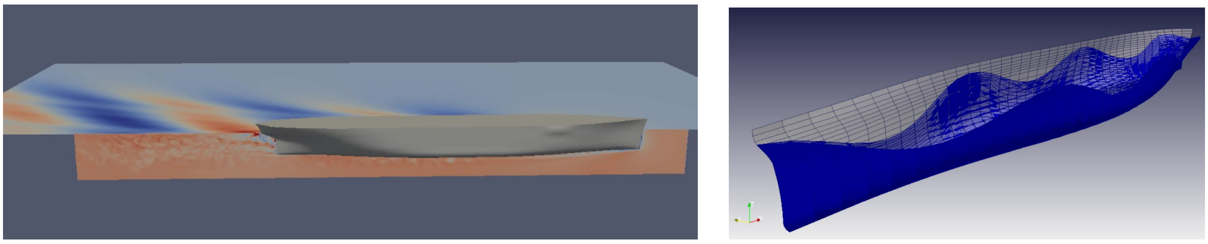
In the hybrid LBM scheme, the DFs are decomposed into their inviscid and perturbation parts, $f(\mathbf{x}, t, \xi) = f^I(\mathbf{x}, t, \xi) + f^P(\mathbf{x}, t, \xi)$, where the inviscid component can be constructed from the potential flow solution. The coupled Eqs. 1 are satisfied by modeling the particle collisions within Ω_α using the equilibrium function,

$$f_\alpha^{eq,P} = w_\alpha \left[\rho^P + \rho_o \left(3\frac{\mathbf{u}^P \cdot \mathbf{e}_\alpha}{c^2} + \frac{9}{2}\frac{(\mathbf{e}_\alpha \cdot \mathbf{u}^P)^2 + 2(\mathbf{e}_\alpha \cdot \mathbf{u}^P)(\mathbf{e}_\alpha \cdot \mathbf{u}^I)}{c^4} - \frac{3}{2}\frac{(\mathbf{u}^P)^2 + 2\mathbf{u}^P \cdot \mathbf{u}^I}{c^2} \right) \right] \tag{4}$$

with a lattice dependent directional weighting w_α, particle speed c, and density ρ. One can show that Eqs. 1 are recovered by using a Champan-Enskog expansion, after applying traditional LBM theory which relates f_α to hydrodynamic quantities such as the velocity, $\mathbf{u}$.

The perturbation LBM model uses a large eddy simulation method to model the subgrid scale turbulence. There is also a specialized wall function used to reduce the grid resolution requirements near a solid boundary (e.g., ship hull). It uses a generalized log-layer velocity profile from Musker [14] to relate the velocity in the fluid domain neighboring a wall to the stress on the wall. The implicit relation is solved using a Newton iteration scheme; it is used to determine the velocity and eddy viscosity within the boundary layer and the shear force on the boundary. This wall model has been generalized for curved boundaries that may be represented using NURBS surfaces (Fig. 2a).

A more complete description of the LBM model used in the coupled perturbation equations can be found in O'Reilly et al. [15]. The free surface is resolved with a volume-of-fluid (VOF) approach, thus can handle wave breaking. It has been developed considering the free surface boundary conditions of the hybrid decomposition in which the inviscid and perturbation free surfaces may not be coincident at all time steps.



(a) (b)

Figure 2: Hybrid LBM result for flow around a DTMB series 60 ship at $Fr = 0.3$; and demonstration of the regridding required for fully nonlinear BEM modeling.

Results

Interaction with large or thin monopiles A classic problem is to consider the diffraction of regular waves by a bottom-mounted vertical cylinder. With potential flow alone, we are able to easily compute (Fig. 1a) the conditions for a cylinder of 10 m radius in a depth of 40 m; with incoming waves of 1.0 m height and a period of 5.3 s (corresponding to $kR \approx 1.4$ and $kA \approx 0.07$), and we note both good agreement ($< 5\%$ error) with the quadratic transfer function solution given by Kim and Yue [10], and also we note that no filtering is required on the free-surface. In comparison, for thin monopiles, Harris et al. [9] previously reported that filtering was required to prevent the model from crashing. We note that this is because of the surface Keulegan-Carpenter number ($KC = \pi(kA)/(kR)$); for a large monopile, this value is low, but for large KC numbers, viscous forces become more important. If we test a thin monopile, but with no filtering, at high-resolution with cubic B-spline elements, we obtain a local wave breaking (Fig. 1c), of a form similar to the highly nonlinear type-2 waves observed by Swan and Sheikh [17].

For comparison, it is possible with the LBM model to model wave impact on a monopile directly; in Fig. 1b, initial results are shown using the hybrid approach, but for the inviscid approach, stream function wave theory (i.e., without including the inviscid diffraction from the cylinder) is used directly insteady of the BEM solution, to avoid issues of differing free-surfaces between the inviscid and viscous models. Much of this difference is due to local wave breaking that is seen; in two-dimensions, damping terms for breaking waves in inviscid models has been considered by Subramani et al. [16], or by Guignard and Grilli [7]; this type of absorption has not been extensively studied in three dimensions, though. Future improvements of the hybrid model should include two-way coupling to extract the proper amount of physical energy out of the inviscid BEM model, particularly to account for local breaking.

Wake of a DTMB series 60 hull For a more complex problem, in Fig. 2, the hybrid LBM model is used to simulate flow past a DTMB series 60 hull at a Froude number of 0.3. In this simulation, instead of the the nonlinear numerical wave tank (NWT) described above, a linear code, AEGIR [11], is used, with the wetted hull discretized with 40 by 10 higher-order elements; initially, the perturbation flow is taken as zero, and then as the LBM moves forward in time, the turbulent wake develops. The reason for using a linear solution in an initial simulation is to provide a simple free-surface definition, with a waterline taken to be at $z = 0$, but for the fully nonlinear NWT, it is necessary to properly regrid the surface of this curved surface at each timestep.

At present, the BEM-NWT solution is setup assuming that the walls are vertical. In this case, we have the traditional kinetic free surface boundary condition which can be written compactly as $\eta_t = \phi_z - \nabla\phi \cdot \nabla\eta$ (not considering surface pressure), but if we have an object with a curved wall, we no longer want to track $\eta(x, y, t)$, but a point which moves along the hull (i.e., with a changing (x,y) coordinate). If the hull is already discretized with some higher order mesh, where we can define a local coordinate system (s, m, n), as many industrial shapes are, then we can write the evolution of the waterline as a function of these local coordinates. If we take the s-direction to be moving up and down, then following the approach of Zhang and Kashiwagi [18], we have:

$$\frac{ds}{dt} = \frac{\eta_t}{z_s - x_s\eta_x - y_s\eta_y} = \frac{\phi_z - \nabla\phi \cdot \nabla\eta}{z_s - x_s\eta_x - y_s\eta_y}. \tag{5}$$

The incorporation of these more general regridding routines into the full NWT, and further into the hybrid model, is ongoing. In Fig. 2b, we demonstrate the concept of the remeshing, whereby we have an original hull mesh, and as we adjust the free-surface by assuming a theoretical variation of η, we see that the mesh retains a reasonable form.

Summary

This model is a work-in-progress for naval hydrodynamics and ocean engineering problems. The main advantage of this hybrid approach is its ability to use a smaller domain to solve the NS equations relative to standard solvers, hence allowing for both higher resolution and efficiency. The ultimate goal here is to model seakeeping problems for multiple degree-of-freedom floating bodies advancing in strongly nonlinear irregular wave fields. Preliminary results show capabilities of capturing the wake of a ship and wave impact on monopiles; identified issues regarding this coupling with nonlinear potential flow depend on the interaction between the free-surfaces in the two models, particularly for locally breaking waves.

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