Wave energy park interactions in short-crested waves

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Highlights

- A semi-analytical multiple scattering method has been extended to include multidirectional waves.
- The method has been used to study the performance of an array of point-absorber wave energy converters in multidirectional and unidirectional waves.
- The shadowing effect is less pronounced in multidirectional waves, and the performance is also affected by the wave spreading parameter and number of incident wave directions.

1 Introduction

Ocean waves provide a promising renewable energy source, and much research has been invested during the last decades into different approaches on how to convert the energy in waves into useful electricity. Many different technologies have been developed, and most concepts require that several or many wave energy converters (WECs) are deployed together in arrays, or parks, if energy is to be extracted on a large scale and to lower the cost of the produced electricity.

The layout of the wave energy park will affect the performance of the devices due to scattered and radiated waves [1, 2], and optimal park performance includes optimizing the array layout. Many works on optimal configurations of wave energy parks have considered only incident regular waves, and even if irregular waves have been considered, almost all have considered only long-crested, unidirectional waves. However, real ocean waves are multidirectional, with irregular waves travelling in different directions simultaneously.

The effect of the directional spreading parameter in multidirectional waves on the motion and performance of an attenuator WEC was studied in [3], and it was seen that less energy could be harnessed in multidirectional waves as compared to unidirectional waves. In [4], an array of 12 oscillating wave surge converters was studied in multidirectional waves and it was found that the array had a slightly lower interaction factor than compared to unidirectional waves. The wake effect behind WEC arrays was studied in [5] and [6], and both papers found that the wake is reduced when directional wave spreading is taken into account. This effect and other wave energy array effects in short-crested waves was also investigated experimentally in [7]. Wave run-up on bottom-mounted cylinders was studied in [8]. The experimental and analytical results gave a good agreement and showed that run-up is more pronounced in multidirectional waves as opposed to unidirectional, and that the largest transverse force occurs on the last cylinder of the cylinder array, which is very different from the behaviour in unidirectional waves.

The mentioned works show that the behaviour of offshore structures is different in multidirectional waves as compared to unidirectional, and that we would expect a different performance of wave energy parks in realistic, multidirectional waves. Here, the semi-analytical multiple scattering method presented in [9, 10] is extended to include multidirectional (short-crested) waves, and applied to study the performance of arrays of point-absorbing wave energy converters in realistic, multidirectional waves.

2 Theory

Multiple scattering theory Consider a fluid that is incompressible and irrotational, implying that it can be described by potential flow theory using a fluid velocity potential satisfying the Laplace equation, $\nabla\Phi = 0$. Further assume that the viscosity of the fluid can be neglected and that the wave height is small compared to the wave length, so that the boundary constraints at the sea surface can be linearized. The fluid is then described by linear potential flow theory, and the details can be found in many text books such as [11].

Consider an array of N point-absorbing wave energy converters, each consisting of a floating truncated cylinder buoy connected to a linear generator at the seabed. At the origin of each buoy (x^i, y^i) we define local cylindrical coordinate systems (r, θ, z). Due to the linearity of the problem, the fluid potential can be decomposed into incident, radiated and scattered waves. The general solution to the Laplace equation and

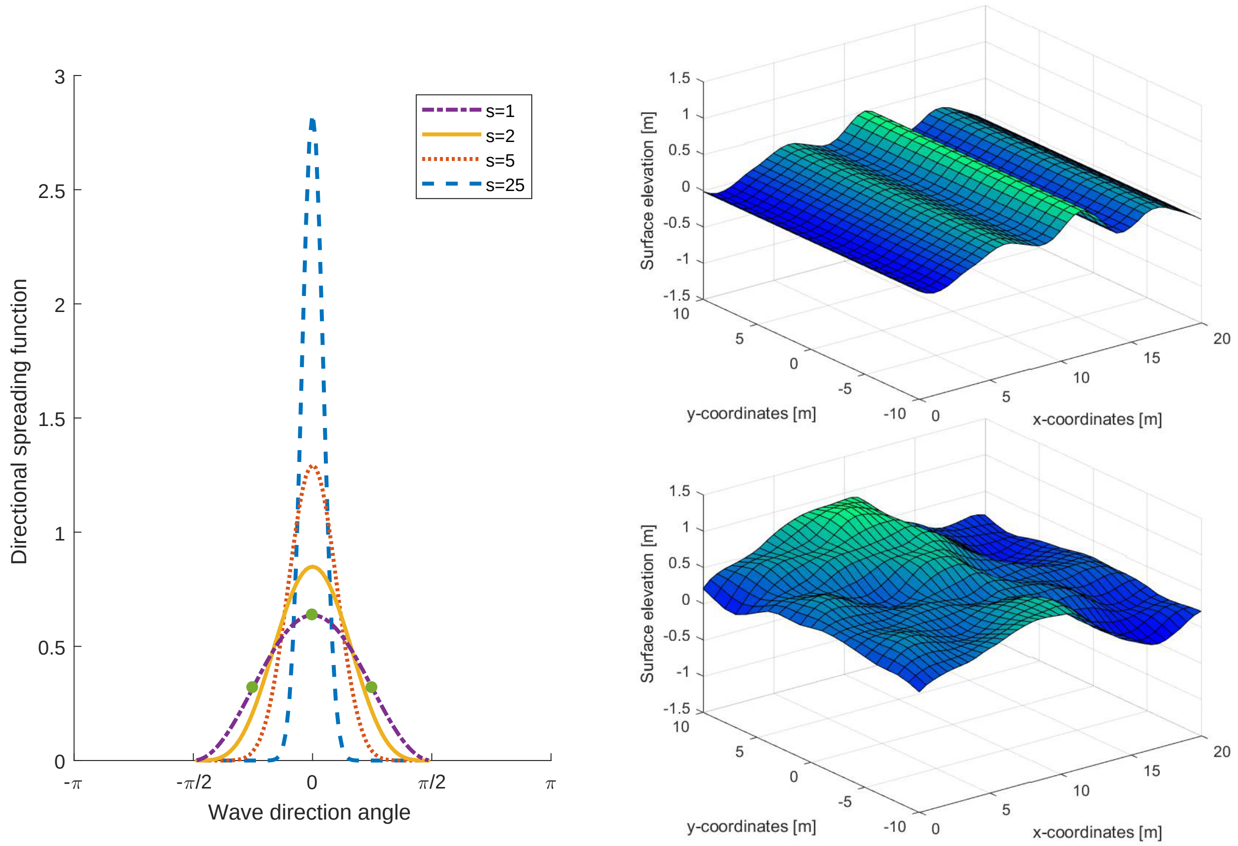

Figure 1: Left: Directional spreading function $D(\chi - \bar{\chi})$ for different values of the spreading parameter s. Right: surface elevation for unidirectional waves (top) and multidirectional waves (bottom) with spreading parameter $s = 1$ and three wave directions $\chi_m = \{0, \pm\pi/4\}$, indicated in the directional spreading function by circles.

the linear boundary constraints at the seabed, sea surface and all rigid boundaries is given in terms of Bessel functions and vertical eigenfunctions. Graf's addition theorem for Bessel functions is used to write the scattered and radiated waves from one buoy as incident waves on the other buoys. By requiring continuity for the potentials at each buoy boundary, the unknown coefficients in the potentials can be solved for by the inversion of a diffraction matrix. For more details, please see [9, 10].

Multidirectional waves Open ocean waves are assumed, i.e. no reflective structures exist so that there are no phase-locked waves – all the phases are randomly distributed, implying that the wave components are independent of each other. Thus, we can treat the irregular multidirectional waves as a superposition of harmonic waves travelling in different directions.

A single harmonic wave travelling in direction χ away from the x-axis can be described by the surface elevation $\eta(x, y, y) = a\cos\left(\omega t - \bar{k}\cdot\bar{x} + \varphi\right)$, where φ the phase, and ω is the angular frequency related to the wave number vector $\bar{k} = (k\cos\chi, k\sin\chi, 0)$ by the dispersion relation for dispersive ocean waves. When the waves are composed of many waves travelling in independent directions, the surface elevation can be written as the superposition

$$\eta(x, y, t) = \mathrm{Re}\left(\sum_{n=-\infty}^{\infty}\sum_{m=1}^{M} a_{mn} e^{i[\omega_n t - \bar{k}_{mn}\cdot\bar{x}]}\right) = \sum_{n=-\infty}^{\infty}\sum_{m=1}^{M} |a_{mn}|\cos(\omega_n t - \bar{k}_{mn}\cdot\bar{x} + \varphi_{mn}) \tag{1}$$

where the phase is $\varphi_{mn} = \arg(a_{mn})$ and $\bar{k}_{mn}\cdot\bar{x} = k_n(x\cos\chi_m + y\sin\chi_m)$. The complex amplitude coefficients can be written in terms of the directional wave spectra as $\left|a_{mn}\right|^2 = S(\omega_n, \chi_m)d\omega d\chi$ where $d\omega = \omega_{n+1} - \omega_n$ and $d\chi = \chi_{m+1} - \chi_m$. The expression for the surface elevation turns into an inverse Fourier integral when $d\omega$ becomes infinitesimal and we write $a_{mn} = A(\omega_n, \chi_m)d\omega d\chi$. The directional wave spectrum can be decomposed into a direction independent spectrum and a directional spreading function with normalization constraint,

$$S(\omega, \chi) = S(\omega)D(\omega, \chi), \qquad \int_{-\pi}^{\pi} D(\omega, \chi)d\chi = 1, \tag{2}$$

as well as periodicity, $D(\omega, 2\pi) = D(\omega, 0)$.

Several different directional spreading functions have been defined and studied in the literature [12, 13]. A common assumption is that the directional spreading function can be described by a unimodal model parametrized by the mean direction $\bar{\chi}$ and another parameter, such as the spreading parameter or a directional width, such that it is independent of the wave frequency. Here, we will use the directional spreading function presented in [14] and used recently in [3, 4],

$$D(\chi - \bar{\chi}) = \begin{cases} F(s)\cos^{2s}(\chi - \bar{\chi}), & |\chi - \bar{\chi}| < \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

with

$$F(s) = \frac{1}{\sqrt{\pi}} \frac{\Gamma(s+1)}{\Gamma(s+\frac{1}{2})} = \frac{1}{\pi} \frac{(2s)!!}{(2s-1)!!} \tag{4}$$

which is simply 1 over the integral in equation (2), hence the normalization constraint is satisfied. However, for discrete wave directions the sum over all wave directions only converges to the integral value when $d\chi$ is infintesimally small. Hence, the coefficient will be defined as

$$F(s) = \frac{1}{\sum_{m=1}^{M} \cos^{2s}(\chi_m - \bar{\chi}) d\chi} \tag{5}$$

which converges to the value in (4) when $d\chi \to 0$.

The principal wave direction will here be considered as $\bar{\chi} = 0$, i.e. moving along the x-direction. The shape of the directional spreading function for different values of the spreading parameter s is shown in figure 1. As can be seen from the figure, the higher the spreading parameter, the more energy in the waves is distributed along the principal wave direction. From (2), the amplitude function can be decomposed into a directional independent part and the directional dependent part. However, only the modulus of the complex amplitude is known, and we add an unknown phase, which is assumed to be uniformly statistically distributed over $(-\pi, \pi)$,

$$A(\omega, \chi) = |A(\omega, \chi)| e^{i\varphi(\omega,\chi)} = \sqrt{S(\omega) D(\omega, \chi) \frac{1}{d\omega d\chi}} e^{i\varphi(\omega,\chi)} = |A(\omega)| \sqrt{D(\omega, \chi) \frac{1}{d\chi}} e^{i\varphi(\omega,\chi)} \tag{6}$$

where $D(\omega, \chi)$ is the directional spreading function defined in (3). In this work, we will use incident irregular *unidirectional* waves for which the modulus $|A(\omega)|$ is known, and compute the complex amplitudes for the multidirectional waves according to the expression in (6).

In the frequency domain, the surface elevation can be written as the product of the amplitude function and a transfer function $H_m(\omega)$, which in the time domain becomes a convolution,

$$\eta(x, y, t) = \sum_{m=1}^{M} D(\chi_m - \bar{\chi}) d\chi \; \eta(0, 0, t) * h_m(t) \tag{7}$$

where $\eta(0, 0, t) = \widehat{A(\omega)}$ is the surface elevation at point $(x, y) = (0, 0)$ and $h_m(t)$ is the inverse Fourier transform of the transfer function $H_m(\omega)$.

Equations of motion When the fluid potentials have been solved for by the above procedure, the hydrodynamical forces on the floats is given in the frequency domain by the surface integral of the potential over the wetted surface of the floats. The excitation force is obtained from the case when all buoys are held fixed and there are incident (multidirectional) waves, and the radiation force is obtained from the case when all buoys are free to oscillate and there are no incident waves. When the hydrodynamical forces have been obtained, the equations of motion for the float and the connected linear generator are solved in the frequency domain and the motion in time is obtained by inverse Fourier transform [9, 10]. The absorbed power for each wave energy converter is computed in the time domain and showed in figure 2.

3 Results

The results for wave energy parks in unidirectional vs multidirectional waves is shown in figure 2. The shadowing effect in the multidirectional waves is less pronounced than for the unidirectional waves, where the WECs in the first rows perpendicular to the incident principal wave direction have a higher energy absorption than the ones in the last rows. Figure 2b) shows one simulation with spreading parameter $s = 5$ and 15 wave directions $\chi_m = \{0, \pm m\pi/16\}$. Due to the randomness of the phase in equation (6), the power absorption is not symmetric over the x-axis. When averaging over a large number of simulations, the results for the multidirectional waves will become symmetric over the principal wave direction. Similar results are obtained for different values of the spreading parameter and number of incident wave directions, which will be discussed at the workshop.

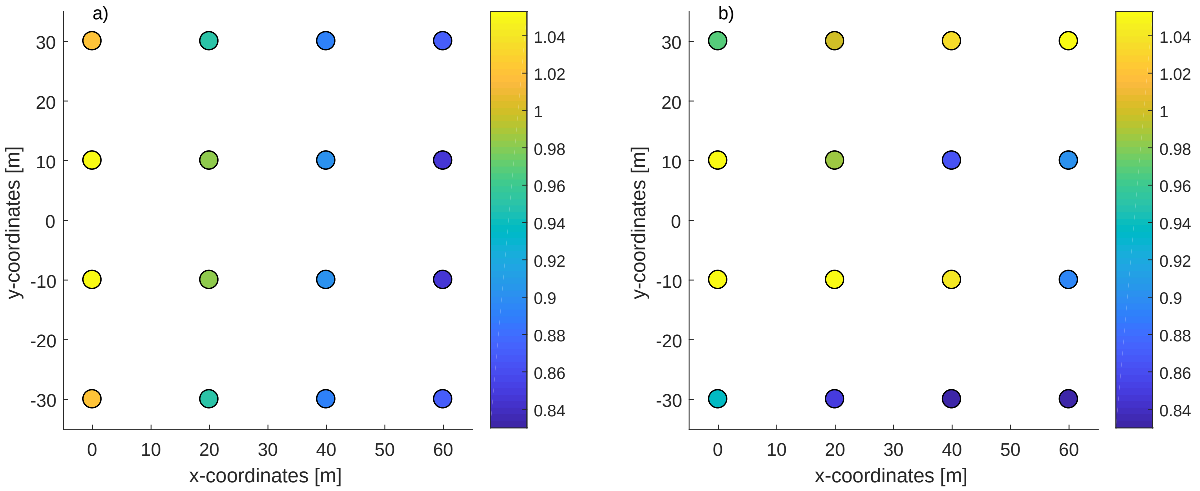

Figure 2: Park performance for an array of 16 WECs in a) unidirectional (long-crested) waves and b) multidirectional (short-crested) waves. Spreading parameter $s = 5$ is used for the multidirectional waves and there are 15 incident wave directions $\chi_m = \{0, \pm m\pi/16\}$, $m = 1, \ldots, 7$. The color shows the ratio of the time averaged power by the device and a device in isolation.

4 Conclusions

Realistic waves are irregular and short-crested, which must be taken into account when studying and optimizing the performance of wave energy systems. Here, the performance of arrays of 16 point-absorber wave energy converters has been studied in unidirectional (long-crested) and multidirectional (short-crested) waves. An analytical multiple scattering theory has been used to evaluate the hydrodynamical interactions in the array. The results show that the shadowing effect is less pronounced in the short-crested waves, which is of relevance when designing optimal layouts for wave power parks.

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