Scattering of Periodic Surface Waves by a Two Dimensional Platform Supported by a Pile Array

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HIGHLIGHT

A generalized pressure equation, in form of a Laplace equation with complex coefficients obtained, is derived from linearized Navier Stokes equations through homogenization theory. Free surface wave scattering around a two-dimensional platform is investigated by considering the viscous effects due to pile array.

1 INTRODUCTION

Very large scale floating structures supported by small diameter piles are a common form of practical structures in coastal and offshore engineering, e.g. offshore airports, large storage facilities and wave energy converters (Kashiwagi, 2000; Singh, 2014). The scattering problems of non-viscous water waves by multiple bodies have been well resolved using different methods in context of potential flow theory (Kagemoto& Yue, 1986; Spring & Monkmeyer, 1974; Simon, 1982; Linton & Evans, 1990). As the number of piles dramatically increases and more realistic flow conditions need to be considered, viscous effects may significant affect the wake regions of each members in the pile array and dissipate the wave energy along the floating bodies. In this study, we propose a semi-analytical approach to examine the viscous effects on wave scatterings around a two dimensional platform supported by arrays of a large number of piles, as shown in Fig.1.

2 STATEMENT OF THE PROBLEM

We consider a water region of finite and constant depth h'. Infinitesimal waves of characteristic frequency ω' and amplitude A' enter region beneath the platform from the open sea. $k'_0 = \omega'^2/g$ is the characteristic macro-length scale. The water depth is comparable with the wave length, $k'_0 h' = \mathcal{O}(1)$, which is suitable for wave length about 100 meters in a water region with depth of 20 meters. The width of the platform is $2L'$ in x-direction. The draft is D'. Over a large horizontal area $2L'$, vertical cylinder plies of diameter a' are built at the horizontal separation of $\mathcal{O}(\ell')$, where the cylinder spacing is much smaller than the typical wave length, i.e. $k'_0\ell' \ll 1$.

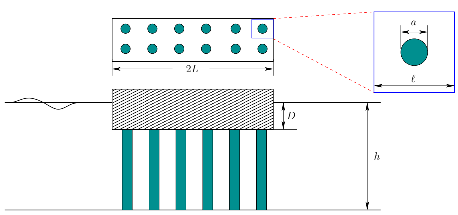


Figure 1: Illustration of long platform supported by piles for 2D problem

The incoming waves dictate the scales of the dynamics pressure $p' \sim \rho g A'$. Using the micro length ℓ' and h'_0 to normalize the spatial coordinates we introduce the following change of variables

$$x_i = x'_i/\ell',\ Z = k'_0 z',\ t = \omega' t',\ h = k'_0 h',\ p = p'/\rho g A',\ (u_i, w) = (u'_i, w')/\omega' A',\ \zeta = \zeta'/A' \tag{1}$$

2.1 Open water region

In the open water region away from the platform, we ignore turbulence and viscosity. The dimensionless potential function ϕ satisfies the Laplace equation $\nabla^2\phi = 0$. Considering harmonic waves, linearity of the problem allows separation of the time factor as $(\phi, p) = (\Phi, P)e^{-it}$. The incoming wave is from $X = -\infty$ towards the platform located between $X = \pm L$. Considering an incident wave in form of $\zeta = \mathcal{A}e^{ik(X+L)-it}$. In terms of eigenfunctions, the potential in the exterior regions, $\Phi = \Phi_I + \Phi_R$ and $\Phi = \Phi_T$ in each sides, can be expresses as

$$\Phi_I = -f_0 e^{-k_0(X+L)}, \quad \Phi_R = -\sum_{n=0}^{\infty} c_n f_n e^{k_n(X+L)} \quad \Phi_T = -\sum_{n=0}^{\infty} b_n f_n e^{-k_n(X-L)} \tag{2}$$

with $k_0 = -ik$, $k_n \tan k_n h = -k \tanh kh$, $n = 1, 2, \cdots$. c_0 and b_0 represent the reflection and transmission coefficients, respectively.

2.2 Viscous flow region beneath the platform

We assume constant eddy viscosity ν_e for turbulent flow inside cylinder array region. For infinitesimal waves, the three dimensional flows are governed by the linearized Reynolds equations. Then the normalized continuity equation reads:

$$\frac{\partial u_i}{\partial x_i} + \epsilon \frac{\partial w}{\partial Z} = 0 \tag{3}$$

where we define $\epsilon \equiv k_0' \ell' = \omega'^2 \ell'/g \ll 1$ is a small ratio of micro-to-macro lengths, and $\sigma = \nu_e/\omega' \ell'^2 = \mathcal{O}(1)$. The dimensionless horizontal and vertical momentum equations are

$$\epsilon \frac{\partial u_i}{\partial t} = -\frac{\partial p}{\partial x_i} + \sigma\epsilon \left(\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \epsilon^2 \frac{\partial^2 u_i}{\partial Z^2} \right), \quad \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial Z} + \sigma \left(\frac{\partial^2 w}{\partial x_j \partial x_j} + \epsilon^2 \frac{\partial^2 w}{\partial Z^2} \right) \tag{4}$$

Two-scale expansions are introduced, $x_i \to x_i + \epsilon X_i$, $u_i \to u_i^{(0)} + \epsilon u_i^{(1)}$, $w \to w^{(0)} + \epsilon w^{(1)}$, $p \to p^{(0)} + \epsilon p^{(1)}$. In view of linearity of Eqs (4), we assume the following solution for the micro-scale flow in the cell

$$\widetilde{u}_i^{(0)} = -K_{ij}(\vec{x}) \frac{\partial \widetilde{p}^{(0)}}{\partial X_j}, \quad \widetilde{p}^{(1)} = -A_j(\vec{x}) \frac{\partial \widetilde{p}^{(0)}}{\partial X_j}, \quad \widetilde{w}^{(0)} = -W(\vec{x}) \frac{\partial \widetilde{p}^{(0)}}{\partial Z} \tag{5}$$

Substitute these assumptions into (3) and (4), we have

$$\frac{\partial K_{ij}}{\partial x_j} = 0, \quad -iK_{ij} = \delta_{ij} + \sigma \frac{\partial^2 K_{ij}}{\partial x_k \partial x_k} - \frac{\partial A_j}{\partial x_i}, \quad x_i \in \Omega_f \tag{6}$$

(6) must be solved in the fluid part Ω_f in the unit cell of area Ω, subjecting to the boundary conditions on the cylinders $K_{ij} = 0$, $x_i \in S_B$ and periodicity boundaries $K_{ij}, A_j :\ \Omega-$ periodic. For uniqueness, we require $\langle A_j \rangle = 0$. The leading order of (4) becomes

$$-iW = 1 + \sigma \frac{\partial^2 W}{\partial x_j \partial x_j}, \quad x \in \Omega_f, \quad -h < Z < -D \tag{7}$$

with boundary conditions $W = 0$, $(x_i, Z) \in S_B$ and $z = -D, -h$ and $W : \Omega-$periodic, which can be solved by the finite element method (FEM). Equation (7) is solved in horizontal two-dimensional space, i.e. $W = W(x_i)$, $i = 1, 2$, and is irrelevant to macro-coordinate Z.

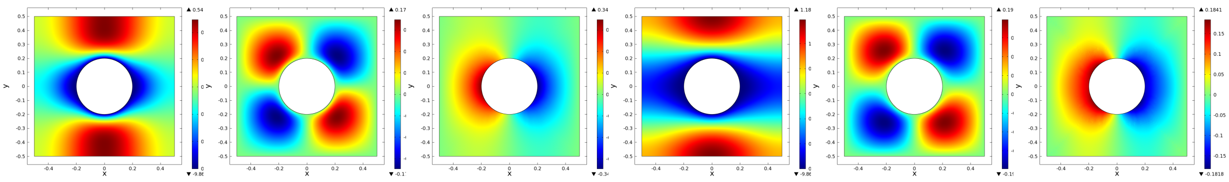
Examples of the FEM results are shown in Fig.2 for the cell problem governed by (6). After solving the canonical cell problem by FEM, we take the cell average for K_{ij} and W, defined by $\langle f \rangle = \iint_{\Omega_f} f dxdy/\Omega$. Then we get Darcy's law $\langle \widetilde{u}_i^{(0)} \rangle = -\mathcal{K}_{ij} \partial \widetilde{p}^{(0)}/\partial X_j$ and the complex permeability defined by $\mathcal{K}_{ij} = \langle K_{ij} \rangle$. Similar numerical results for $\mathcal{W} = \langle W \rangle$ can be obtained when σ is specified. For different wave parameters and eddy viscosity, results of $\mathcal{K}$ and $\mathcal{W}$ are shown in Fig.3.

Leading order velocities $\langle \widetilde{u}_i^{(0)} \rangle$ and $\langle \widetilde{w}^{(0)} \rangle$ are macro-scale variables relating X_i and Z. Cell average of continuity equation (3) gives

$$\Omega_f \left(\frac{\partial \langle u_i^{(0)} \rangle}{\partial X_i} + \frac{\partial \langle w^{(0)} \rangle}{\partial Z} \right) + \iint_{\Omega_f} \frac{\partial u_i^{(1)}}{\partial x_i} dxdy = 0 \tag{8}$$

Using Gauss theorem, periodical and no-slip boundary conditions, the integral above is zero. Hence we obtain the cell averaged continuity equation

$$\frac{\partial \langle \widetilde{u}_i^{(0)} \rangle}{\partial X_i} + \frac{\partial \langle \widetilde{w}^{(0)} \rangle}{\partial Z} = 0 \tag{9}$$



(a) Real(K_{11}) (b) Real(K_{21}) (c) Real(A_1) (d) Imag(K_{11}) (e) Imag(K_{21}) (f) Imag(A_1)

Figure 2: Illustration of FEM solutions for K_{ij} and A_j, $j = 1$. From numerical results, $\mathcal{K} = \mathcal{K}_{11} = \mathcal{K}_{22} = 0.3134 + 0.5764i$, $\mathcal{K}_{12} = \mathcal{K}_{21} = 0$. For A_j, we compare the gradient value, we obtained $\langle \partial A_1/\partial x\rangle = \langle \partial A_2/\partial y\rangle = 0.2506 + 0.1416i$. $\mathcal{K}_{21}$ represents the y-direction velocity caused by the macro-scale pressure gradient in x-axis. Parameters are: porosity $\mathcal{N} = 0.8743$, and dimensionless viscous coefficient $\sigma = 0.0259$.

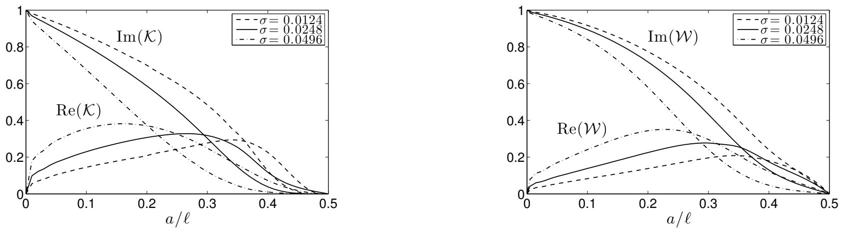


Figure 3: Comparison of magnitude of $\mathcal{K}$ and $\mathcal{W}$. $\sigma = \nu_e/\omega'\ell'^2$. Taking laboratary values: using constant $\nu_e = 1.0$E-3 m^2/s, $\ell' = 8$ cm. $h' = 0.5$ m. $\omega' = 4\pi,\ 2\pi,\ \pi$ s^{-1}, corresponding to $kh = 8.05,\ 2.08,\ 0.77$.

Cell average of horizontal and vertical momentum equations,

$$-i\langle \widetilde{u}_i^{(0)}\rangle = -\mathcal{N}\frac{\partial \widetilde{p}^{(0)}}{\partial X_i} - \alpha_{ik}\frac{\partial \widetilde{p}^{(0)}}{\partial X_k}, \quad -i\langle \widetilde{w}^{(0)}\rangle = -\mathcal{N}\frac{\partial \widetilde{p}^{(0)}}{\partial Z} - \beta\frac{\partial \widetilde{p}^{(0)}}{\partial Z} \tag{10}$$

where

$$\alpha_{ik} = \frac{1}{\Omega}\oint ds\left[\sigma\left(\frac{\partial K_{ik}}{\partial x_j} + \frac{\partial K_{jk}}{\partial x_i}\right) - A_k\delta_{ij}\right]n_j, \quad \beta = \frac{\sigma}{\Omega}\oint ds\frac{\partial W}{\partial x_j}n_j \tag{11}$$

does not depend on the vertical coordinate Z. $\mathcal{N} = \Omega_f/\Omega = 1 - \pi a^2/4\ell^2$ is the porosity of the cell. The values of $\alpha_{11} = \alpha_{22} = -\mathcal{N} - i\mathcal{K}$, $\alpha_{12} = \alpha_{21} = 0$ and $\beta = -\mathcal{N} - i\mathcal{W}$. Substituting the momentum equations (10) into the continuity equation (9), we have the general pressure equation in form of the Laplace equation as

$$\frac{\partial}{\partial X_i}\left(\mathcal{K}\frac{\partial \widetilde{p}^{(0)}}{\partial X_i}\right) + \mathcal{W}\frac{\partial^2 \widetilde{p}^{(0)}}{\partial Z^2} = 0 \tag{12}$$

In usual isotropic porous media, where there is no contrast of length scale in all directions, the macro-scale equation is Laplace equation. Herein x_i and Z are in sharp contrast, hence (12) is not isotropic. $\mathcal{K}(X_i)$ and $\mathcal{W}(X_i)$ vary with horizontal macro-coordinate, and are irrelevant to vertical coordinate Z.

The value of dimensionless viscosity σ depends on the eddy viscosity coefficient inside the viscous flow region. A simple energy balance model had been proposed for a similar problem (Mei et al, 2014). Note from (6), the dimensionless viscosity affects the complex number $\mathcal{K}$. Therefore the turbulent viscosity ν_e affects not only the dissipation but also the phase difference between the velocity and the pressure gradient.

2.3 Matching conditions

For a linear problem, we expect a solution in form of $\widetilde{p}^{(0)} = Pe^{-it}$. We require the velocity to be continuous at $X = \pm L$ for $-h \le Z < -D$, and to vanish for $-D \le Z \le 0$. At cylinder array region, the average horizontal velocity is $\langle \widetilde{u}^{(0)}\rangle$. Combing with the definition $\langle \widetilde{u}^{(0)}\rangle = -\mathcal{K}\partial P/\partial X$, so we have $\partial\Phi/\partial X = -\mathcal{K}\partial P/\partial X$ for $-h \le Z < -D$ and $\partial\Phi/\partial X = 0$ for $-D \le Z < 0$. Another matching condition is continuous of potential or pressure, i.e. $i\Phi = P$ for $-h \le Z < -D$.

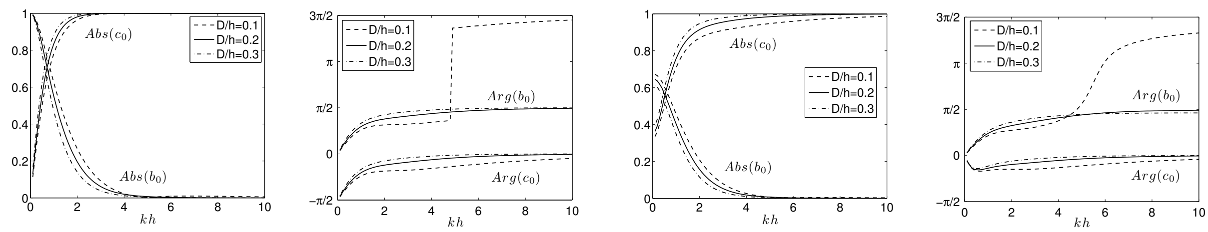
3 NUMERICAL PROCEDURE

For finite length platform in 2D space, the general solution of (12) $P = \sum_{n=0}^{\infty} P_n$ is rewritten as $P_0(X) = p_0X + q_0$, $P_n(X) = p_n \sinh\lambda_n X + q_n\cosh\lambda_n X$, $n = 1, 2, ...$ with $\lambda_n = K_n\sqrt{\mathcal{W}/\mathcal{K}}$, $K_n = n\pi/(h - D)$. p_n and

q_n for $n = 0, 1, 2, 3...$ are complex coefficients to be determined from the other boundary conditions. Using the matching conditions at $X = \pm L$, we can obtained the unknowns through Ritz method numerically. Convergence tests have been carried out to find the order n in series expansions of Φ and P. Preliminary verifications of the calculation procedure are examined through comparison with open water flows by setting $\mathcal{K} = \mathcal{W} = i$ and the porosity $\mathcal{N} = 1$. To verify present numerical solution, we compare the numerical results with Stoker's solution for a thin plate slab with $D = 0$. By replacing the value of $\mathcal{K}$, $\mathcal{W}$, and $\mathcal{N}$ in the code, we can calculate the full problem straightforward.

4 TRANSMISSION AND REFLECTION

For the case platform without pile array, the phase difference between refection and transmission wave is $(-1)^n\pi/2$ for $k_n < k < k_{n+1}$. When there is pile array, the difference is $\pi/2$ for short waves when the draught is large, e.g. $D/h = 0.2 \sim 0.3$. However, the phase difference is zero for infinite long waves. The phase difference is gradually increase to $\pi/2$ when kh increases, as shown in Fig.4. For no piles cases, the phase angle of transmission waves has a sharp jump π. When pile array is in existence, there is a smooth transition. The transmission coefficient b_0 is smaller in for pile supported platform than that of no piles, indicating the eddy viscosity can dissipate a lot of wave energy. Due to the existing of cylinders, the reflection coefficient c_0 is larger in cylinder region.



(a) platform without pile array (b) platform with pile array

Figure 4: Comparison of reflection and transmission coefficient. $L/h = 1$.

5 CONCLUSIONS AND FUTURE WORKS

Using homogenization theory, the scattering problem of free surface waves around cylinder arrays and platform is investigated. Preliminary results of a two dimensional problem are obtained, and viscous effects on the transmission and reflection coefficients are discussed. The proposed semi-analytical method can be easily extended to three dimensional problems with complex geometry profile of the platform. More numerical results will be reported in the conference.

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