On resonant modes in moonpools with restrictions or recesses

Bernard Molin[1*], Xinshu Zhang[2]
[1] Ecole Centrale Marseille & IRPHE, [2] Shanghai Jiao Tong University
bernard.molin@centrale-marseille.fr, xinshuz@sjtu.edu.cn

1 INTRODUCTION

Moonpools can be found in drillships and some other offshore floating supports. They usually have vertical walls, from deck to keel. However some moonpools in drillships have "recesses" (see Figure 1). The floating production platform MONOBR, proposed by Sphaier *et al.* (2007), has a large axisymmetric moonpool with a "restriction". Similar restrictions are found in some turrets of FPSOs or FLNGs (see Molin *et al.* 2014).

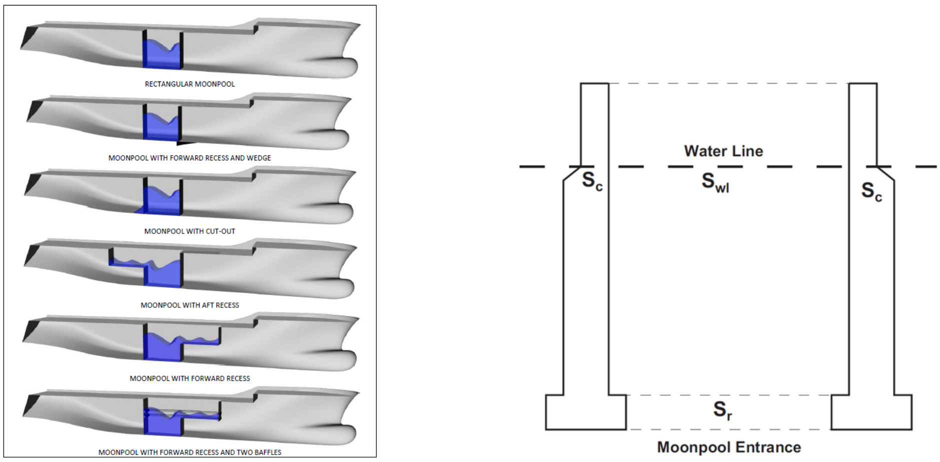

Figure 1: Types of moonpools on drillships (left, taken from Hammargren & Törnblom 2012). The MONOBR concept (right, taken from Sphaier *et al.* 2007).

Moonpools are prone to undesirable resonance problems, under outer wave action or forward speed, and it is desirable, at the design stage, to be able to predict their resonant frequencies. Natural modes in moonpools consist in the so-called piston mode, up and down motion of the entrapped water, and in sloshing modes, similar to the sloshing modes in a tank.

Molin (2001) proposed a theoretical frame to derive the resonant frequencies for rectangular moonpools with vertical walls. His work was based on some simplifying assumptions: the floating support is motionless, the waterdepth is infinite, the length and breadth of the support are infinite. The fluid domain is then decomposed into two parts: the moonpool and a semi-infinite fluid domain below the keel level. Linearized potential flow theory is used, the velocity potential being written as an eigenfunction expansion in the moonpool. The matching condition with the lower fluid domain is written as an integral equation relating the potential and its vertical derivative.

In this paper we follow the same procedure, with the moonpool being decomposed into two parts where different eigen-function expansions are used and need to be matched on the common boundary. We first consider the axisymmetric case (MONOBR) and we focus on axisymmetric resonant modes. Then we address rectangular moonpools with one or two recesses.

2. AXISYMMETRIC CASE

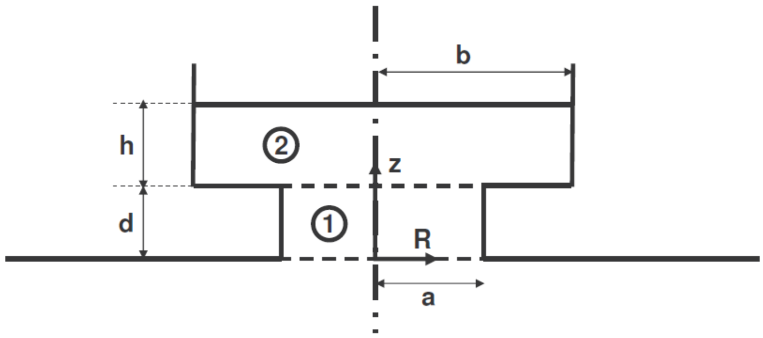

Figure 2: Geometry.

The geometry is illustrated in figure 2. The linearized velocity potential $\Phi(R, z, t)$ is supposed to be harmonic in time at a frequency ω: $\Phi(R, z, t) = \varphi(R, z) \cos(\omega t + \psi)$. The reduced potential φ verifies the Laplace equation $\Delta\varphi = 0$, the linearized free surface equation $g\,\varphi_z - \omega^2\,\varphi = 0$ at $z = d + h$, no-flow conditions at the solid walls and the following matching condition with the lower fluid domain (see Molin 2001):

$$\varphi(x, y, 0) = \frac{1}{2\pi} \iint_S \frac{\varphi_z(x', y', 0)}{\sqrt{(x - x')^2 + (y - y')^2}}\, \mathrm{d}x'\, \mathrm{d}y' \tag{1}$$

with S the surface of the moonpool $R \leq a$.

The interior of the moonpool is decomposed into two rectangular (in cylindrical coordinates) subdomains: subdomain 1 for $0 \leq z \leq d,\ \ 0 \leq R \leq a$, and subdomain 2 for $d \leq z \leq h,\ \ 0 \leq R \leq b$.

The following eigen-function expansions apply:

$$\varphi_1(R, z, \theta) = A_0 + B_0\,\frac{z}{d} + \sum_{n=1}^{\infty} (A_n \cosh k_n z + B_n \sinh k_n z)\, J_0(k_n R) \tag{2}$$

$$\varphi_2(R, z, \theta) = C_0 + D_0\,\frac{z - d}{h} + \sum_{n=1}^{\infty} (C_n \cosh \lambda_n (z - d) + D_n \sinh \lambda_n (z - d))\, J_0(\lambda_n R) \tag{3}$$

with J_0 the Bessel function and k_n, λ_n the roots of $J_0'(k_n a) = J_0'(\lambda_n b) = 0$.

The bottom boundary condition writes

$$A_0 + \sum_{m=1}^{\infty} A_m\, J_0(k_m R) = \frac{1}{2\pi} \int_0^a r\, \mathrm{d}r \int_0^{2\pi} \frac{B_0/d + \sum_n B_n k_n\, J_0(k_n r)}{\sqrt{R^2 + r^2 - 2Rr\, \cos(\theta - \alpha)}}\, \mathrm{d}\alpha \tag{4}$$

Taking advantage of the orthogonality of the set $[1,\ J_0(k_n R)]$ over $[0,\ a]$, that is multiplying each side with R (then $R\, J_0(k_m R)$) and integrating over the disc, a vectorial equation can be derived:

$$\vec{A} = \mathbf{M_{AB}} \cdot \vec{B} \tag{5}$$

with $\vec{A} = (A_0, ...A_n, ...)$, $\vec{B} = (B_0, ...B_n, ...)$.

Matching of φ_1 and φ_2 on their common boundary, then of their vertical derivatives (together with enforcing $\varphi_{2z} = 0$ for $a \leq R \leq b,\ z = d$) results into two more vectorial equations

$$\vec{A} + \mathbf{D_B}\, \vec{B} = \mathbf{M_C}\, \vec{C} \tag{6}$$

$$\vec{D} = \mathbf{M_{DA}}\, \vec{A} + \mathbf{M_{DB}}\, \vec{B} \tag{7}$$

with $\vec{C} = (C_0, ...C_n, ...)$, $\vec{D} = (D_0, ...D_n, ...)$.

The free surface equation $g\,\varphi_{2z} - \omega^2\,\varphi_2 = 0$ gives

$$\mathbf{D_1}\,\vec{C} + \mathbf{D_2}\,\vec{D} = \omega^2\,\left(\vec{C} + \mathbf{D_4}\,\vec{D}\right) \tag{8}$$

with $\mathbf{D_1}$, $\mathbf{D_2}$, $\mathbf{D_4}$ diagonal matrices.

From (5) and (6) we get:

$$\vec{B} = (\mathbf{M_{AB}} + \mathbf{D_B})^{-1}\,\mathbf{M_C}\,\vec{C} \tag{9}$$

and then, from (7):

$$\vec{D} = (\mathbf{M_{DA}}\,\mathbf{M_{AB}} + \mathbf{M_{DB}})\,(\mathbf{M_{AB}} + \mathbf{D_B})^{-1}\,\mathbf{M_C}\,\vec{C} = \mathbf{M_{DC}}\,\vec{C} \tag{10}$$

and, from (8), the following eigen-value problem is obtained

$$(\mathbf{D_1} + \mathbf{D_2}\,\mathbf{M_{DC}})\,\vec{C} = \omega^2\,(\mathbf{I} + \mathbf{D_4}\,\mathbf{M_{DC}})\,\vec{C} \tag{11}$$

with $\mathbf{I}$ the identity matrix. The resolution of this eigen-value problem gives the resonant frequencies ω_i and the associated eigen-vectors $\vec{C_i}$. Practically the series are truncated after 10 or 20 terms, which provides a sufficient accuracy.

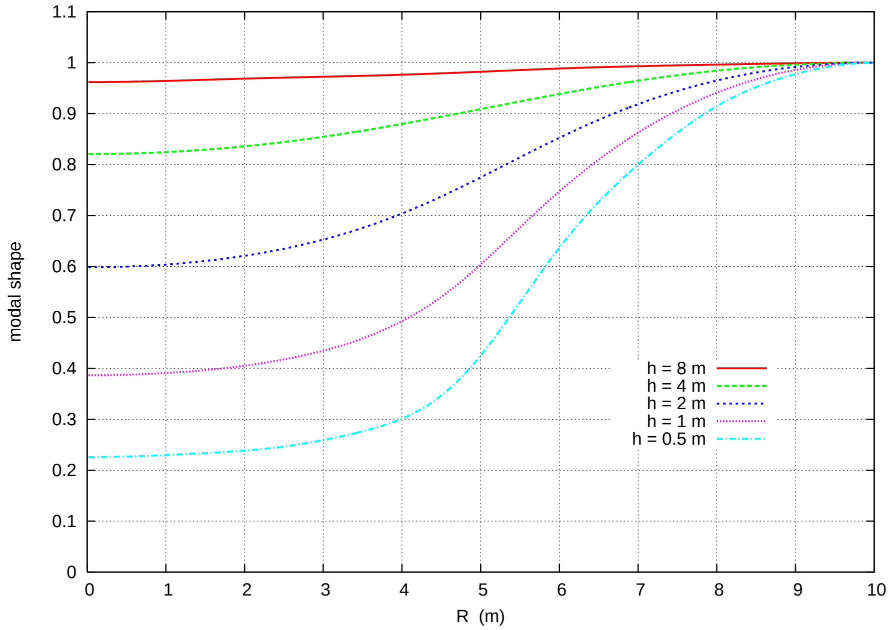

Figure 3: Piston modal shapes for different water heights h. $b = 10$ m, $a = 5$ m, $d = 1$ m.

As an application we take an outer radius b equal to 10 m, an inner radius a equal to 5 m, a restriction height d equal to 1 m, and we vary the waterheight h in the upper compartment of the moonpool. Figure 3 shows the modal shapes of the free surface obtained for the piston mode. They are normalized with a maximum value of 1. It is striking that the maximum wave elevation takes place at the outer wall, and that, as the waterheight h decreases, the free surface motion gets lower and lower in the center, down to values near zero.

3. RECTANGULAR CASE

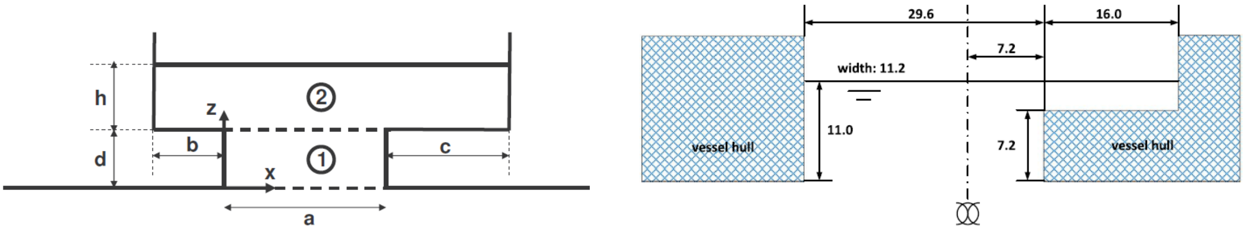

Figure 4: Geometry (left). Moonpool dimensions (from Guo *et al.* 2017) (right).

Now we consider the rectangular case of a moonpool with one or two recesses, as shown in Figure 4 (left). The moonpool is assumed to be narrow enough that the flow inside can be considered as two-dimensional. It is straightforward to adapt the procedure presented in the axisymmetric case to the rectangular case: one just has to replace the Bessel functions $J_0(k_n R)$ and $J_0(\lambda_n R)$ by $\cos k_n x$ and $\cos \lambda_n (x+b)$ where $k_n = n\pi/a$ and $\lambda_n = n\pi/(a+b+c)$. For the matching with the lower fluid domain and deriving the matrix $\mathbf{M_{AB}}$ the same routine as in Molin (2001) is used.

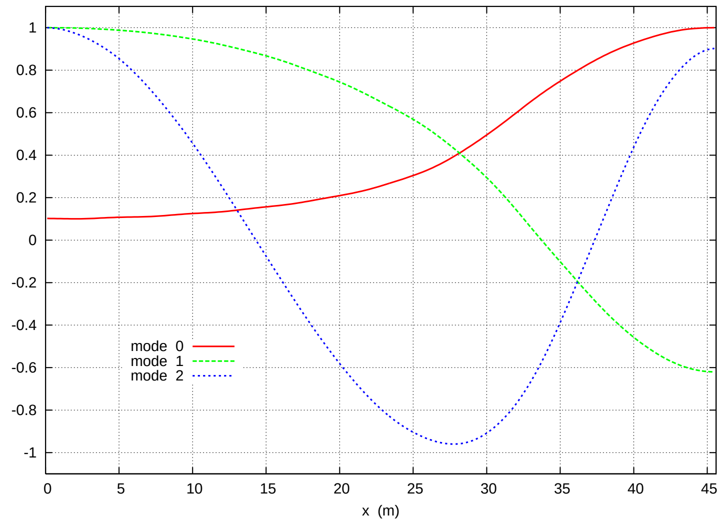

Figure 5: Modal shapes.

Figure 5 shows the modal shapes obtained for the piston mode (mode 0) and for the first two sloshing modes. Their natural frequencies agree fairly well with the values derived from the experiments of Guo *et al.* (2016). It is striking that, alike in the axisymmetric case, for the piston mode, the maximum elevation is obtained in the recess. It is the other way around for the first sloshing mode. These features are in agreement with the experimental results of Guo *et al.* (2016).

REFERENCES

Guo X., Lu H., Yang J., Peng T. 2016 Study on hydrodynamic performances of a deep-water drillship and water motions inside its rectangular moonpool, in *Proceedings of the Twenty-sixth (2016) International Ocean and Polar Engineering Conference*, ISOPE, Rhodes.

Guo X., Lu H., Yang J., Peng T. 2017 Resonant water motions within a recessing type moonpool in a drilling vessel, *Ocean Engineering*, **129**, 228–239.

Hammargren E., Törnblom J. 2012 Effect of the moonpool on the total resistance of a drillship, M.Sc. Thesis, Chalmers University of Technology.

Molin B. 2001 On the piston and sloshing modes in moonpools, *J. Fluid Mech.*, **430**, 27–50.

Spaier S.H., Torres F.G.S., Masetti I.Q., Costa A.P. & Levi C. 2007 Monocolumn in waves: Experimental analysis, *Ocean Engineering*, **34**, 1724–1733.