Asymmetric Water-Entry of a Wedge with Rolled-up Vortex Sheet

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1. INTRODUCTION

Asymmetric water entry of a wedge into a liquid free surface is a typical representation of the initial stage of fluid/structure impact in various engineering applications. The geometry of the body enables self-similar variables to be used, which incorporate the time variable into spatial ones. The problem has already been extensively considered. The work over the last decade includes those by Semenov & Iafrati (2006), Semenov & Wu (2012) based on the integral hodograph method and those by Xu, Duan & Wu (2008, 2010) based on the BEM. These solutions clearly identified a singular behaviour of the velocity and pressure at the corner point of the wedge apex. Attempts to deal with the singularity through the Kutta condition were made by Riccardi & Iafrati (2004) using point vortices while the free surface remains unchanged and by Xu & Wu (2015) using the method of discrete vortexes.

In this study we extend the integral hodograph method developed previously for the vortex-free water entry problems to include a rolled-up vortex sheet shed from the edge of the wedge. The vortex sheet in ideal fluid can be considered as the limit of infinite Reynolds number and away from the vortex sheet the flow will remain irrotational. To deal with the vortex sheet, we adopt the general approach in Pullin (1978) who considered the self-similar flow due to vortex sheet shed from the corner of an infinite wedge in the unbounded fluid domain. The Kutta condition applied at the wedge apex removes the velocity singularity and determines the total circulation of the vortex sheet.

In the presentation, we focus our attention on the shape of the vortex line and the free surfaces as well as on the pressure distribution along the wedge surface. The integral hodograph method is used to derive analytical expressions for the derivative of the complex potential and for the complex velocity, both of which are defined in the first quadrant of a parametric plane. It enables the original partial differential equation with nonlinear boundary conditions on the unknown free surface and on the unknown vortex sheet to be reduced to a system of integro-differential equations along the axes of the first quadrant and a line which is the mapping of the vortex line in the parametric plane, respectively. The motion of the vortex sheet is governed by Birkhoff-Rott integro-differential equation. The coupled systems of integral equations are then solved through successive approximations. The calculated free surface shape, streamlines and pressure distributions on the wedge surface are presented and discussed.

2. FORMULATION OF THE PROBLEM

We consider a wedge of inner angle 2α with its tip fixed at the origin, as shown in Fig.1*a*. The flow comes from infinity to the wedge with speed V and the velocity forms an angle γ_{in} with the $x-$axis. A stagnation point A is expected on the windward side of the wedge, where the zero streamline splits. In previous studies (Semenov & Iafrati, 2006, Semenov & Wu, 2012) it was assumed that toward the apex C, the flow accelerates to an infinite speed and negotiates around the sharp corner. Then, it decelerates on the leeward side. Although such flow configuration is possible in ideal fluid, it is very different from real situations especially at local areas.

We will seek the solution of the problem through superposition of two potentials

$$W(Z,t) = W_1(Z,t) + W_2(Z,t) = V^2 t w(z) = V^2 t\left[w_1(z) + w_2(z)\right], \tag{1}$$

where $W_1(Z,t) = V^2 t w_1(z)$ is the complex potential corresponding to water entry of the wedge without vortex sheet, $W_2(Z,t) = V^2 t w_2(z)$ is the complex potential of the vortex sheet, and $z = Z/Vt$ is the self-similar variable. The potential $w_2(z)$ is determined in such a way to provide zero normal velocitycomponent on whole boundary of the flow domain including the wedge surface and the free surface. We choose the first quadrant of the $\zeta -$plane in Fig.1*b* to formulate the boundary-value problems for the complex velocity, dw/dz, and for the function $dw/d\zeta$ on the real and imaginary axes of the first quadrant.

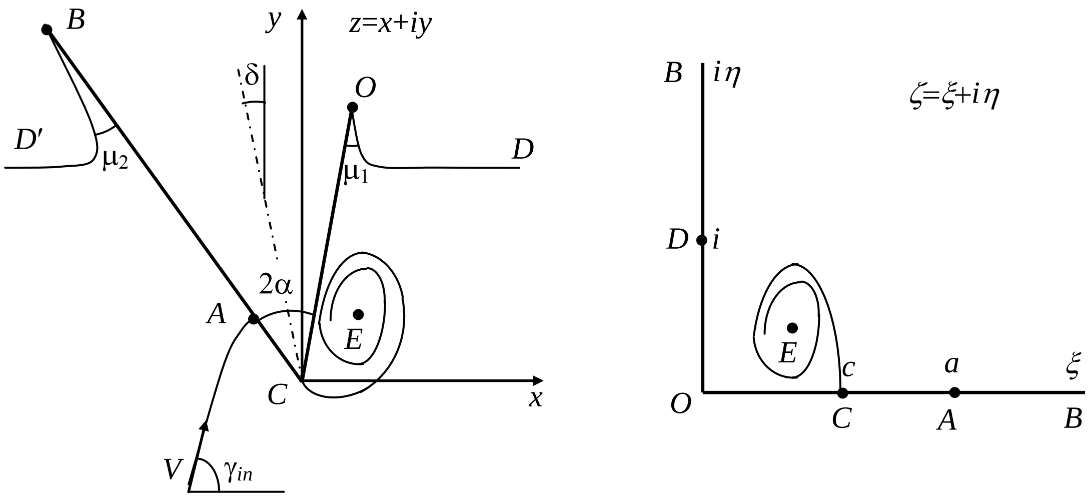

Fig. 1. (*a*) The similarity plane $z = x + iy$, (*b*) the $\zeta -$plane.

According to Eq. (1) we can write

$$\frac{dw}{d\zeta} = \frac{dw_1}{d\zeta} + \frac{dw_2}{d\zeta} = \frac{dw_1}{d\zeta}\left(1 + \frac{dw_2}{d\zeta} \Big/ \frac{dw_1}{d\zeta}\right), \tag{2}$$

$$\frac{dw}{dz} = \frac{dw_1}{dz}\left(1 + \frac{dw_2}{dz} \Big/ \frac{dw_1}{dz}\right) = \frac{dw_1}{dz}\left(1 + \frac{dw_2}{d\zeta} \Big/ \frac{dw_1}{d\zeta}\right), \tag{3}$$

The potential w_1 is chosen as the reference potential as it is affected by the vortex sheet potential w_2 on the nonlinear free surface boundary conditions only, while on the wedge surface both w_1 and w_2 satisfy the impermeable boundary condition independently. Furthermore dw_1/dz may be singular at the tip of the wedge, but it is continuous across the vortex sheet. w_2 due to vortex sheet is then to ensure that the total velocity dw/dz is finite at the tip. From equations (2) and (3) it follows that the derivative of the mapping function,

$$\frac{dz}{d\zeta} = \frac{dw}{d\zeta} \Big/ \frac{dw}{dz} = \frac{dw_1}{d\zeta} \Big/ \frac{dw_1}{dz}, \tag{4}$$

can be written based on the reference potential w_1 only.

The free surface boundary conditions for self-similar flows can be written in the following form (Semenov & Iafrati, 2006):

$$(v + s\cos\theta)\frac{d\ln v}{ds} = s\sin\theta\frac{d\theta}{ds}, \tag{5}$$

$$\frac{d\beta}{ds} = -\frac{1}{\tan\theta}\frac{d\ln v}{ds}, \tag{6}$$

where $v(\eta)$ and $\theta(\eta)$ are the velocity magnitude and angle to the free surface, $s(\eta)$ is the arc length coordinate of the free surface, with $s = 0$ at tip of the thin jet (point O), and $\beta(\eta)$ is the angle between the velocity and the $x-$axis. Taking into account Eqs.(1) - (3), we can write the following relations

$$v(\eta)=\left|\frac{dw_1}{dz}\right|_{\zeta=i\eta}\left|1+\frac{dw_2}{d\eta}/\frac{dw_1}{d\eta}\right|, \qquad \theta(\eta)=\theta_1(\eta)+\theta_2(\eta), \qquad \beta=-\mathrm{Im}\left(\ln\left.\frac{dw}{dz}\right|_{\zeta=i\eta}\right),$$

$$\theta_1=\arg\left(\frac{dw_1}{ds}\right) \quad \text{and} \quad \theta_2=\arg\left(1+\frac{dw_2}{d\eta}/\frac{dw_1}{d\eta}\right).$$

The derivative of the complex potential, $dw_2/d\zeta$, can be found using the Chaplygin singular point method (see Chapter 1 in Gurevich, 1965). According to the Plemelj formula, on the vertex sheet we have

$$\left.\frac{dw_2}{d\zeta}\right|_{\zeta=\zeta(\lambda)}=-J\left[\pm\frac{1}{2}\left(\frac{d\zeta}{d\lambda}\right)^{-1}+\frac{\zeta}{\pi i}\mathrm{P.V.}\int_0^1\frac{\zeta^2(\lambda')-\bar{\zeta}^2(\lambda')}{(\zeta^2(\lambda)-\zeta^2(\lambda'))(\zeta^2(\lambda)-\bar{\zeta}^2(\lambda'))}d\lambda'\right], \tag{7}$$

where $\zeta(\lambda)$ is the mapping of the vortex sheet in the $\zeta-$plane, the sign "+" and "-" correspond to each side of the vortex sheet, as the tangential velocity is discontinuous across the vortex sheet, respectively. The parameter $\lambda=1-\gamma$, in which γ is the normalized circulation along the vortex sheet, is scaled as follows: $\gamma=0$ at point E and $\gamma=1$ at point C. J is the total circulation along the vortex sheet, which is determined from Kutta condition imposed at the corner point C.

The Birkhoff-Rott equation is based on the fact that the circulation Γ at a fixed point Z on the vortex sheet does not vary with time and it relates the motion of vortex sheet, whose shape may be written as $Z(\Gamma,t)$, to the average of local fluid velocities on both sides of the vortex sheet (Moore, 1975),

$$\left.\frac{\partial\bar{Z}}{\partial t}\right|_{\Gamma}=\frac{1}{2}\left[\left(\frac{\partial W}{\partial Z}\right)^{+}+\left(\frac{\partial W}{\partial Z}\right)^{-}\right], \tag{8}$$

where $\pm$ corresponds to the same sign in Eq.(7) and the bar denotes the complex conjugate. Eq.(8) automatically satisfies the conditions of continuous normal velocity and pressure across the vortex sheet (Pullin, 1978). In the similarity plane, using self-similar variable $z=Z/Vt$, Eq.(8) takes the form

$$\bar{z}[\zeta(\lambda)]+(1-\lambda)\frac{d\bar{z}}{d\bar{\zeta}}\frac{d\bar{\zeta}}{d\lambda}=\frac{1}{2}\left[\left(\frac{dw}{dz}\right)^{+}_{\zeta=\zeta(\lambda)}+\left(\frac{dw}{dz}\right)^{-}_{\zeta=\zeta(\lambda)}\right], \tag{9}$$

from which the shape of the vortex sheet in the $\zeta-$plane, $\zeta(\lambda)$, and in the similarity plane, $z[\zeta(\lambda)]$, can be determined.

In Fig.2 are shown results for the wedge angle $2\alpha=90^0$ turned by angle $\delta=25^0$ relative vertical axis and entering vertically into the free surface. The streamlines are shown in Fig.2*a*. The stagnation point is clearly seen. On the leeward side of the wedge there is a relatively small vortex sheet region which is shown by a larger scale in Fig.2*b*. The dashed line corresponds to the vortex line which is truncated after rotating 5 rounds. The rest of the circulation is modelled as an isolated vortex of the strength $1-\lambda_N$, where the value $\lambda_N=0.69$ and $J=0.33$ are obtained for the case shown in Fig.2. The solid lines in Fig.2*b* are the streamlines with the increment 0.005 of the stream function. It can be seen that larger difference between results with and without vortex sheet occurs near the wedge apex, where $c_p\to-\infty$ for the vortex-free flow, and $c_p\approx 0$ for the present case. On the leeward side the pressure coefficient reaches its minimal value $c_{p\min}\approx-8.3$ at the distance $s_m=0.088$ from the wedge apex. Thus, from the present method, the pressure coefficient may still take negative value relative to the ambient pressure, but it is finite. The minimum pressure occurs at a point very close to the centre of the vortex, as can be seen from Fig.2*b*. These results are qualitatively similar to those obtained by Riccardi & Iafrati (2004), who considered the impact of an asymmetric floating wedge and studied flow details close to the apex, neglecting the effects of the free surface deformation.

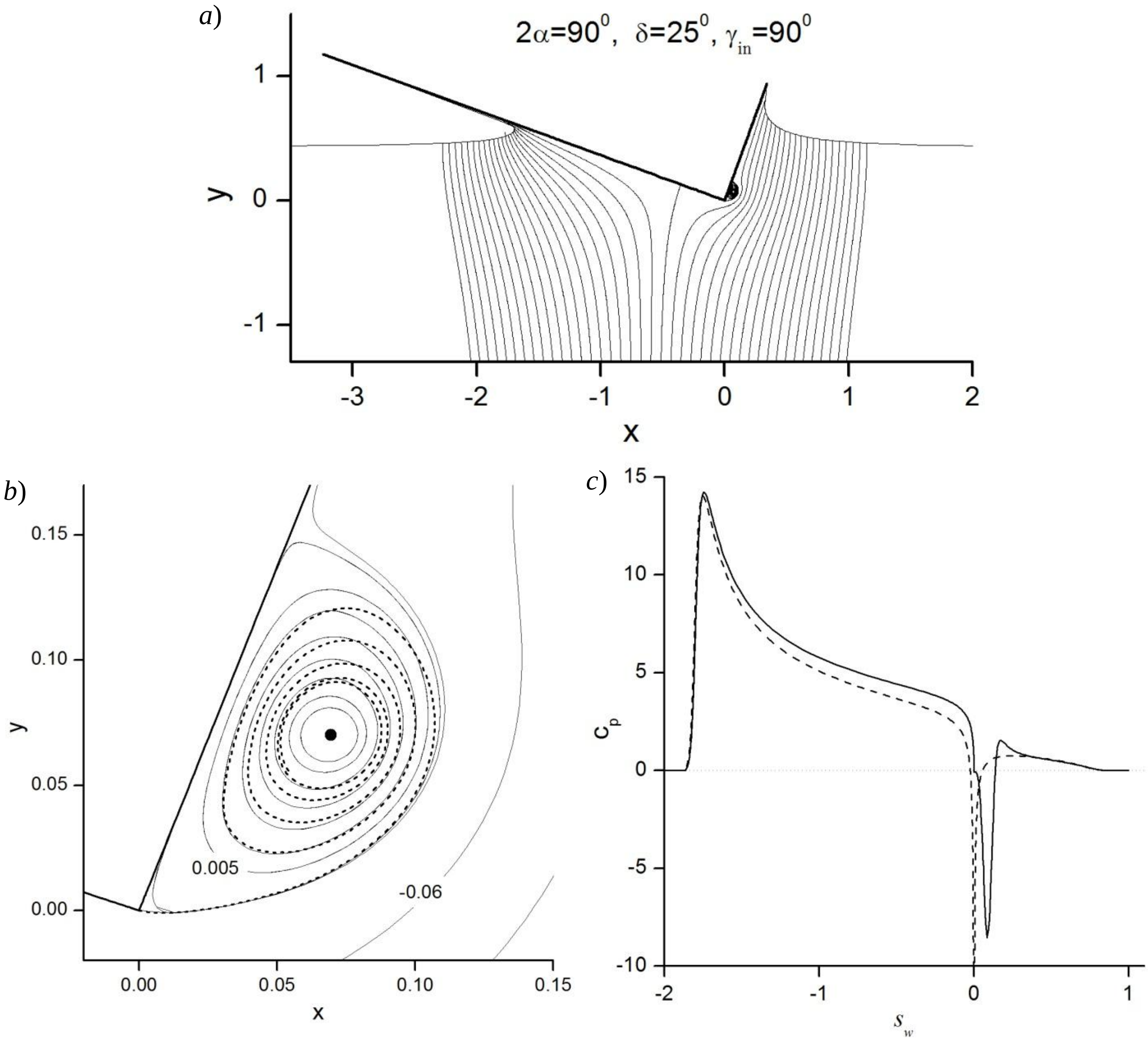

Fig.2. (*a*) the free surface shape and streamlines, (*b*) vortex sheet region: vortex sheet (dashed line) and streamlines (solid lines), c) pressure distributions on the windward side (s_w<0) and leeward (s_w>0): present solution with vortex sheet (solid line), vortex - free solution (dashed line).

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REFERENCES

1. Gurevich, M. I. 1965. Theory of jets in ideal fluids. Academic Press, 585p.
2. Moore, D. W. 1975 The rolling up of a semi-infinite vortex sheet. Proc. Roy. SOC. A 345, 417-430.
3. Pullin, D.I. 1978. The large-scale structure of unsteady self-similar rolled-up vortex sheets. *JFM*, 88, (3), 401 - 430.
4. Riccardi, G. & Iafrati, A. 2004. Water impact of an asymmetric floating wedge. *JEM*, 49, 19 - 39.
5. Semenov, Y.A. & Cummings, L.J. 2006. Free boundary Darcy flows with surface tension.... *EJAM*, 1**7**, 607 – 631.
6. Semenov, Y. A. & Iafrati, A. 2006 On the nonlinear water entry problem... *JFM*, 547, 231–256.
7. Semenov, Y.A. & Wu, G.X. 2012. Asymmetric impact between liquid and solid wedges. *Proc. of RS, A. 469.*
8. Xu, G.D., Duan, W.Y. & Wu, G.X. 2008. Numerical simulation of oblique .... Ocean Engin., 35, 1597 - 1603.
9. Xu, G.D., Duan, W.Y. & Wu, G.X. 2010. Simulation of water entry of a wedge.... *Proc. of RS A. 466, 2219 - 2239.*
10. Xu, G.D. & Wu, G.X. 2015. Oblique water entry of a wedge with vortex shedding. 30th IWWWFB, Bristol, UK.