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Hydrodynamics of Side Wall Effects through Image Green Function based TEBEM

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Highlights:

- A novel numerical model based on image Green function and Taylor Expansion Boundary Element Method (TEBEM) has been developed to calculate the side wall effects on first-order motion responses and second-order drift loads upon offshore structures in wave tank.
- Drift loads involving side wall effects can only be obtained by near-field formula. Attention should be payed to the accuracy and convergence of drift force especially for non-smooth offshore structures. TEBEM improves the accuracy of induced velocity, Good agreement can be obtained, compared to experimental results.

1. Introduction

Generally, the results exhibit huge difference from the expected in open sea at some frequencies due to side wall effects. How to avoid or clarify the effects of side wall interference is essential for estimating the hydrodynamics on offshore structures or ships in open sea, therefore the hydrodynamic problems under the effects of side wall has been investigated by many researchers.

Clément[1] measured the first-order and second-order quantities on hemisphere and box-shaped barge in wave tank, and it was shown that the reflection due to the side walls affects the measurements of the first-order vertical motions and second-order drift loads. McIver[2] gave the theoretical models for diffraction and radiation solutions on truncated vertical cylinder including side wall effects through multipoles expansion method. However, this method is limited to bodies of simple geometry such as truncated vertical cylinders. Therefore, Green function composed of infinite images is assumed a powerful method to solve boundary-value problems. Kashiwagi[3] divided the Green function involved in side wall effects into open sea Green function and other term, and the slowly convergent was replaced by a double integral over a semi-infinite domain. Chen[4] took advantage of source method to develop tank Green function(TGF), which can be written as the sum of two parts based on the convergence. Due to the convergence of Green function is too slow, Xia[5] divided the Green function into three parts in near, middle and far field respectively. Newman[6] developed Image Green Function(IGF) method, which is composed of a series of open sea Green function satisfying free-surface condition, and also modified WAMIT procedure to calculate an elongated ship-like body in a channel.

Generally, there are two possible formulations to predict the second-order hydrodynamics on offshore structures or ships in open sea. So-called near-field formulation and far-field formulation. The far-field formulation, velocity potential is needed only, is restricted to give only surge, sway forces and yaw moment. The near-filed formulation can avoid this limitation, which can obtain all components of second-order wave loads. Compared to the far-filed formulation, the near-filed formulation is widely used to deal with the side wall effects as the results of former is not effective enough for this issue. However, the near-filed formulation relies on several first-order quantities, including velocity potential, velocity and motion response et al, which may result in numerous consumption of computation.

Although source method (SM) or mixed distribution method can solve velocity potential and motion response effectively on the side wall effects, the tangential velocity and second-order loadings on offshore structures with sharp corners cannot be solved accuracy, such as the interaction between floating bodies and column on platform, bilge keel or bulbous bow on ships. For this issue, Duan [7] employed Taylor Expansion Boundary Element Method (TEBEM), which can improve the computation accuracy of low-order BEM for the solution of non-smooth boundary problem and avoid the difficulty in high-order BEM, and this method were proved effective enough to deal with two-dimension and three-dimension problems[8-9]. In the present study, the first-order TEBEM and IGF method are used to deal with offshore structures hydrodynamics on the side wall effects.

2. Numerical Method

2.1 Boundary-Value Problems

The Cartesian right-handed coordinate system $O-xyz$ is shown in Fig.1. The vertical reflection of center of gravity is in the origin O, and the $O-xy$ is placed on the undisturbed free surface with positive z axis pointing upwards. The wave tank is of width b and has a center line coinciding with the x axis.

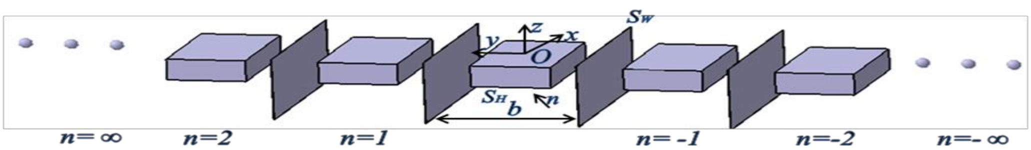

Fig.1 The sketch of side wall effects (four images)

When the fluid is assumed incompressible and inviscid, and the flow is irrotational, the motion of the fluid can be described by the velocity potential in frequency domain as $\Phi(x, y, z, t) = \mathrm{Re}\left[\varphi(x, y, z)\, e^{-\mathrm{i}\omega t}\right]$. The spatial velocity potential φ can be divided into two parts: the incident potential φ_0, and disturbed potential φ_P.

By the superposition principle [10], the disturbed potential is the summation of radiation potentials $\varphi_j (j = 1,\sim 6)$ and diffraction potential. Then the disturbed potentials $\varphi_j (j = 1,\sim,7)$ can be solved by the following boundary value problem.

$$\begin{cases} \nabla^2 \varphi_j = 0 & (in\ D) \quad (j = 1,2,\mathrm{L}\ 7) \\ \dfrac{\partial \varphi_j}{\partial y} = 0 & (on\ S_W) \quad (j = 1,2,\mathrm{L}\ 7) \\ \dfrac{\partial \varphi_j}{\partial z} - \upsilon \varphi_j = 0 & (z = 0) \quad (j = 1,2,\mathrm{L}\ 7) \\ \dfrac{\partial \varphi_j}{\partial n} = n_j \quad (on\ S_H) \quad (j = 1,2,\mathrm{L}\ 6); & \dfrac{\partial \varphi_7}{\partial n} = -\dfrac{\partial \varphi_0}{\partial n} \quad (on\ S_H) \\ \varphi_j = \mathrm{O}\left(\dfrac{1}{\sqrt{r}} e^{\mathrm{i}k_0 r}\right) & (j = 1,2,\mathrm{L}\ 7) \end{cases} \tag{1}$$

Where n points into the offshore structures, D means the fluid domain, $\upsilon = \omega^2/g$ 、 ω 、 g are wave number, wave frequency and gravity acceleration respectively. S_W means the side wall of wave tank, and S_H means the average wetted surface.

Above hydrodynamic problems of offshore structures can be solved through TEBEM, whose principle and numerical method can refer to Chen [11]. Once the disturbed potentials and velocities have been solved, the mean drift force acting on the floating body can be obtained by pressure integral on the wetted surface can be expressed as follows:

$$\begin{aligned} \boldsymbol{F}^{(2)} = & \frac{\rho g}{4} \int_{wl} (\zeta_3 - \chi_3)(\zeta_3 - \chi_3)^* \frac{\boldsymbol{n} dl}{\sqrt{1 - n_3^2}} - \frac{\rho}{4} \iint_{S_H} \left(\nabla \varphi \cdot \nabla \varphi^*\right) \boldsymbol{n} ds - \\ & \frac{\mathrm{i}\omega\rho}{2} \iint_{S_H} \left(\chi \cdot \nabla \varphi^*\right) \boldsymbol{n} ds + \frac{1}{2} \boldsymbol{\eta}_{\boldsymbol{R}}^* \times \left[\boldsymbol{F}^{(1)} - \rho g A_{wp} \left(\eta_3 - \eta_5 x_f\right) \boldsymbol{k}\right] \end{aligned} \tag{2}$$

Where "(2)" is mean drift force; and the superscript * means the conjugate function; $\zeta_3 = i\omega\varphi / g$ is the wave elevation. A_{wp}, x_f, r and wl represent the waterplane area, the coordinate of the center of offshore structures, the position vector of the fluid partical and waterline respectively. $\boldsymbol{T} = \mathrm{Re}\left(\boldsymbol{\eta_T} e^{-i\omega t}\right)$, $\boldsymbol{\eta_T} = (\eta_1, \eta_2, \eta_3)$ is translational displacement. $\boldsymbol{\Omega} = \mathrm{Re}\left(\boldsymbol{\eta_R} e^{-i\omega t}\right)$, $\boldsymbol{\eta}_R = (\eta_4, \eta_5, \eta_6)$ is rotational displacement. $\boldsymbol{X} = \mathrm{Re}\left(\boldsymbol{\chi} e^{-i\omega t}\right)$, where $\boldsymbol{\chi} = (\chi_1, \chi_2, \chi_3) = \boldsymbol{\eta_T} + \boldsymbol{\eta_R} \times \boldsymbol{r}$ means the total displacement.

2.2 Image Green function

To satisfy the side wall boundary condition in Eq. (1), the traditional three-dimension frequency Green function is modified to obtain image Green function. The IGF can be written by a series of three-dimension Green function in open sea representing the infinite images of the side walls to satisfy the side wall condition.

Image Green function G can be expressed as follow:

$$G(p,q) = \sum_{n=-\infty}^{\infty} G_n^O(p,q) \tag{3}$$

Where G_n^O represents the complex infinite Green function in frequency domain satisfying linear free surface condition [10]. $q(\xi, \eta_n, \zeta)$ and $p(x, y, z)$ are source point and field point respectively. As the result of that the source point $q(\xi, \eta_n, \zeta)$ is needed to obtain the image point about side wall, the nth image position associated with side wall can be written as follow:

$$\eta_n = (-1)^n \eta + nb \tag{4}$$

The complex infinite Green function in frequency domain expressed as follow:

$$G_n^O = 1/r_n + 1/r'_n + 2\upsilon \int_L \frac{e^{k(z+\zeta)}}{k - \upsilon} J_0(kH_n)\, dk \tag{5}$$

Where $r_n = \sqrt{(x-\xi)^2 + (y-\eta_n)^2 + (z-\xi)^2}$ represents the distance between source point $q(\xi, \eta_n, \zeta)$ and field point $p(x, y, z)$. $r'_n = \sqrt{(x-\xi)^2 + (y-\eta_n)^2 + (z+\xi)^2}$ represents the distance between field point and the image point $q'(\xi, \eta_n, -\zeta)$ about free surface of source point. $H_n = \sqrt{(x-\xi)^2 + (y-\eta_n)^2}$ means the horizontal distance between field point and source point, and J_0 is the Bessel function of the first kind.

Generally, if the $n = 0$ in Eq.（5）, the image Green function represent the three-dimension Green function in open sea, and the hydrodynamic problems can be solved based on this Green function.

3. Numerical Results and Discussion

3.1 Cone-Shaped Floating Body

In this session, the cone-shaped floating body is used to calculate the second-order horizontal drift loads, vertical drift loads and drift moments, which is used to validate the accuracy of TEBEM.

The structure and main parameters of the cone-shaped floating body is shown in Fig.2. The experiment is implemented in wave tank with depth h=1.0m. More detail of the experiment can be found in the thesis of Koichi Masuda[12] .

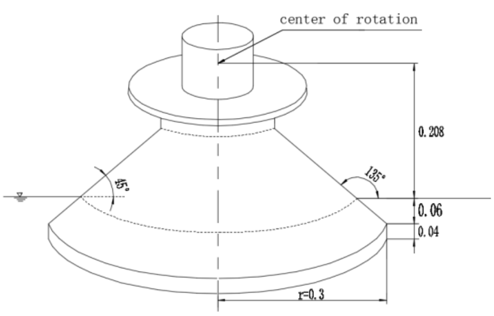

Fig.2 Cone-Shaped Floating Bodies

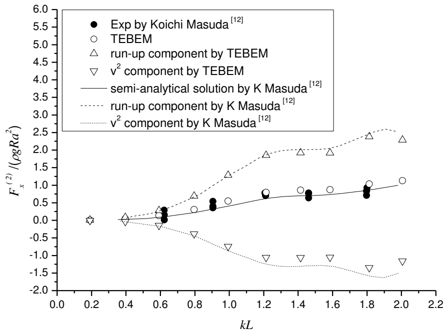

Fig.3 Second-order horizontal drift loads on Cone-Shaped Floating Bodies

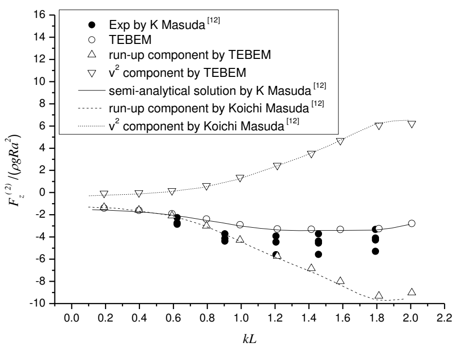

Fig.4 Second-order vertical drift loads on Cone-Shaped Floating Bodies

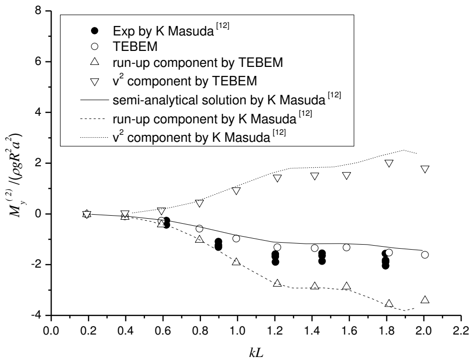

Fig.5 Second-order drift moments on Cone-Shaped Floating Bodies

The drift loads and moment are shown in Fig. 3 to 5. Meanwhile, the results are compared with the experimental results and semi-analytic solutions results from Koichi Masuda[12]. The second-order drift loads and drift moment are no-dimensional by $\rho g R a^2$ and $\rho g R^2 a^2$ respectively.

As shown in Fig.3-5, the present results of drift forces and moments are in good agreement with the experiment and semi-analytic solutions results obtained by Koichi Masuda. Therefore, the correctness of TEBEM is verified.

3.2 Box-Shaped Barge

In this session, the box-shaped barge is used to calculate the hydrodynamic performance in wave tank including hydrodynamic coefficients, first-order motion response and second-order horizontal drift loads, which is used to validate the accuracy of TEBEM associated with IGF method.

A box-shaped barge is placed in the wave tank. The main parameters of box-shaped barge are $L \times B \times T = 0.608m \times 0.608m \times 0.270m$, and the experiment is implemented in wave tank with width b=3.0m and depth h=5.0m, and other parameters can be found in the thesis of Clément[1]. The boxed-shape barge is the meshed with 225 panels on the wetted surface and 81 panels in the inner free surface.

The heave RAO (Response Amplitude Operator) and drift loads are shown in Figs6 and 7 with taking into account side wall effects. Meanwhile, the results are compared with the experimental results from Clément[1] and numerical results from Chen[4] based on source method (SM). The heave motion and second-order horizontal drift loads are no-dimensional by wave amplitude a and $\rho g L a^2$ respectively.

The convergent hydrodynamics of boxed-shape barge can be gotten, when the image number of IGF is selected about 120 to 160, ensuring that side wall effects are taken into account. As shown in Fig.6, the present results of first-order heave response are in good agreement with the experiment by Clément[1] and numerical results obtained through TGF (whose kernel is same with IGF) by SM from Chen[4]. Unlike the first-order heave response, the second-order horizontal drift loads are more sensitive to the side wall effects, especially at high frequencies. Compared to the results obtained from Chen[4] through source method, present results based on first-order TEBEM and IGF method are in good agreement with the experimental results from Clément[1]. As in Fig.7, the differences between the source method and experimental results are much bigger than first-order TEBEM at nondimensionalized wave number near 1.6. Similarly to the comparison between source method and TEBEM in open sea condition, the first-order TEBEM can obtain highly accurate velocity of offshore structures with sharp corners like boxed-shape barge, which can improve the numerical accuracy of second-order horizontal drift load. Therefore, the numerical results are much more associated with the experimental results than the results derived from source method.

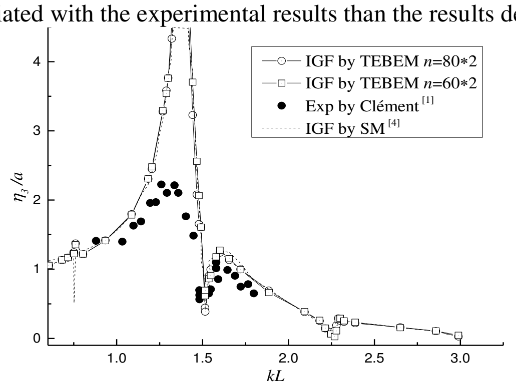

Fig.6 First-order heave response on box-shaped barge in wave tank with width 5.0m

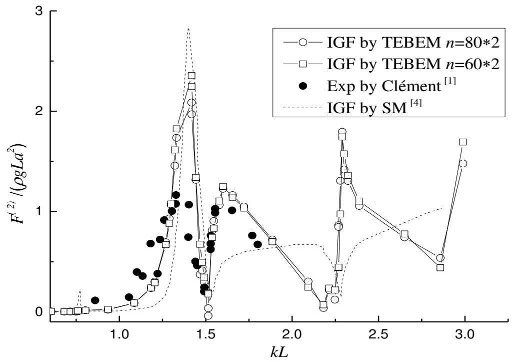

Fig.7 Second-order horizontal drift loads on box-shaped barge in wave tank with width 5.0m

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