Second Order Wave Induced Actions for Parametric Rolling

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INTRODUCTION

The main issue that must be addressed in modeling the Parametric Roll as a stochastic process is that it is a highly nonlinear phenomenon. Therefore, spectral and statistical techniques employed in linear systems cannot be freely applied. The objective of the present work is to include the nonlinear influence of the wave passage into the computation of the wave induced actions in a numerical model for the assessment of Parametric Roll in both regular and irregular head seas. The model presented here is a time efficient derivative model sophisticated enough to capture the non-ergodicity behavior of the roll stochastic process reported in literature.

SECOND ORDER RESPONSES DUE TO LINEAR STOCHASTIC EXCITATION

The time domain input-output relation of a linear system is represented by the convolution integral. For a nonlinear systems, it is assumed that its response can be represented by a Volterra series (which is a series of convolution integrals of higher order) of sufficiently high order terms. Then using Fourier analysis the nonlinear system response can be expressed as function of high order response operators, referred as Generalized Frequency Response Functions or GFRF, and the input spectrum, see Billings (2013). Neal (1974), showed that the second order time domain response due to Gaussian excitation with Spectrum S_X, can be represented by the real part (denoted by $\Re$) of the following series:

$$y_2(t) = \frac{1}{2}\Re\left\{\sum_{m=1}^{R}\sum_{n=1}^{R} H_2(\omega_n,\omega_m)e^{j(\omega_n+\omega_m)t-j(\epsilon(\omega_n)+\epsilon(\omega_m))}\sqrt{2S_X(\omega_n)2S_X(\omega_m)\Delta\omega_n\Delta\omega_m}\right\} + \frac{1}{2}\Re\left\{\sum_{m=1}^{R}\sum_{n=1}^{R} H_2(\omega_n,-\omega_m)e^{j(\omega_n-\omega_m)t-j(\epsilon(\omega_n)-\epsilon(\omega_m))}\sqrt{2S_X(\omega_n)2S_X(\omega_m)\Delta\omega_n\Delta\omega_m}\right\} \quad (1)$$

where (ω_1,ω_2) is the frequency space and ϵ represents random phase. The GFRF of second order or second order transfer function is denoted by:

$$H_2(\omega_1,\omega_2) = |H_2(\omega_1,\omega_2)|e^{j\varphi(\omega_1,\omega_2)} \quad (2)$$

Note that if $n = m$, there is an oscillation frequency $2\omega_1$ and a mean term that does not depend on time.

PROBING METHOD

It has been proven in Rugh (1981) that the steady state response of a nonlinear Volterra system with an input of Rth-complex exponentials or probes, with corresponding amplitudes X_i, defined as:

$$x(t) = \sum_{i=1}^{R}(X_i e^{j\omega_i t} + X_i e^{-j\omega_i t}) = \sum_{i=1}^{R}(\,2X_i\cos(\omega_i)) \quad (3)$$

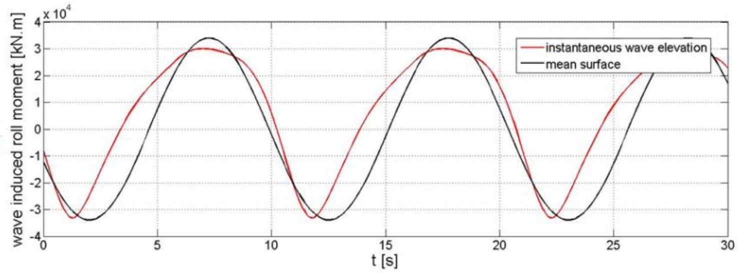
can approximately be defined by a sequence of generalized response transfer functions. For $R = 2$ the second order steady state response becomes:

$$y_2(t) = X_1^2H(\omega_1,-\omega_1) + X_2^2H(\omega_2,-\omega_2) + 4X_1X_2\,|H(-\omega_1,\omega_2)|\cos\left[(\omega_2-\omega_1)t+\varphi_{-1,2}\right] + 4X_1X_2|H(\omega_1,\omega_2)|\cos\left[(\omega_1+\omega_2)t+\varphi_{1,2}\right] + 2X_1^2|H(\omega_1,\omega_1)|\cos\left[(2\omega_1)t+\varphi_{1,1}\right] + 2X_2^2|H(\omega_2,\omega_2)|\cos\left[(2\omega_2)t+\varphi_{2,2}\right] \quad (4)$$

The 2nd-order GFRF can be obtained by comparison between equation (4) and a known solution. Then the coefficients of equation (4) can be obtained. For example, the 2nd-order GFRF $H_2(j\omega_1, j\omega_2)$ can be obtained by extracting the coefficient of the term $\cos(\omega_1+\omega_2)\,t$ of the known solution. It should be mentioned that the 1st-order GFRF's $H_1(j\omega_1)$, $H_1(j\omega_2)$ and the 2nd-order GFRF's $H_2(j\omega_1, j\omega_1)$ and $H_2(j\omega_2, j\omega_2)$ can be obtained with the single tone probes $cos\omega_1$ and $cos\omega_2$. If the excitation has stochastic nature as in equation (1), the 2nd-order GFRF's $H_2(j\omega_1, j\omega_2)$ should be computed for an adequate range of ω_1 and ω_2 with different probes $2X_1cos\omega_1 + 2X_2cos\omega_2$.

NUMERICAL APPROACH

The wave induced forces and moments can be obtained by integrating the pressure over the exact wetted surface of a fixed vessel using a 3D panel method. For simplicity, the wave pressure at the free surface boundary is computed by performing extrapolation of the linear theory of waves. This technique provides results quite similar to those obtained with wave profile stretching methods. Considering the domain of integration limited by the exact wave elevation the forces and moment calculated do not have a pure cosine harmonic oscillation behavior, see figure 1b. Now it is possible to compare the results obtained from the wave pressure integration with equation (4), for different wave inputs. All computations and results shown in the present work will be performed on the containership called NTU (figure 1a), see details in Holden et. al (2008).



(a) (b)

Fig. 1 (a) NTU Containership. (b) Time series of wave induced roll moment

Fourier analysis can be used on the wave passage dependent actions. So, for a regular wave the wave induced roll moment can be represented as:

$$M_{roll}(t) = M_{roll_0} + M_{roll_1}\cos(\omega - \alpha_1) + M_{roll_2}cos(2\omega - \alpha_2) + \cdots \quad (5)$$

So by performing the Fourier analysis up to the second harmonic for different roll positions and taking the slope of the infinitesimal roll moment with the infinitesimal roll position as:

$$\frac{\partial FK_{roll}(t)}{\partial\phi} = \frac{\partial FK_{roll\_0}}{\partial\phi} + \frac{\partial FK_{roll\_1}}{\partial\phi}\cos(\omega - \alpha_{\phi1}) + \frac{\partial FK_{roll\_2}}{\partial\phi}cos(2\omega - \alpha_{\phi2}) \quad (6)$$

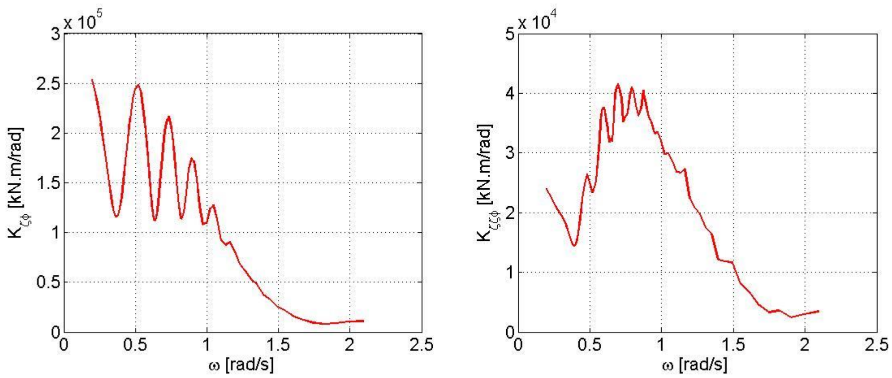
The following derivatives can be defined:

$$K_{\zeta\phi0} = \frac{\partial FK_{roll\_0}}{\partial\phi}, K_{\zeta\phi} = \frac{\partial FK_{roll\_1}}{\partial\phi}, K_{\zeta\zeta\phi} = \frac{\partial FK_{roll\_2}}{\partial\phi} \quad (7)$$

where $\alpha_{\phi1}$ and $\alpha_{\phi2}$ are the phases of the wave frequency and double wave frequency oscillation terms.

In a similar manner derivatives can be obtained for the other degrees of freedom of the vessel. By obtaining these derivatives with their corresponding phases the wave induced forces and moments for a particular vessel position can be obtained using Taylor series exapansion representation.

By executing the same procedure for a range of appropriate wave frequencies and using the probing method previously described, the derivatives can be used to obatain transfer functions between the wave input and the wave induced forces and moment. The derivative $K_{\zeta\phi}$ with wave frequency oscillation will represent a linear transfer function. Meanwhile derivative $K_{\zeta\zeta\phi}$ with double wave frecuency oscilation will represent a second order transfer function with $\omega_1 = \omega_2$, which is in the main diagonal of the frequency space. Finally, the time independent derivative $K_{\zeta\phi0}$ will represent the constant term in equation (4). Derivatives $K_{\zeta\phi}$ and $K_{\zeta\zeta\phi}$ in figure 2 were computed using a wave with 1m of amplitude. In order to obtain the complete second order transfer function as defined in equation (2), the wave input must be similar to the multi tone input of equation (3). Then the coefficients of equation (4) can be obtained by using Fourier analysis in the results from the wave pressure integration for the terms with oscillation frequency $(\omega_1 + \omega_2)$ and $(\omega_2 - \omega_1)$, often called as intermodulation terms. Results have shown that the main diagonal terms are significantly higher than the intermodulation terms. Hence, off-diagonal terms will be disregarded for simplicity. Finally, wave amplitude must be addressed; this is done by computing the derivatives shown above for various wave amplitudes. Then interpolation is performed when required.



(a) (b)

Fig. 2 (a) derivative $\boldsymbol{K_{\zeta\phi}}$vs frequency. (b) derivative $\boldsymbol{K_{\zeta\zeta\phi}}$vs frequency.

PARAMETRIC ROLL

Neves and Rodriguez (2006a) introduced a Heave-Roll-Pitch coupled model for Parametric Rolling in head seas where the restoring actions and the wave induced actions are modelled as a Taylor series expansion up to the third order. By modifying that model by introducing the wave-induced actions as equations (5), and taking their corresponding derivatives, the corresponding roll coupled equation will be:

$$(I_{xx}+A_{44})\ddot{\phi}+B_{14}\dot{\phi}+B_{24}\dot{\phi}|\dot{\phi}|+K_{\phi}\phi+K_{z\phi}+K_{\phi\theta}\phi\theta+\frac{1}{2}K_{zz}z^2\phi+\frac{1}{6}K_{\phi\phi\phi}\phi^3+\frac{1}{2}K_{\theta\theta\phi}\theta^2\phi$$
$$+K_{z\phi\theta}z\phi\theta+K_{\zeta\phi}(t)\phi+K_{\zeta z\phi}(t)z\phi+K_{\zeta\phi\theta}(t)\phi\theta=0 \quad (8)$$

where:

$$K_{\zeta\phi}(t)=K_{\zeta\phi0}(\omega_e,\zeta)+K_{\zeta\phi}(\omega_e,\zeta)\cos(\omega_e t-\varphi_{\phi1})+K_{\zeta\zeta\phi}(\omega_e,\zeta)\cos(2\omega_e t-\varphi_{\phi2}) \quad (9)$$

$$K_{\zeta z\phi}(t)=K_{\zeta z\phi0}(\omega_e,\zeta)+K_{\zeta z\phi}(\omega_e,\zeta)\cos(\omega_e t-\varphi_{z1}) \quad (10)$$

$$K_{\zeta\phi\theta}(t)=K_{\zeta\phi\theta0}(\omega_e,\zeta)+K_{\zeta\phi\theta}(\omega_e,\zeta)\cos(\omega_e t-\varphi_{\theta1}) \quad (11)$$

Notice that the heave and pitch equations will be represented in a similar fashion. Numeral 4 indicate roll direction. A and B represent the added mass and damping hydrodynamic coefficients. K is roll moment derivative; the indexes indicate the derivative direction. Hydrostatic restoring actions are represented by the displacement derivatives (heave z, roll ϕ and pitch θ), while the wave induced restoring action are represented by the wave index ζ. Note that derivatives are dependent on the wave amplitude and they need to be transformed into the encounter frequency ω_e. Also the double frequency terms for the coupled derivatives $K_{\zeta z\phi}$ and $K_{\zeta\phi\theta}$ were omitted.

A mixed methodology which combined the classical seakeeping approach and time domain simulations techniques were used to apply the Neves and Rodriguez model for irregular waves, see Rodriguez et al (2015). The methodology is based on the fact that the oscillatory behaviour of the time dependent terms of equation (8) is now a random process. Therefore, it is assumed that random processes can be introduced in equation (8) by generating realizations based on the linear and second order transfer function with the wave spectrum excitation. In particular, the realization of the second order part has the form of the time series of equation (1), excluding the intermodulation terms here.

The model described in this section is sophisticated enough to represent the reported bifurcation of the parametric rolling in regular seas. Additionally, it is a very time efficient model. A comparison between a realization of the roll process produced with the presented derivative model and a realization produced by a state of the art hybrid simulation code called “DSSTAB” (which computes the hydrostatic and wave induced actions in each time step considering both instantaneous vessel positions and wave elevation, see details in Pasquetti et al 2012) for a given wave condition of Hs=7m and Tp=10.54s is shown in figure 3.

Figure 4 shows that the derivative model can accurately predict the parametric roll in irregular waves. More important, the computation time of the derivative model was less than 2 seconds (after the pre-computation of the derivatives) while the realizations with the DSSTAB took approximately an hour. Taking advantage of this feature of the model, 500 realizations of the same wave condition were generated. Figure 4a shows the probability density function of all roll realizations. It is clear that the roll process is non-Gaussian and non-ergodic as reported in literature. Moreover, extreme value distribution of roll from the 500 realizations is plotted in figure 4b using block maxima principle, see Kim and Troesch (2013). This process took only a couple of hours with the derivative model instead of the several hours (probably months) needed with models similar to the DSSTAB.

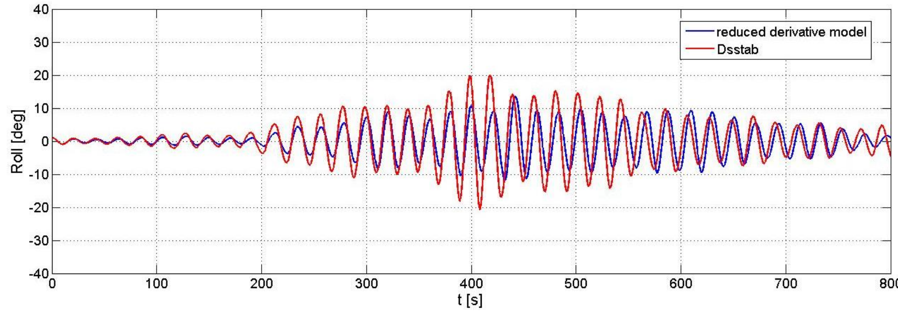
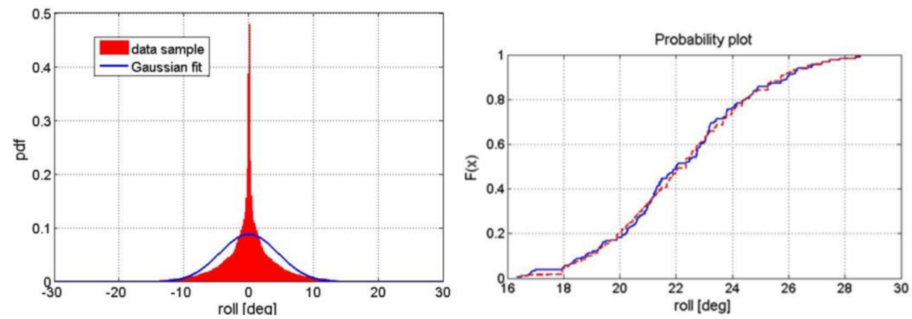


Fig. 3 One Roll realization for Hs=7m and Tp=10.54s **.**



(a) **(b)**

Fig. 4 (a) Probability density function of roll. (b) Extreme value distribution of Roll from 500 realizaions.

CONCLUSIONS

A methodology for computing second order transfer functions of the wave induced forces and actions have been presented. Regardless of the simplification of the second order transfer function (by disregarding intermodulation terms), implementation of time domain realizations of the second order transfer functions in a heave-roll-pitch coupled model produces results compatible with results obtained with a more accurate model.

The main advantage of this new heave-roll-pitch coupled model is its time efficiency. Producing 500 realizations of the movement processes takes merely few hours. This feature makes this new model appropriate for performing statistics and probability analysis of the parametric rolling under stochastic conditions.

REFERENCES

[1] BILLINGS, S.A., 2013, *Nonlinear System Identification: NARMAX Methods in the Time, Frequency, and Spatio-Temporal Domains*, John Wiley & Sons, UK.

[2] Neal, E., 1974. Second order hydrodynamic forces due to stochastic excitation. In *Proceedings 10th ONR Symposium*. Cambridge, Mass.

[3] RUGH, W.J., 1981, *Nonlinear System Theory*, Web version (2002), ISBN 0-8018-2549-0, Baltimore, USA.

[4] HOLDEN, C., GALEAZZI, R., RODRIGUEZ, C., PÉREZ, T., FOSSE, T.I., BLANKE, M., NEVES, M. A. S., 2008, "Nonlinear Container Ship Model for the Study of Parametric Roll Resonance", *Modeling, Identification and Control*, vol. 28, no. 4, pp.87-103.

[5] NEVES, M. A. S., RODRIGUEZ, C. A., 2006a, "On Unstable Ship Motions Resulting from Strong Non-Linear Coupling". Ocean Engineering, vol. 33, no. 14 (Oct), pp. 1853-1883.

[6] Claudio A. Rodríguez, Polo, J. C. F., Neves M.A.S., 2016. A Time-Efficient Approach for Nonlinear Hydrostatic and Froude-Krylov Forces for Parametric Roll Assessment in Irregular Seas. Ocean Engineering, Volume 120, 1 July 2016, Pages 246–255

[7] Eduardo Pasquetti, COELHO, L. C. G., Neves M.A.S., Mauro C Oliveira, Paulo T. T. Esperança, Claudio A. Rodríguez, Miguel Angel Celis, Polo, J .C. F., 2012. A Nonlinear Numerical Algorithm For Time-Domain Hydrodynamic Simulations Of Vessel Motions In The Presence Of Waves. Proceedings Of The Asme 2012 31st International Conference on Ocean, Offshore and Arctic Engineering Proceedings of the ASME 2012 31st International Conference on Ocean, Offshore and Arctic Engineering, Rio de Janeiro, 1 1-14 Houston: ASME.

[8] D.-H. Kim, A. Troesch, 2013, Statistical estimation of extreme roll responses in short crested irregular head seas, in: Transactions of SNAME, Vol. 121, Society of Naval Architects and Marine Engineers, pp. 123–160.