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Accurate evaluation of the Kochin function for added resistance using a high-order finite difference-based seakeeping code

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1 INTRODUCTION

At the 32nd IWWWFB in Dalian, we presented our implementation of the far-field method for second-order wave drift forces based on the Kochin function, using the open-source seakeeping code *OceanWave3D-Seakeeping*. In that work we used Maruo's method (Maruo, 1960), and calculated the added resistance by a line integral along the azimuthal angle θ around the body in the far field. Some difficulties were encountered with regard to evaluating the singular and improper integrals, together with identifying the highest frequency limit where we can practically and reliably calculate the Kochin function by a numerical integration over the surface of the body. Motivated by discussions with Prof. Kashiwagi during this workshop (Kashiwagi, 2017), we subsequently applied the Hanaoka transformation (Maruo, 1960) to change the integration domain from θ to a wave-number like variable m. This allows a method developed by Prof. Kashiwagi to be used to evaluate the relevant singular integrals, leading to more robust and accurate results. In this abstract, we outline the numerical method and present new calculations for the added resistance of a submerged and a floating spheroid, These results are compared with near-field solutions, and calculations using boundary element codes where applicable.

2 MARUO'S FORMULATION

The far-field formulation to calculate the wave drift force, requires the knowledge of the wave kinematics and the surface elevation in the far field. Thanks to the ingenious work by Maruo, all this information can be obtained using the Kochin function, and an integration over the surface of the body. The wave drift force (added resistance) can then be calculated by a line integral over the azimuthal angle θ around the body as follows:

$$R_w = \frac{\rho}{8\pi}\left\{\left[\int_{-\pi/2}^{-\theta_0} + \int_{\theta_0}^{\pi/2} - \int_{\pi/2}^{3\pi/2}\right] F_1(\theta)d\theta + \int_{\theta_0}^{2\pi-\theta_0} F_2(\theta)d\theta\right\}. \tag{1}$$

Where $F_j(\theta) = |H_1(\kappa_j,\theta)|^2 \kappa_j \left(\kappa_j \cos\theta - k\cos\beta\right)/\sqrt{1-4\tau\cos\theta}$. The variable κ_j is given by $\kappa_{1,2} = \kappa_0\left[1 \pm \sqrt{1-4\tau\cos\theta}/(2\cos\theta)\right]^2$, where the incident wave number is $k = 2\pi/\lambda$, $\omega = k\left(c - U\cos\beta\right)$ is the encounter frequency with $c = \sqrt{g/k}$ the phase speed (in deep water), while $\kappa_0 = g/U^2$ and $\tau = U\omega/g$. Here g is acceleration due to gravity, ρ is the water density and U is the forward speed of the body. The angle of the incident wave is given by β with $\beta = \pi$ corresponding to head seas. In the integral bounds, when $\tau \leq 1/4$, $\theta_0 = 0$ otherwise $\theta_0 = \cos^{-1}(1/4\tau)$. The Kochin function is also denoted by $H_1(\kappa_j,\theta)$. According to (Maruo, 1960), the Hanaoka transformation can be applied to recast the integration domain from θ to a wave-number like parameter denoted by $m^{\pm} = \kappa_0\left(1 - 2\tau\cos\theta \pm \sqrt{1-4\tau\cos\theta}\right)/\left(2\cos\theta\right)$. Note that in this case $\cos\theta = \kappa_0 m/\left(m+\kappa_0\tau\right)^2$. According to Figure 1, the integration bounds can be computed from: $\bar{k}_1 = m^+(\theta=\pi)$, $\bar{k}_2 = m^-(\theta=\pi)$, $\bar{k}_3 = m^-(\theta=\theta_0)$ and $\bar{k}_4 = m^+(\theta=\theta_0)$. Moreover $m^+ \to \infty$ as $\theta \to -\pi/2^+$ or $\theta \to \pi/2^-$, and $m^+ \to -\infty$ as $\theta \to \pi/2^+$ or $\theta \to 3\pi/2^-$. This leads to:

$$\bar{k}_1 = -\frac{\kappa_0}{2}\left(1+2\tau+\sqrt{1+4\tau}\right), \quad \bar{k}_3 = \frac{\kappa_0}{2}\left(1-2\tau-\sqrt{1-4\tau}\right) \quad \text{if } \tau > \frac{1}{4} \quad \text{then} \quad \bar{k}_3 = \kappa_0\tau,$$
$$\bar{k}_2 = -\frac{\kappa_0}{2}\left(1+2\tau-\sqrt{1+4\tau}\right), \quad \bar{k}_4 = \frac{\kappa_0}{2}\left(1-2\tau+\sqrt{1-4\tau}\right) \quad \text{if } \tau > \frac{1}{4} \quad \text{then} \quad \bar{k}_4 = \kappa_0\tau.$$

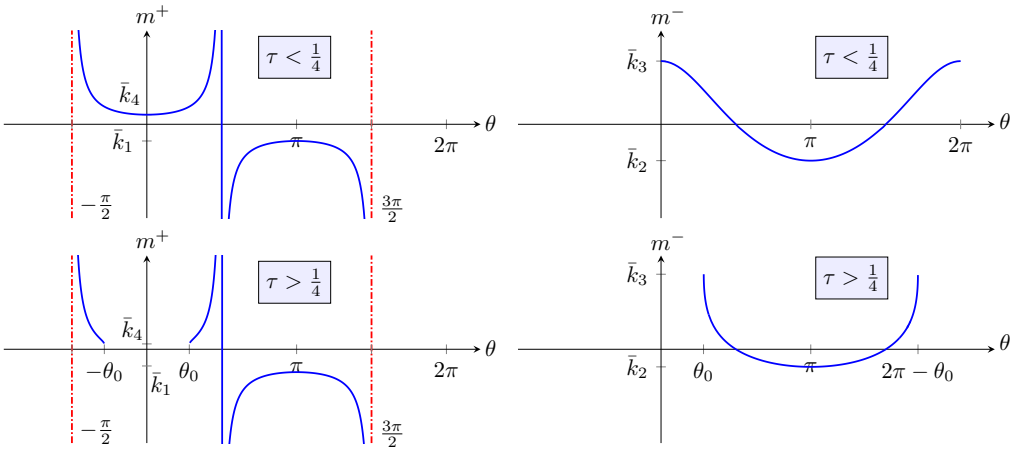

Figure 1: Variable m vs. θ (top: $\tau < 1/4$), (bottom: $\tau > 1/4$)

Note also, that when $\tau = 1/4$ then $\bar{k}_3 = \bar{k}_4 = \kappa_0/4$. Considering the added resistance from (1) it is easy to verify that:

$$\int_{-\pi/2}^{-\theta_0} + \int_{\theta_0}^{\pi/2} \cdots d\theta = 2\int_{\bar{k}_4}^{\infty} \cdots dm\,, \int_{\pi/2}^{3\pi/2} \cdots d\theta = 2\int_{-\infty}^{\bar{k}_1} \cdots dm \text{ and } \int_{\theta_0}^{2\pi-\theta_0} \cdots d\theta = 2\int_{\bar{k}_2}^{\bar{k}_3} \cdots dm.$$

If according to (Kashiwagi, 2013) we define $\bar{\kappa}(m) = (\omega + m\,U)^2/g$, then the added resistance can be re-expressed by an integration over m as:

$$R_w = \frac{\rho}{4\pi}\left\{-\int_{-\infty}^{\bar{k}_1} + \int_{\bar{k}_2}^{\bar{k}_3} + \int_{\bar{k}_4}^{\infty}\right\} \frac{\bar{\kappa}\,(m - k\cos\beta)}{\sqrt{\bar{\kappa}^2 - m^2}}\,|H_1(m)|^2\,dm\,. \tag{2}$$

Note that $\bar{k}_1$ and $\bar{k}_2$ are the solutions to the equation $\bar{\kappa} = -m$ (both when $\tau < 1/4$ and $\tau > 1/4$), and $\bar{k}_3$ and $\bar{k}_4$ are the solutions to the equation $\bar{\kappa} = m$ (only when $\tau < 1/4$). It is also straightforward to show that when $U = 0$, only the middle integral contributes to the wave drift force, and the integral bounds $\bar{k}_2$ and $\bar{k}_3$ become in fact equal to $-k$ and k respectively. In this case $\bar{\kappa}$ will be also equal to the wave number k. The Kochin function in (2) as a function of m can be defined by:

$$H_1(m) = \iint_{S_B}\left(\phi_B\frac{\partial}{\partial n} - \frac{\partial\phi_B}{\partial n}\right)\exp\left[\bar{\kappa}z + \mathrm{i}\left(x\,m + y\sqrt{\bar{\kappa}^2 - m^2}\right)\right]\,ds\,. \tag{3}$$

Here S_B denotes the mean surface of the body, and ϕ_B includes the unsteady velocity potentials due to all radiation modes plus the scattering of the incident wave. A right-handed Cartesian coordinate system (x, y, z) is also adopted where x is along the length of the body and z is vertically upward.

3 RESULTS AND DISCUSSION

The time-domain seakeeping solver *OceanWave3D-Seakeeping* (Amini-Afshar et al., 2015) is used to obtain the first-order radiation and the scattering velocity potentials for calculating the Kochin function. This solver is based on the finite-difference method, hence it requires the spacial discretisation of the whole 3D computational domain. We use a hyperbolic grid generation technique to build an overlapping grid for the bodies. Practically we are not able to calculate the Kochin function (3) by a numerical integration, beyond a Nyquist wave number given by $k_{Nyq} = \pi/l_a$. This is defined based on the average spacing of the computational grid l_a along the waterline of the body. For the accurate calculation of the Kochin function both m and $\sqrt{\bar{\kappa}^2 - m^2}$ in (2) should be below k_{Nyq}, and the bounds of the line integrals are all truncated at this limit. The presented results in this abstract are for a submerged and a floating spheroid. These calculations are all based on the Neumann-Kelvin linearisation and with $\beta = \pi$.

3.1 Submerged spheroid

We consider first a submerged spheroid with length over breadth ratio given by $L/B = 5$. The body is fixed, the submergence depth is $d = 0.75B$ and the Froude number is given by $U/\sqrt{gL} = 0.2$. The results based on the far-field and the near-field methods are shown in Figure 2 (right). The calculations based on the boundary element method by (Iwashita and Ohkusu, 1989) are also shown. Note that the contribution of the improper integrals for this case is almost zero, but they are included in the final calculations and are also shown separately in the figure. For $\lambda/L = 1$ the Kochin function and the integrand of the middle integral from (2) are also plotted in Figure 2 (left). This might seem to confirm the commonly applied assumption that these contributions are negligible and can be neglected. However, as shown below, this does not seem to be a general result.

3.2 Floating spheroid

The added resistance for the floating spheroid is presented in Figure 3 (left) for several Froude numbers. The body is free in the surge, heave and pitch modes of motion. The presented results in this figure show only the contribution from the middle integral. Very good agreement is observed between the far-field and the near-field solutions. The Kochin function and the integrand from (2) for $\lambda/L = 2$ are also presented in Figure 3 (left). In Figure 4, we examine the behaviour of the improper integrals for the two cases of $Fr = 0.05$ and $Fr = 0.25$. As can be seen, the contributions in the case of low Froude number is negligible except in very short-wave range. At the larger Froude number however, the contribution from these integrals becomes as large as that of the middle integral. It is common practice in the literature to neglect these integrals, see for example (Liu et al., 2011), at this moment we have no clear explanation as to why based on our calculations these contributions are not evaluated properly for the case of floating spheroid.

4 CONCLUSIONS

The calculation of the added resistance based on the far-field method and using the Kochin function is presented. The line integrals are evaluated in the wave-number m domain. All improper integrals are taken into consideration, and their contributions to the added resistance is demonstrated. As the oscillatory integrand in the Kochin function integral (3) can not be practically resolved beyond the maximum wave number k_{Nyq}, the integral bounds in (2) are truncated at this limit. A very good agreement can be seen between the near-field, the far-field methods and a boundary element solution for the case of a submerged spheroid. For the floating spheroid, a perfect match between the far-field and the near-field is observed only when the contribution from the improper integrals is neglected. Although not shown here, this is however not true of other bodies like the Wigley hull where neither results including all contributions nor those including only the middle integral agree with pressure integration results. Investigation is underway to understand the reasons behind these unexpected results.

5 ACKNOWLEDGMENT

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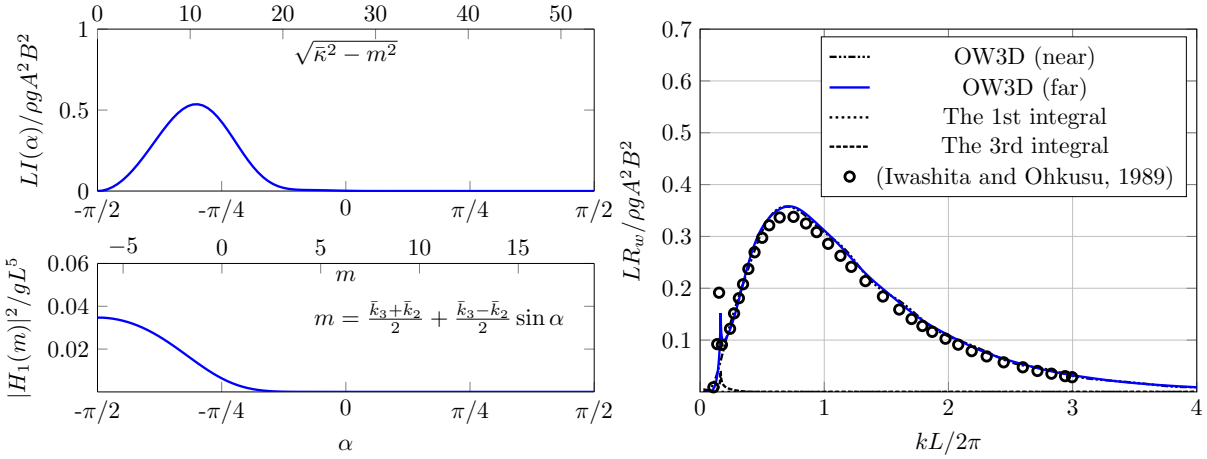

Figure 2: Added resistance for the fixed submerged spheroid (right). The integrand $I(\alpha)$ in equation (2) (left-top), and the Kochin function $|H_1(m)|^2$ (left-bottom), corresponding to $\lambda/L = 1$. $k_{Nyq} = 155$.

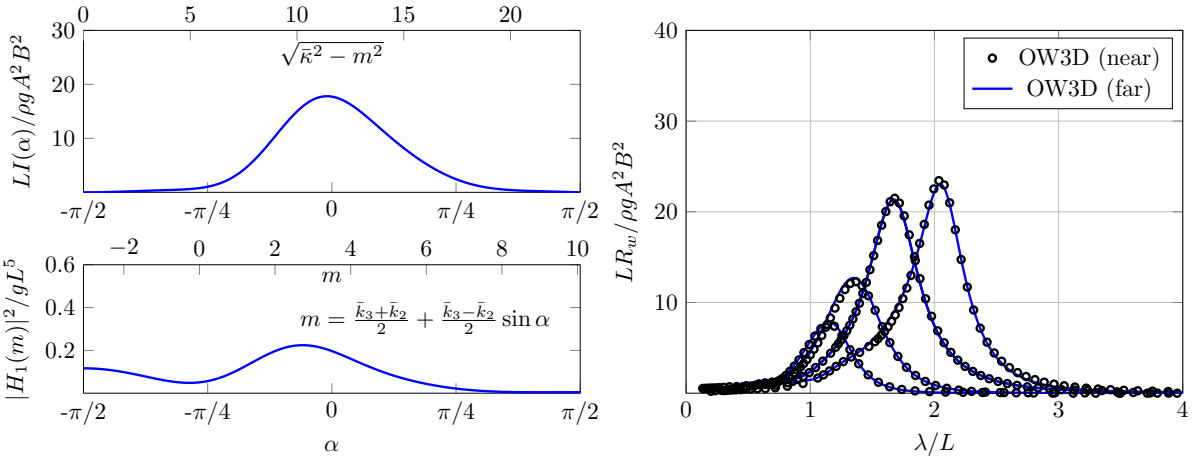

Figure 3: Added resistance for the floating spheroid (right) using only the middle integral for several Froude numbers from left to right given by $Fr = 0.0$, 0.05, 0.15 and 0.25. The integrand $I(\alpha)$ in equation (2) (left-top), and the Kochin function $|H_1(m)|^2$ (left-bottom), corresponding to $\lambda/L = 2$ and $Fr = 0.25$. $k_{Nyq} = 154$.

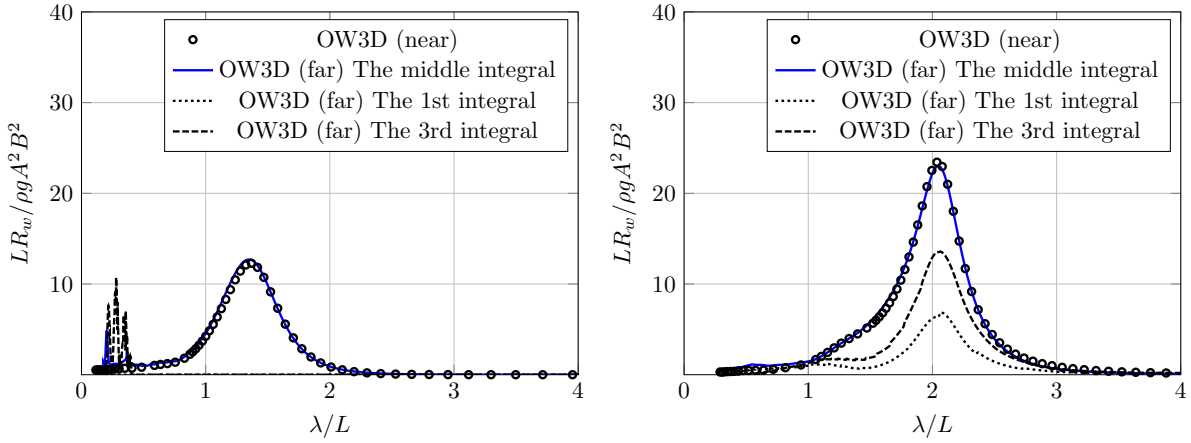

Figure 4: The separate contributions due to each of the integrals in equation (2) for the added resistance of the floating spheroid. ($Fr = 0.05$, left) and ($Fr = 0.25$ right).