# Local-flow component in the Green function for diffraction radiation of water waves

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**Highlights**

The Green function of the theory of diffraction radiation of time-harmonic waves by an offshore structure, or a ship at low speed, in deep water is considered. The Green function $G$ and its gradient $\nabla G$ are expressed in the usual manner as the sum of three components that correspond to the fundamental free-space singularity, a non-oscillatory local flow, and waves. Simple approximations that only involve elementary continuous functions (algebraic, exponential, logarithmic) of real arguments are given for the local flow components in $G$ and $\nabla G$. These approximations are global approximations valid within the entire flow region, rather than within complementary contiguous regions as can be found in the literature.

## 1. Introduction

Diffraction radiation of time-harmonic water waves by an offshore structure, or a ship at low speed, within the classical framework of linear potential flow theory and the Green function method, is routinely used to predict added-mass and wave-damping coefficients, motions, and wave loads. The Green function, which represents the velocity potential due to a pulsating source at a singular point under the free surface as is well known, is an essential element of this method. Accordingly, the Green function has been studied in a broad literature, especially for the simplest case of deep water that is considered here.

The Green function $G$ can be expressed as the sum of the fundamental free-space singularity and a flow component that accounts for free-surface effects. Moreover, this free-surface component is commonly decomposed into a wave component $W$ that represents the waves radiated by the pulsating source, and a non-oscillatory local flow component $L$. This basic decomposition into a wave and a local flow component is not unique. Indeed, three alternative decompositions and related single-integral representations of the Green function $G$ are given in Noblesse (1982).

Several alternative mathematical representations and approximations of $G$ and $\nabla G$ that are well suited for numerical evaluation can be found in the literature. In particular, complementary near-field and far-field asymptotic expansions and Taylor series are given in Martin (1980), Noblesse (1982) and Telste & Noblesse (1986). Several practical approximate methods for computing $G$ and $\nabla G$ have also been given. These alternative methods include polynomial approximations within complementary contiguous flow regions, given in Newman (1984a,1985), Wang (1992) and Zhou et al. (1999), and table interpolation associated with function and coordinate transformations, given in Ba et al. (1992) and Ponizy et al. (1994). Other useful practical methods can be found in the literature, notably in Peter & Meylan (2004), Yao et al. (2009), D'elía et al. (2011) and Shen et al. (2015).

Accuracy and efficiency are essential requirements of methods for numerically evaluating $G$ and $\nabla G$, and these important aspects are considered in the practical approximate methods listed in the foregoing. Indeed, the alternative methods proposed in these studies provide accurate and efficient methods for computing $G$ and $\nabla G$.

Numerical errors associated with potential-flow panel methods stem from several well-known sources, including:

(i) discretization of the wetted hull surface of an offshore structure or a ship; i.e. the number and the type (flat or curved) of panels,

(ii) approximation of the variations (piecewise constant, linear, quadratic, or higher-order) of the densities of the singularity (source, dipole) distributions over a surface panel,

(iii) numerical integration of the Green function and its gradient over a (flat or curved) panel, and

(iv) numerical evaluation of the Green function and its gradient.

Moreover, the Green function $G$ (as well as its gradient $\nabla G$) is given by the sum of the fundamental free-space singularity, a wave component $W$ and a non-oscillatory local flow component $L$ as was already noted. Thus, numerical errors that stem from an approximation of the local flow components in the representations of $G$ and $\nabla G$ are only one part among several sources of errors associated with panel methods. While the ideal approximations to $G$ and $\nabla G$ are highly accurate and efficient as well as very simple, this ideal goal is hard to reach in practice because accuracy, efficiency and simplicity are competing requirements.

The level of accuracy that is actually required for useful practical approximations to $G$ and $\nabla G$ therefore is a fairly complicated issue. This issue is partly considered in Wu et al. (2016a) for the similar theory of steady ship waves (linear potential flow around a ship hull that advances at a constant speed in calm water). Specifically, the errors due to a simple analytical approximation to the local flow component $L$ in the Green function for steady ship waves are considered in that study. This approximate local flow component $L$, given in Noblesse et al. (2011), is very simple and highly efficient, but not particularly accurate. Yet, this simple relatively crude approximation to the Green function for steady ship waves is found in Wu et al. (2016a) to yield predictions of sinkage, trim and drag that do not differ appreciably from the predictions obtained if the Green function is computed with high accuracy. This finding suggests that highly accurate approximations to the local flow components in the Green function $G$ and its gradient $\nabla G$ for the theory of wave diffraction radiation similarly may not be necessary for practical purposes.

The approximations to the local flow components in the representations of $G$ and $\nabla G$ for wave diffraction radiation considered in Wu et al. (2016b) are given here. These approximations are based on a pragmatic hybrid approach that combines numerical approximations with near-field and far-field analytical expansions, in a manner similar to that used in Noblesse et al. (2011) for the Green function of the theory of ship waves. The approximations given here are valid within the entire flow region, i.e. are global approximations, unlike the approximations for complementary contiguous flow regions that can be found in the literature. The approximations to the local flow components given here only involve elementary continuous functions (algebraic, exponential, logarithmic) of real arguments, and provide an efficient and particularly simple method for numerically evaluating the Green function $G$, and its gradient $\nabla G$, for diffraction radiation of time-harmonic waves in deep water. The global approximations to the local flow components in $G$ and $\nabla G$ given here are similar to, but considerably more accurate than, the approximations given in Wu et al. (2016c).

## 2. Basic integral representations

A Cartesian system of coordinates $\mathbf{X} \equiv (X, Y, Z)$ is used. The $Z$ axis is vertical and points upward, and the undisturbed free surface is taken as the plane $Z = 0$. Diffraction radiation of time harmonic waves with radian frequency $\omega$ and wavelength $\lambda = 2\pi g/\omega^2$, where $g$ denotes the gravitational acceleration, is considered. Nondimensional coordinates

$$\mathbf{x} \equiv (x, y, z) \equiv (X, Y, Z)\omega^2/g \tag{1}$$

are defined.

The Green function $G(\mathbf{x}, \tilde{\mathbf{x}})$ corresponds to the spatial component of a nondimensional velocity potential

$$\mathrm{Re}\,[G(\mathbf{x}, \tilde{\mathbf{x}})\, e^{-\mathrm{i}\omega T}] \tag{2}$$

where $T$ denotes time. Expression (2) represents the potential of the flow created at the point $\mathbf{x} \equiv (x, y, z \leq 0)$ by a pulsating source located at the point $\tilde{\mathbf{x}} \equiv (\tilde{x}, \tilde{y}, \tilde{z} < 0)$, or by a flux through the free surface at the point $\tilde{\mathbf{x}} \equiv (\tilde{x}, \tilde{y}, \tilde{z} = 0)$.

The nondimensional distances between the flow-field point $\mathbf{x}$ and the source point $\tilde{\mathbf{x}}$ or its mirror image $\tilde{\mathbf{x}}_1 \equiv (\tilde{x}, \tilde{y}, -\tilde{z})$ with respect to the undisturbed free-surface plane $z = 0$ are denoted as $r$ and $d$, and are given by

$$r \equiv \sqrt{(x-\tilde{x})^2 + (y-\tilde{y})^2 + (z-\tilde{z})^2} \tag{3a}$$

$$d \equiv \sqrt{(x-\tilde{x})^2 + (y-\tilde{y})^2 + (z+\tilde{z})^2} \tag{3b}$$

The horizontal and vertical components of the distance $d$ between the points $\mathbf{x}$ and $\tilde{\mathbf{x}}_1$ are given by

$$0 \leq h \equiv \sqrt{(x-\tilde{x})^2 + (y-\tilde{y})^2} \qquad v \equiv z + \tilde{z} \leq 0 \tag{4}$$

The Green function $G$ is expressed as

$$4\pi G = -1/r + L + W \tag{5}$$

where $-1/r$ is the fundamental free-space Green function, and $L$ and $W$ represent a local flow component and a wave component that account for free-surface effects. The component $L$ corresponds to a non-oscillatory local flow and the component $W$ represents circular surface waves radiated by the pulsating singularity located at the source point $\tilde{\mathbf{x}}$. The basic decomposition (5) into a local flow and waves is non unique, as was already noted. Three alternative decompositions and related integral representations are given in Noblesse (1982).

The so-called near-field integral representation in Noblesse (1982) is considered here. The wave component $W$ in this representation is given by

$$W(h, v) \equiv 2\pi\,[\widetilde{H}_0(h) - \mathrm{i}\,J_0(h)]\,e^v \tag{6}$$

where $\widetilde{H}_0(\cdot)$ and $J_0(\cdot)$ denote the zeroth-order Struve function and the zeroth-order Bessel function of the first kind. The corresponding local flow component $L$ is given by

$$L(h, v) \equiv -\frac{1}{d} - \frac{4}{\pi}\int_0^{\frac{\pi}{2}} \mathrm{Re}\, e^M E_1(M)\, d\theta \quad \text{where} \quad M \equiv v + \mathrm{i}\,h \tag{7}$$

and $E_1(\cdot)$ is the complex exponential integral function.

The gradient $\nabla G \equiv (G_x, G_y, G_z)$ of the Green function $G$ is expressed in Noblesse (1982) as

$$4\pi G_z \equiv \frac{z-\tilde{z}}{r^3} + L_z + W \quad \text{where} \quad L_z = \frac{v}{d^3} - \frac{1}{d} + L \tag{8a}$$

$$4\pi G_h \equiv \frac{h}{r^3} + L_h + W_h \quad \text{where} \quad L_h = \frac{h}{d^3} + L_* \tag{8b}$$

$$4\pi G_x \equiv G_h\,\frac{x-\tilde{x}}{h} \quad \text{and} \quad 4\pi G_y \equiv G_h\,\frac{y-\tilde{y}}{h} \tag{8c}$$

The wave component $W_h$ in (8b) is given by

$$W_h(h, v) \equiv 2\pi\,[2/\pi - \widetilde{H}_1(h) + \mathrm{i}\,J_1(h)]\,e^v \tag{9}$$

where $\widetilde{H}_1(\cdot)$ and $J_1(\cdot)$ denote the first-order Struve function and the first-order Bessel function of the first kind. The local flow component $L_*$ in (8b) is given by

$$L_*(h, v) \equiv \frac{4}{\pi}\int_0^{\frac{\pi}{2}} \mathrm{Im}\,[e^M E_1(M) - 1/M]\cos\theta\, d\theta \tag{10}$$

where $M$ is defined in (7).

The exponential function $e^v$ and the Bessel and Struve functions in expressions (6) and (9) for the wave components $W$ and $W_h$ are infinitely differentiable. Moreover, several practical and efficient alternative approximations for the Bessel and Struve functions are given in the literature; notably in Hitchcock (1957), Abramowitz & Stegun (1965), Luke (1975) and Newman (1984b).

## 3. Near-field and far-field approximations

The variables $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$ defined as

$$\alpha \equiv -v/d \equiv \sqrt{1-\beta^2} \qquad \beta \equiv h/d \equiv \sqrt{1-\alpha^2} \tag{11}$$

are used hereafter. The related variable $0 \leq \sigma \leq 1$ defined as

$$\sigma \equiv h/(d-v) \equiv \beta/(1+\alpha) \equiv \sqrt{(1-\alpha)/(1+\alpha)} \tag{12}$$

is also used in this section and the next.

The behaviors of the local flow component $L$ defined by the integral representation (7) in the near-field and far-field limits $d \to 0$ and $d \to \infty$ are considered in Noblesse (1982). In particular, expressions (7.7), (7.8), (7.22) and (7.23) in Noblesse (1982) yield the near-field approximation

$$L = -\frac{1}{d} + 2\left(\log\frac{d-v}{2} + \gamma\right) + 2v\left(\log\frac{d-v}{2} + \gamma - 1\right) + 2h\,(\sigma - 2) + O(d^2\log d) \text{ as } d \to 0 \tag{13}$$

where $\gamma = 0.577\ldots$ is Euler's constant. For $\alpha \neq 0$, expression (6.14) in Noblesse (1982) yields the far-field approximation

$$L = \frac{1}{d} + \frac{2\alpha}{d^2} - \frac{2-6\alpha^2}{d^3} + O\left(\frac{1}{d^4}\right) \text{ as } d \to \infty \text{ if } \alpha \neq 0 \tag{14a}$$

For $\alpha = 0$, Eqs (6.8) in Noblesse (1982) and (12.1.30) in Abramowitz & Stegun (1965) yield the far-field approximation

$$L = -\frac{3}{d} + \frac{2}{d^3} + O\left(\frac{1}{d^5}\right) \text{ as } d \to \infty \text{ if } \alpha = 0 \tag{14b}$$

Expression (8b) and the partial derivatives, with respect to $h$, of expressions (7.7), (7.8), (7.22) and (7.23) in Noblesse (1982) yield the near-field approximation

$$L_* = \frac{2\sigma}{d} + 2\,(\sigma - 2) - h\left(\log\frac{d-v}{2} + \gamma + \sigma\beta + 2\alpha - \frac{3}{2}\right) - 4v + O(d^2\log d) \text{ as } d \to 0 \tag{15}$$

For $0 < \alpha < 1$, expressions (8b) and (14a) similarly yield the far-field approximation

$$L_* = -\frac{2\beta}{d^2} - \frac{6\alpha\beta}{d^3} + O\left(\frac{1}{d^4}\right) \text{ as } d \to \infty \text{ if } 0 < \alpha < 1 \tag{16a}$$

One has $L_* \equiv -4e^v$ in the special case $\alpha = 1$. In the special case $\alpha = 0$, expression (14b) and Eq. (12.1.31) in Abramowitz & Stegun (1965) yield the far-field approximation

$$L_* = \frac{2}{d^2} + O\left(\frac{1}{d^4}\right) \text{ as } d \to \infty \text{ if } \alpha = 0 \tag{16b}$$

## 4. Approach

The infinite flow region $0 \le h < \infty, -\infty < v \le 0$ is mapped onto the unit square

$$0 \le \rho \equiv d/(1+d) \le 1 \qquad 0 \le \beta \equiv h/d \le 1 \tag{17a}$$

via the relations

$$d = \rho/(1-\rho) \qquad h = \beta d \qquad v = -\sqrt{1-\beta^2}\, d \tag{17b}$$

As was already noted in the introduction, a pragmatic hybrid approach similar to that used in Noblesse et al. (2011) for the Green function of steady ship waves is adopted here. This approach combines numerical approximations with the near-field and far-field analytical expansions given in the previous section.

### 4.1. The function L

The local flow component $L$ defined by (7) is expressed as

$$L = -\frac{1}{d} + \frac{2P}{1+d^3} + 2L' \tag{18a}$$

$$\text{where } P \equiv e^v\left(\log\frac{d-v}{2} + \gamma - 2d^2\right) + d^2 - v \tag{18b}$$

$$\text{and } L' \equiv \frac{-P}{1+d^3} - \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \mathrm{Re}\, e^M E_1(M)\, d\theta \tag{18c}$$

Moreover, $\gamma$ in (18b) is Euler's constant. The decomposition (18a) expresses the local flow $L$ as the sum of two dominant terms that are defined analytically and are related to the near-field and far-field expansions (13) and (14), and the correction $2L'$. The correction term $2L'$ defined by (18c) is numerically approximated below.

Eqs (13), (18a) and (18b) yield the near-field approximation

$$L' \sim d(\sigma\beta - 2\beta) \text{ as } d \to 0 \tag{19}$$

Eqs (14), (18a) and (18b) yield the far-field approximations

$$L' \sim \frac{1}{d^3}(3\alpha^2 - 1) \text{ as } d \to \infty \text{ if } \alpha \ne 0 \tag{20a}$$

$$L' \sim -\frac{1}{d^3}\left(\log\frac{d}{2} + \gamma - 1\right) \text{ as } d \to \infty \text{ if } \alpha = 0 \tag{20b}$$

The near-field approximations (13) and (19) and the far-field approximations (14) and (20) yield

$$L'/L = O(d^2) \text{ as } d \to 0 \tag{21a}$$

$$L'/L = O(1/d^2) \text{ as } d \to \infty \text{ if } \alpha \ne 0 \tag{21b}$$

$$L'/L = O(\log d/d^2) \text{ as } d \to \infty \text{ if } \alpha = 0 \tag{21c}$$

These relations show that $L' \ll L$ in both the near field $d \to 0$ and the far field $d \to \infty$. Expressions (17) and the asymptotic approximations (19) and (20) show that the function $L'$ is asymptotically similar to $\rho(1-\rho)^3$ in the limits $\rho \to 0$ and $\rho \to 1$.

An approximation to the function $L'$ of the form

$$L'(\rho,\beta) \approx \rho(1-\rho)^3 R \text{ where} \tag{22a}$$

$$R \equiv (1-\beta)A - \beta B - \frac{\alpha C}{1+6\alpha\rho(1-\rho)} + \beta(1-\beta)D \tag{22b}$$

is considered in Section 5. The terms $A(\rho)$, $B(\rho)$, $C(\rho)$ and $D(\rho)$ in (22b) are polynomials in $\rho$.

### 4.2. The function $L_*$

The local flow component $L_*$ associated with the horizontal derivative $G_h$ of the Green function $G$ in (8b) is now considered. This flow component, defined by (10), is expressed as

$$L_* = \frac{2P_*}{1+d^3} - 4Q_* + 2L'_* \tag{23a}$$

$$\text{where } \begin{cases} P_* \equiv \dfrac{\beta+h}{d-v} - 2\beta + 2e^v d - h \\ Q_* \equiv e^{-d}(1-\beta)\left(1 + \dfrac{d}{1+d^3}\right) \end{cases} \tag{23b}$$

$$\text{and } L'_* \equiv \frac{-P_*}{1+d^3} + 2Q_* + \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \mathrm{Im}\left[e^M E_1(M) - \frac{1}{M}\right]\cos\theta\, d\theta \tag{23c}$$

The decomposition (23a) expresses the local flow $L_*$ as the sum of two dominant terms and the correction $2L'_*$ defined by (23c). This correction term is numerically approximated further on.

Eqs (15), (23a) and (23b) yield the near-field approximation

$$L'_* \sim -d\left[\frac{\beta}{2}\left(\log\frac{d-v}{2} + \gamma + \sigma\beta + 2\alpha - \frac{7}{2}\right) - 2\alpha + 2\right] \text{ as } d \to 0 \tag{24}$$

Eqs (16), (23a) and (23b) yield the far-field approximation

$$L'_* \sim \frac{1}{d^3}(2\beta - \sigma - 3\alpha\beta) \text{ as } d \to \infty \text{ if } \alpha \ne 1 \tag{25}$$

Along the vertical axis $\alpha = 1$, one has $L'_* \equiv 0$.

The near-field approximations (15) and (24) and the far-field approximations (16) and (25) yield

$$L'_*/L_* = O(d^2 \log d) \text{ as } d \to 0 \tag{26a}$$

$$L'_*/L_* = O(1/d) \text{ as } d \to \infty \tag{26b}$$

where the identity $L'_* \equiv 0$ if $\alpha = 1$ was used. The relations (26) show that one has $L'_* \ll L_*$ in both the near field $d \to 0$ and the far field $d \to \infty$. Eqs (17) and the asymptotic approximations (24) and (25) show that the function $L'_*$ is asymptotically similar to $\rho(1-\rho)^3$ in both the limits $\rho \to 0$ and $\rho \to 1$.

An approximation to the function $L'_*$ of the form

$$L'_*(\rho,\beta) \approx \rho(1-\rho)^3 R_* \text{ where} \tag{27a}$$

$$R_* \equiv \beta A_* - (1-\alpha)B_* + \beta(1-\beta)\rho(1-2\rho)C_* \tag{27b}$$

is considered in the next section. The terms $A_*(\rho)$, $B_*(\rho)$ and $C_*(\rho)$ in (27b) are polynomials in $\rho$.

## 5. Practical approximations

The local flow components $L$ and $L_*$ defined by the integral representations (7) and (10) and the related local flow components $L_z$ and $L_h$ are approximated as

$$L \approx L^a \quad \text{and} \quad L_* \approx L_*^a \tag{28a}$$

$$L_z \approx L_z^a \equiv \frac{v}{d^3} - \frac{1}{d} + L^a \quad \text{and} \quad L_h \approx L_h^a \equiv \frac{h}{d^3} + L_*^a \tag{28b}$$

Hereafter, $L^a$, $L_*^a$, $L_z^a$ and $L_h^a$ denote approximations to the local flow components $L$, $L_*$, $L_z$ and $L_h$, respectively.

Expressions (18a), (18b) and (22) define the approximate local flow component $L^a$ as

$$L^a \equiv -\frac{1}{d} + \frac{2P}{1+d^3} + 2\rho(1-\rho)^3 R \tag{29a}$$

where $P$ and $R$ are defined by (18b) and (22b) as

$$P \equiv e^v\left(\log\frac{d-v}{2} + \gamma - 2d^2\right) + d^2 - v \tag{29b}$$

$$R \equiv (1-\beta)A - \beta B - \frac{\alpha C}{1+6\alpha\rho(1-\rho)} + \beta(1-\beta)D \tag{29c}$$

Here, $\gamma = 0.577\ldots$ is Euler's constant, and $\alpha, \beta, \rho$ are defined by (11) and (17a). Moreover, the polynomials $A(\rho)$, $B(\rho)$, $C(\rho)$ and $D(\rho)$ in (29c) are defined as

$$A \equiv 1.21 - 13.328\rho + 215.896\rho^2 - 1763.96\rho^3 + 8418.94\rho^4 - 24314.21\rho^5 + 42002.57\rho^6 - 41592.9\rho^7 + 21859\rho^8 - 4838.6\rho^9 \quad (29d)$$

$$B \equiv 0.938 + 5.373\rho - 67.92\rho^2 + 796.534\rho^3 - 4780.77\rho^4 + 17137.74\rho^5 - 36618.81\rho^6 + 44894.06\rho^7 - 29030.24\rho^8 + 7671.22\rho^9 \quad (29e)$$

$$C \equiv 1.268 - 9.747\rho + 209.653\rho^2 - 1397.89\rho^3 + 5155.67\rho^4 - 9844.35\rho^5 + 9136.4\rho^6 - 3272.62\rho^7 \quad (29f)$$

$$D \equiv 0.632 - 40.97\rho + 667.16\rho^2 - 6072.07\rho^3 + 31127.39\rho^4 - 96293.05\rho^5 + 181856.75\rho^6 - 205690.43\rho^7 + 128170.2\rho^8 - 33744.6\rho^9 \quad (29g)$$

Expressions (23a), (23b) and (27) define the approximate local flow component $L_*^a$ as

$$L_*^a \equiv \frac{2P_*}{1+d^3} - 4Q_* + 2\rho(1-\rho)^3 R_* \quad (30a)$$

where $P_*$, $Q_*$ and $R_*$ are defined by (23b) and (27b) as

$$P_* \equiv \frac{\beta+h}{d-v} - 2\beta + 2e^v d - h \quad (30b)$$

$$Q_* \equiv e^{-d}(1-\beta)\left(1 + \frac{d}{1+d^3}\right) \quad (30c)$$

$$R_* \equiv \beta A_* - (1-\alpha)B_* + \beta(1-\beta)\rho(1-2\rho)C_* \quad (30d)$$

The polynomials $A_*(\rho)$, $B_*(\rho)$ and $C_*(\rho)$ in (30d) are defined as

$$A_* \equiv 2.948 - 24.53\rho + 249.69\rho^2 - 754.85\rho^3 - 1187.71\rho^4 + 16370.75\rho^5 - 48811.41\rho^6 + 68220.87\rho^7 - 46688\rho^8 + 12622.25\rho^9 \quad (30e)$$

$$B_* \equiv 1.11 + 2.894\rho - 76.765\rho^2 + 1565.35\rho^3 - 11336.19\rho^4 + 44270.15\rho^5 - 97014.11\rho^6 + 118879.26\rho^7 - 76209.82\rho^8 + 19923.28\rho^9 \quad (30f)$$

$$C_* \equiv 14.19 - 148.24\rho + 847.8\rho^2 - 2318.58\rho^3 + 3168.35\rho^4 - 1590.27\rho^5 \quad (30g)$$

The approximations $L^a$ and $L_*^a$ given by (29) and (30) hold within the entire flow region $0 \leq d$, and only involve elementary continuous functions (algebraic, exponential, logarithmic) of real arguments. The errors associated with the approximations (29) and (30) are analyzed in Wu et al. (2016b). This analysis shows that the approximations (29) and (30) are sufficiently accurate for practical purposes.

## 6. Conclusion

The Green function $G$ in the classical theory of wave diffraction radiation by an offshore structure, or a ship at low forward speed, in deep water is expressed as the sum of the fundamental free-space Green function $-1/r$, a non-oscillatory local flow component $L$ and a wave component $W$, in the usual manner. The gradient of $G$ is similarly expressed as the sum of three basic components. The wave components $W$ and $W_h$ in these basic decompositions of $G$ and its gradient $\nabla G$ are expressed in terms of real functions of one variable, specifically the exponential function $e^v$, the Bessel functions $J_0(h)$ and $J_1(h)$ and the Struve functions $\widetilde{H}_0(h)$ and $\widetilde{H}_1(h)$. These functions are infinitely differentiable and can be readily evaluated; e.g. Hitchcock (1957), Abramowitz & Stegun (1965), Luke (1975), Newman (1984b).

The approximations to the local flow components in the expressions for $G$ and $\nabla G$ given here are global approximations that are valid within the entire flow region, unlike the approximations for complementary contiguous flow regions that can be found in the literature, and only involve elementary continuous functions (algebraic, exponential, logarithmic) of real arguments. The analysis of the errors associated with these approximations to the local flow components that is given in Wu et al. (2016b) shows that the approximations are sufficiently accurate for practical purposes. The global approximations given here provide a particularly simple and highly efficient way of numerically evaluating the Green function and its gradient for diffraction radiation of time-harmonic waves in deep water.

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