

## An energy dissipation model for wave resonance problems in narrow gaps formed by floating structures

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### HIGHLIGHT

A simple conceptual energy dissipation model is proposed for the piston mode wave resonance in narrow gaps formed by floating bodies. Fundamental physical mechanisms for a number of physical phenomena observed in model tests are understood with the aid of the new conceptual model. An approach to determine artificial damping term in the modified potential flow model is proposed.

### 1 INTRODUCTION

The gap resonance problem has attracted extensive attentions in the past decades due to its engineering significance. For this class of problems, potential flow models generally over-predict resonant response amplitudes significantly due to an inherent limitation where energy dissipations due to viscous effects are neglected. In the frame of linear potential flow theory, [Chen \(2004\)](#) modified the free surface boundary condition via introducing a linear artificial dissipative term. The modified potential flow models are able to predict resonant response amplitudes accurately and efficiently with a damping coefficient being tuned against experimental data.

A conceptual model based on control volume concept is proposed in this study for the motion of the oscillating fluid bulk in a fixed narrow gap. A linearized damping coefficient, which is dependent on body geometries, response amplitude and friction and local loss coefficients similar to those in pipe flows, is introduced to account for the energy dissipations in gap resonance problems. A quantitative link is established between the damping coefficient in the present conceptual model and that used in the modified potential models by [Chen \(2004\)](#) and [Lu et al. \(2011\)](#).

### 2 A CONCEPTUAL MODEL FOR GAP OSCILLATIONS

The gap resonance problem considered in this study is illustrated in Fig. 1. To analyze the wave induced motion of the fluid in the gap, the energy conservation of the fluid enclosed by a control volume (*CV*) is considered. The *CV* includes three regions, denoted as region 1, 2 & 3 as shown in Fig. 1. The region 3 represents a zone outside the gap that is influenced by the gap flow, which is an unknown a priori. In the dimension ( $s^*$  and  $B^*$ ) of the region 3, depth  $s^*$  can be determined through experimental calibrations based on a principle that the flow velocity in/out of the exterior boundaries of region 3 is significantly smaller than the flow velocity in the gap. The energy conservation equation over the *CV* reads

$$\frac{d(K+U)}{dt} = W + \iint_{CS} \dot{Q} dS \quad (1)$$

where  $K$  and  $U$  are the total kinetic and potential energy contained in the *CV*, which can be approximated respectively as

$$K = \sum_{j=1}^3 K_j = \frac{\rho}{2} \sum_{j=1}^3 S_j V_j^2, \quad U = \frac{\rho g B_g}{2} \eta(t)^2 \quad (2)$$

where  $\rho$  is the density of fluid,  $K_j$ ,  $S_j$  and  $V_j$  are the kinematic energy, fluid volume and mean velocity of the  $j$ -th region, respectively,  $g$  is acceleration due to gravity and  $\eta(t)$  is the amplitude of

fluid oscillation in the gap relative to the mean water level.

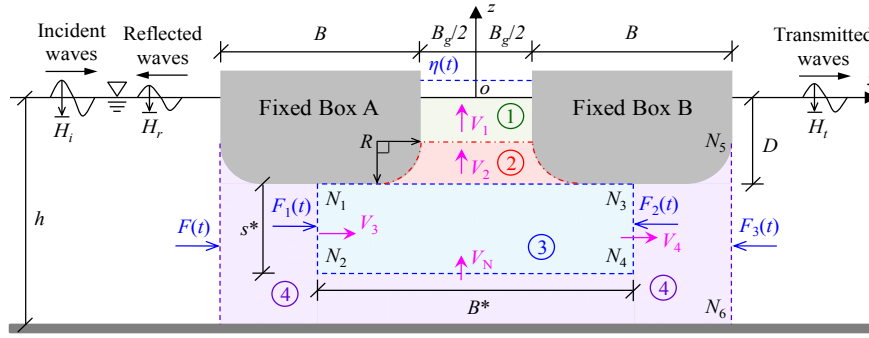


Fig. 1 Sketch of gap resonance between two fixed identical boxes with a curved gap entrance

The rate of work done on the  $CV$  and rate of energy flux can be respectively approximated as

$$W = F_1 V_3 - F_2 V_4 - W_D, \quad \iint_{CS} \dot{Q} dS = \frac{\rho}{2} s^* (V_3^3 - V_4^3) \quad (3)$$

where  $F_1$  and  $F_2$  are forces acting on surface  $N_1N_2$  and  $N_3N_4$  respectively;  $W_D$  represents the summation of energy dissipation due to friction forces on the  $CV$  surfaces (body surfaces and seabed) and viscous flows inside the  $CV$ . Please note  $V_N/V_1 \approx 0$  is assumed in driving Eq. 2. By expressing the area of the regions as  $S_1 = (D - R)B_g$ ,  $S_2 = (2 - \pi/2)R^2 + RB_g$ ,  $S_3 = s^*B^*$ , and further considering the continuity conditions at the interfaces between regions 1 & 2 and regions 2 & 3, we have  $V_1 = d\eta/dt$ ,  $S_2 V_2/R = B_g V_1$ , and  $s^* V_3 = B_g V_1 + s^* V_4$ . Therefore, based on Eqs. 1~3, we can obtain the equation of motion for the fluid in the gap

$$\rho \left[ B_g (D - R) + \frac{B_g R}{1 + (2 - \pi/2)R/B_g} + B^* s^* \left( \frac{B_g}{s^*} + \frac{V_4}{V_1} \right)^2 \right] \frac{d^2 \eta}{dt^2} + \rho g B_g \eta(t) + \frac{W_D}{d\eta/dt} = F_1 \frac{B_g}{s^*} + (F_1 - F_2) \frac{V_4}{V_1} + \frac{\rho}{2} s^* \frac{V_3^3}{V_1} \left( 1 - \left( \frac{V_4}{V_3} \right)^2 \right) \quad (4)$$

Assuming  $V_4/V_1$  and  $V_3/V_1$  are small, and applying the same energy conservation to region 4 in Fig. 1, it is straightforward to establish a relationship between  $F_1$  and the wave excitation force  $F$ , as shown in Fig. 1. Thus, Eq. 4 can be re-formulated as blow.

$$m^* \frac{d^2 \eta}{dt^2} + \rho g B_g \eta(t) + \frac{W_D}{d\eta/dt} = \gamma F, \quad \text{where } m^* = \rho \left[ B_g (D - R) + \frac{B_g R}{1 + (2 - \pi/2)R/B_g} + \frac{B^* B_g^2}{s^*} \right] \quad (5)$$

In Eq. 5,  $\gamma$  is a coefficient and  $W_D$  includes the energy dissipations from frictions ( $E_{d-f}$ ) and the turbulence ( $E_{d-t}$ ) in the  $CV$ . It is anticipated that the energy dissipation near the gap entrance plays a dominate role and it can be approximated as  $E_{d-t} = 0.5 \zeta B_g D V_1^2 V_1$ . Here  $\zeta$  is a local loss coefficient mainly associated with the energy loss around the gap entrance, which is often used in hydraulics as a minor loss coefficient. The friction-induced dissipation  $E_{d-f}$  includes the contributions from the shear force ( $\tau_g$ ) acting on the side surfaces of floating bodies in the gap, as well as the shear stress ( $\tau_d$ ) acting on the underside surfaces of bodies and the seabed. The contribution from  $\tau_d$  is expected to be insignificant, and is hence ignored in the following derivations. Thus we have  $E_{d-f} = 2\tau_g D V_1$ . The shear force on the wall in gap, i.e., induced by the oscillatory motion of the water columns, can be approximated by  $\tau_g = 0.5f\rho V_1^2$ , where  $f$  is a friction coefficient. Therefore, the dissipation reads

$$W_D = E_{d-f} + E_{d-t} = \frac{\rho}{2} (2fD + \zeta B_g) V_1^2 V_1 \quad (6)$$

Substituting Eq. 6 into Eq. 5 and noting that  $V_1 = d\eta/dt$ , we obtain a linearized equation of motion:

$$m^* \frac{d^2 \eta}{dt^2} + \varepsilon^* B_g \frac{d\eta}{dt} + \rho g B_g \eta(t) = \gamma F, \quad \text{where } \varepsilon^* = \frac{\rho}{2} \left( 2f \frac{D}{B_g} + \zeta \right) \left| \frac{d\eta}{dt} \right| \quad (7)$$

where  $\varepsilon^*$  represents a linearized damping coefficient. According to Eqs. 7 and 5, the natural frequency  $\omega_n$  of the water oscillation in the gap can be derived as

$$\omega_n = \sqrt{\frac{\rho g B_g}{m^*}} = \sqrt{g \left/ \left[ D - \frac{(2 - \pi/2) R^2}{B_g + (2 - \pi/2) R} + \frac{B^* B_g}{s^*} \right]} \right. \quad (8)$$

Assuming a harmonic wave motion in gap  $\eta(t) = \eta_A \sin(\omega t)$  that are induced by a harmonic excitation force  $F = F_A \sin(\omega t + \varphi)$  with an amplitude  $F_A$  and a phase angle  $\varphi$ , Eq. 7 becomes

$$-\eta_A \omega^2 \sin(\omega t) + \varepsilon \eta_A \omega^2 \cos(\omega t) + \eta_A \omega_n^2 \sin(\omega t) = F_A \frac{\gamma}{m^*} \sin(\omega t + \varphi), \text{ where } \varepsilon = \frac{\varepsilon^* B_g}{m^* \omega} \quad (9)$$

Employing the harmonic analysis, the wave amplitude  $\eta_A$  and phase shift  $\varphi$  can be obtained as

$$\eta_A = \gamma F_A \left/ \left( m^* \sqrt{(\omega_n^2 - \omega^2)^2 + (\varepsilon \omega^2)^2} \right) \right., \quad \varphi = \tan^{-1} \left( \frac{\varepsilon \omega^2}{\omega_n^2 - \omega^2} \right) \quad (10a,b)$$

With Eq. 10a, the resonant frequency can be obtained by the fact that  $d\eta_A/d\omega = 0$ , and the resonant amplitude can be approximated by  $\omega \rightarrow \omega_R$ . Therefore we have

$$\omega_R = \frac{\omega_n}{\sqrt{1 + \varepsilon^2}}, \quad \eta_{A-Res} = \frac{\gamma F_A}{\rho g B_g} \sqrt{1 + \frac{1}{\varepsilon^2}} \quad (11a,b)$$

Considering Eqs. 7 and 9, the damping coefficient  $\varepsilon$  can be further written as  $\varepsilon = \varepsilon_A |\cos(\omega t)|$ , where

$$\varepsilon_A = \eta_A \left( 2f \frac{D}{B_g} + \zeta \right) \left/ \left[ 2 \left( D - \frac{0.43 R^2}{B_g + 0.43 R} + \frac{B^* B_g}{s^*} \right) \right] \right. = \frac{\eta_A \omega_n^2}{2g} \left( 2f \frac{D}{B_g} + \zeta \right) \quad (12)$$

Possible methods for determining the friction coefficient  $f$  can be found in [Soulsby \(1997\)](#) while the energy loss coefficient  $\zeta$  will be determined by using available experimental data.

### 3 EXPERIMENTAL SET-UP AND RESULTS

Physical model tests were conducted in a wave flume of 56 m in length, 0.7 m in width and 0.7 m in depth. Two boxes with draft  $D = 0.252$  m, gap spacing  $B_g = 0.05$  m and breadth  $B = 0.5$  m were fixed in the wave flume. Different edge shapes of the twin boxes were considered in the tests. The edge configuration was measured by using a non-dimensional parameter of roundness, defined as  $R/B_g$ . Regular waves with a period ranging from 0.90 s to 1.50 s were used in the tests. The incident wave height  $H_i$  was fixed at 0.024 m for various  $R/B_g$  values.

Fig. 2a shows that the resonant wave height increases sharply as the corner shapes change from sharp to round shapes. The observed variation of resonant frequency with the roundness is consistent with the understanding base on Eq. 8. The larger value of roundness  $R/B_g$  leads to the higher resonant wave frequency. Fig. 2b shows the phase difference between the free surface motions measured at G4 and G3. It is believed that the excitation force  $F(t)$  is in phase with the free surface elevation at G3. Based on Eq. 10b, the damping coefficient  $\varepsilon_A$  can be calibrated via correlation analysis by using the least-square method. The results of  $\varepsilon_A$  are listed in Table 1. The observed substantial increase in the resonant wave height from sharp corner ( $R/B_g = 0$ ) to round corners ( $R/B_g = 0.5-3.0$ ) (see Fig. 2a) is related to the sudden decrease of  $\varepsilon_A$ . As suggested by Eq. 11b, the decrease of  $\varepsilon_A$  will give rise to an increase in the resonant amplitude  $\eta_{A-Res}$ .

The lines in Fig. 2b are the corresponding fitting curves, through which  $\varepsilon_A$  was obtained. Based on Eq. 12 and the results in Table 1, the energy loss coefficient  $\zeta$  was evaluated ( $f$  was estimated by [Soulsby \(1997\)](#)). Fig. 2c shows that the values of  $\zeta$  is much larger than  $f$ , which suggests that the damping induced by the turbulence/vortex shedding is dominant in comparison with wall frictions. Through conducting model tests for oscillating flow passing sudden expanding pipe, [Smith & Swift \(2003\)](#) quantified a general minor loss coefficient. The obtained values of  $\zeta$  in this study are close to the results by [Smith & Swift \(2003\)](#).

Further studies confirm a quantitative link between the present damping coefficient  $\varepsilon_A$  and the

artificial damping coefficient  $\mu$  used in the modified potential models (Chen 2004; Lu et al. 2011), namely  $\mu = \varepsilon_A \omega_n$ . Details of the establishment of this link will be presented in the Workshop. Fig 3 shows the good agreement between the modified potential solutions with  $\mu = \varepsilon_A \omega_n$  and the experimental results.

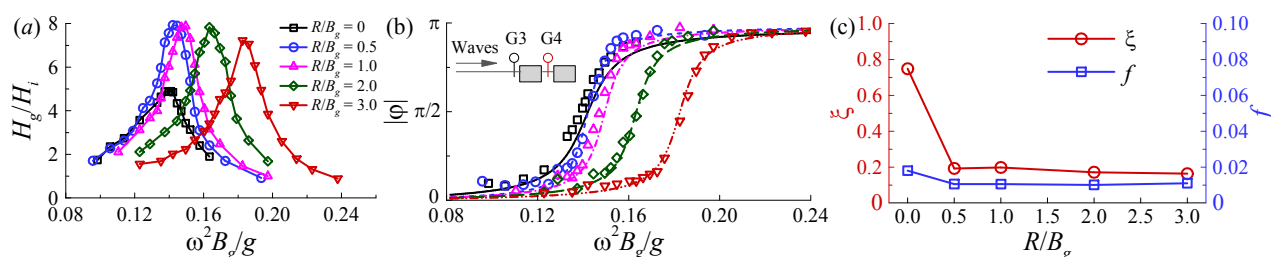
Based on Eq. 12 and the link of  $\mu = \varepsilon_A \omega_n$ , the artificial damping coefficient used in the modified potential flow models (Chen 2004; Lu et al. 2011) can be formulated as follows,

$$\mu = \frac{\eta_A \omega_n^3}{2g} \left( 2f \frac{D}{B_g} + \xi \right) \quad (13)$$

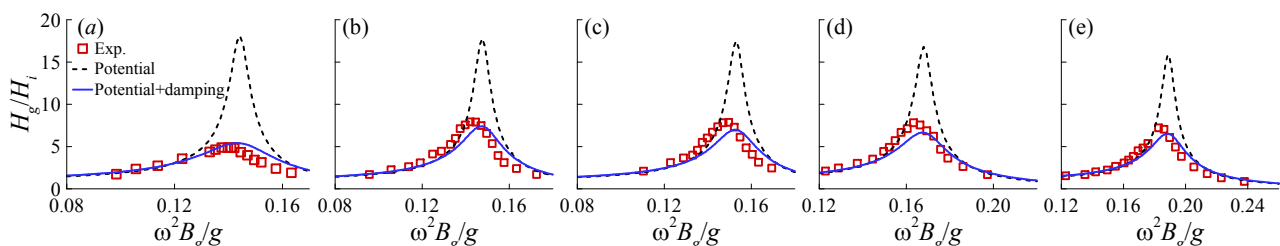
With the aid of Eq. 13, an iterative modified potential model is proposed in this study, by which the resonant wave amplitude in gap can be iteratively predicted given a known loss coefficient  $\xi$ . In addition to the previously examined gap resonance under constant wave amplitude, the cases with varied incident wave heights were also calculated by the iterative modified potential model. Comparisons with the experiments of this study suggest the promising agreement. Moreover, the iterative modified potential model is used to predict the piston-mode fluid resonance in an oscillating moonpool. The present numerical solutions are found to be in good agreement with the experimental data in Faltinsen & Timokha (2015). More details will be presented in the Workshop.

**Table 1 Damping coefficient  $\varepsilon_A$  for different edge roundnesses of  $R/B_g$**

$R/B_g$	0	0.5	1.0	2.0	3.0
$\varepsilon_A$	0.0729	0.0408	0.0435	0.0432	0.0392



**Fig. 2 (a) Variation of the relative wave height in gap with incident wave frequency at different edge roundness; (b) Phase shift of free surface motions at G4 and G3 versus incident wave frequency for different edge roundnesses; (c) Local (minor) energy loss coefficient  $\xi$  and friction coefficient  $f$  versus edge roundness.**



**Fig. 3 Comparison of numerical predictions of potential flow model (with/without damping term) with experimental results for different roundnesses: (a)  $R/B_g = 0$ ; (b)  $R/B_g = 0.5$ ; (c)  $R/B_g = 1.0$ ; (d)  $R/B_g = 2.0$ ; (e)  $R/B_g = 3.0$ . Damping coefficient  $\mu = \varepsilon_A \omega_n$ , where  $\varepsilon_A$  is listed in table 1 and  $\omega_n$  is the resonant frequency.**

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