A hybrid method for wave interacting with a body floating on polynya confined between two semi-infinite ice sheets

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1 INTRODUCTION

There has been an increasing interest in Arctic engineering due to possible new routes for shipping and sources for resource extraction. In such a case, the water surface is covered or partly covered by ice. A typical case is that a water channel opened up by an icebreaker, in which a strip of water surface is confined between two semi-infinite ice sheets. The Green function which satisfies the boundary conditions on the free surface and ice sheet is less straightforward to obtain than the free surface only problem (Wehausen & Laitone, 1960) or the ice cover only problem (Sturova, 2013). Usually it is obtained from the matched eigen function method. Different series are used in sub-regions. The unknown coefficients are found by imposing continuity condition on the interface. Sturova (2015) solved the problem for the Green function with two ice sheets of zero draught. This was then used for a submerged body in polynya and only the body surface discretization was needed. The Green function problem would have to be resolved for a different polynya or a different ice sheet thickness.

Therefore, the hybrid method proposed by Yeung & Bouger (1979) for the free surface flow problem is reintroduced in this work for problems in polynya. Eigen function expansion method will be used below the ice sheets while the Rankine source will be used in the open water region. Matching condition will be imposed on the interface to ensure the flow and pressure continuity. The effectiveness of the hybrid method will be illustrated by two typical cases: an elliptic cylinder submerged in polynya and a rectangular body floating on polynya.

2 MATHEMATICAL MODEL

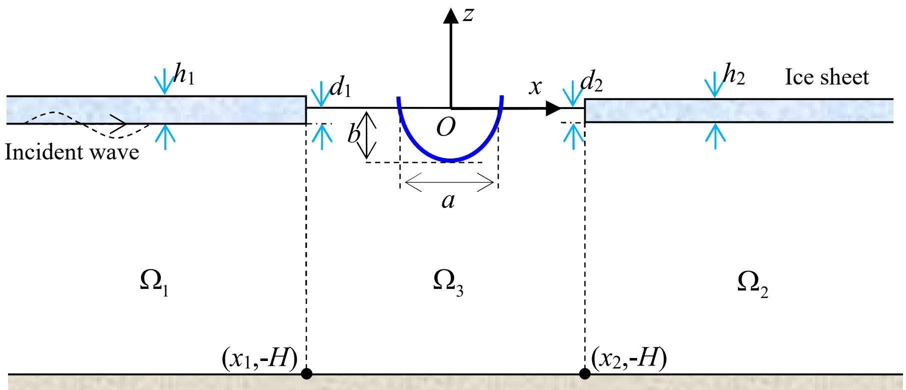

Fig. 1. Definition of the coordinate system and sketch of the problem.

We consider the problem of wave interaction with a body submerged or floating on polynya between two semi-infinite ice sheets, as shown in Fig. 1. The width of the body at still water surface and its draught are a and b respectively. A Cartesian coordinate system $O-xz$ is defined, with the x-axis along the undisturbed mean free surface, and the z-axis pointing vertically upwards. The fluid with density ρ and constant depth H is assumed to be inviscid, incompressible and homogeneous, and its motion to be irrotational. Thus the velocity potential Φ can be introduced to describe the fluid flow. The ice sheet, which begins from x_j to infinity, is modelled as a continuous elastic plate with uniform properties, i.e. thickness h_j, draught d_j, density ρ_j, Young's modulus E_j, Poisson's ration ν_j. Here the subscripts $j=1$, 2 denote the left and right hand side ice sheets respectively.

Based on the linearized velocity potential theory for sinusoidal motion with frequency ω, the total potential can be written as

$$\Phi(x,z,t)=\mathrm{Re}[\alpha_0\phi_0(x,z)\mathrm{e}^{\mathrm{i}\omega t}]+\mathrm{Re}[\sum_{i=1}^{3}\mathrm{i}\omega\alpha_i\phi_i(x,z)\mathrm{e}^{\mathrm{i}\omega t}] \tag{1}$$

where ϕ_0 contains incoming potential ϕ_I and diffracted potential ϕ_D, α_0 is the amplitude of the incident wave; ϕ_i ($i=1$, 2, 3) is the radiation potential due to body oscillation in mode i with complex amplitude α_i. The potentials ϕ_i should satisfy the Laplace's equation throughout the fluid, or

$$\nabla^2\phi_i=0\,,\ (i=0,\ 1,\ 2,\ 3) \tag{2}$$

The linearized free surface boundary condition in region Ω_3 is

$$-\omega^2\phi_i+g\phi_{i,z}=0\,,\ (x_1<x<x_2,\ \ z=0) \tag{3}$$

where g is the acceleration due to gravity. In regions Ω_1 and Ω_2, the boundary condition on the ice sheet is given as (Squire et al., 1995)

$$(L_j\frac{\partial^4}{\partial x^4}-m_j\omega^2+\rho g)\frac{\partial\phi_i}{\partial z}-\rho\omega^2\phi_i=0\,,\ (|x|\ge|x_j|,\ \ z=-d_j) \tag{4}$$

where $L_j=Eh_j^3/[12(1-\nu_j{}^2)]$ and $m_j=h_j\rho_j$ are the effective flexural rigidity and mass per unit area of the ice sheet, respectively. On the vertical surface of the ice sheet edge, the impermeable condition provides

$$\frac{\partial\phi_i}{\partial x}=0\,,\ (x=x_j,\ \ -d_j\le z\le 0) \tag{5}$$

Similarly, the condition to be satisfied on the body surface is

$$\frac{\partial\phi_0}{\partial n}=0 \ \text{ and } \ \frac{\partial\phi_i}{\partial n}=n_i\,,\ (i=1,\ 2,\ 3) \tag{6}$$

where $\vec{n}=(n_1,n_2)$ is the unit normal vector pointing into the body, $n_3=(z-z')n_1-(x-x')n_2$ is the component related to the rotational mode about y-axis pointing into the paper, with (x',z') as the rotational centre. The seabed boundary condition is

$$\frac{\partial\phi_i}{\partial z}=0\,,\ (-\infty<x<+\infty,\ \ z=-H) \tag{7}$$

The radiation condition far away from the body is given as

$$\lim_{x\to-\infty}[\frac{\partial(\phi_i-\delta_{0,i}\phi_I)}{\partial x}-\kappa_0^{(1)}(\phi_i-\delta_{0,i}\phi_I)]=0 \tag{8}$$

$$\lim_{x\to+\infty}(\frac{\partial\phi_i}{\partial x}+\kappa_0^{(2)}\phi_i)=0 \tag{9}$$

where $\delta_{p,q}=1$ if $p=q$ and $\delta_{p,q}=0$ if $p\ne q$, $\kappa_0^{(1)}$ and $\kappa_0^{(2)}$ are the purely positive imaginary roots of the corresponding dispersion equations in regions Ω_1 and Ω_2 respectively.

3 NUMERICAL PROCEDURE

To conduct the numerical procedure, the fluid domain is divided into three sub-regions: the ice-covered regions 1 (Ω_1: $-\infty<x\le x_1$, $-H\le z\le -d_1$) and 2 (Ω_2: $x_2\le x<+\infty$, $-H\le z\le -d_2$) and the free surface region 3 (Ω_3: $x_1<x<x_2$, $-H\le z\le 0$). The velocity potential in region j is denoted as $\phi_i^{(j)}$, which will be solved by the following matching method.

3.1 Eigenfunction expansion in regions 1 and 2

In the ice-covered regions Ω_1 and Ω_2, the velocity potential is written in terms of the corresponding eigenfunctions. We have

$$\phi_i^{(1)} = \delta_{0,i}\phi_I + \sum_{m=-2}^{\infty} R_{i,m}\psi_m^{(1)}\,,\quad \phi_i^{(2)} = \sum_{m=-2}^{\infty} T_{i,m}\psi_m^{(2)} \tag{10}$$

where $\delta_{0,i} = 1$ if $i = 0$ and $\delta_{0,i} = 0$ otherwise,

$$\phi_I = \frac{g}{\mathrm{i}\omega}\mathrm{e}^{-\kappa_0^{(1)}(x-x_1)}\frac{\cos[\kappa_0^{(1)}(H+z)]}{\cos[\kappa_0^{(1)}(H-d_1)]}\,,\quad \psi_m^{(j)} = \mathrm{e}^{\kappa_m^{(j)}(x_j-x)\mathrm{sgn}(x-x_j)}\frac{\cos[\kappa_m^{(j)}(H+z)]}{\cos[\kappa_m^{(j)}(H-d_j)]} \tag{11}$$

with $\mathrm{sgn}(x-x_j)=1$ if $x-x_j>0$ and $\mathrm{sgn}(x-x_j)=-1$ if $x-x_j<0$. In Eq. (11), $\kappa_m^{(j)}$ are the roots of the corresponding dispersion equations (Fox & Squire, 1994). Following Ren et al. (2016) and using the Green's second theorem over the boundary S_i of Ω_i, we have

$$\int_{-H}^{-d_1}(\phi_i^{(1)}\frac{\partial\psi_m^{(1)}}{\partial x} - \frac{\partial\phi_i^{(1)}}{\partial x}\psi_m^{(1)})\,\mathrm{d}z + \frac{L_1}{\rho\omega^2}(\frac{\partial^4\phi_i^{(1)}}{\partial x^3\partial z}\frac{\partial\psi_m^{(1)}}{\partial z} - \frac{\partial^3\phi_i^{(1)}}{\partial x^2\partial z}\frac{\partial^2\psi_m^{(1)}}{\partial z\partial x} + \frac{\partial^3\psi_m^{(1)}}{\partial x^2\partial z}\frac{\partial^2\phi_i^{(1)}}{\partial z\partial x} - \frac{\partial^4\psi_m^{(1)}}{\partial x^3\partial z}\frac{\partial\phi_i^{(1)}}{\partial z})_{z=-d_1}$$

$$= \delta_{0,i}\int_{-H}^{-d_1}(\phi_I\frac{\partial\psi_m^{(1)}}{\partial x} - \frac{\partial\phi_I}{\partial x}\psi_m^{(1)})\,\mathrm{d}z + \frac{\delta_{0,i}L_1}{\rho\omega^2}(\frac{\partial^4\phi_I}{\partial x^3\partial z}\frac{\partial\psi_m^{(1)}}{\partial z} - \frac{\partial^3\phi_I}{\partial x^2\partial z}\frac{\partial^2\psi_m^{(1)}}{\partial z\partial x} + \frac{\partial^3\psi_m^{(1)}}{\partial x^2\partial z}\frac{\partial^2\phi_I}{\partial z\partial x} - \frac{\partial^4\psi_m^{(1)}}{\partial x^3\partial z}\frac{\partial\phi_I}{\partial z})_{z=-d_1}$$

at $x = x_1$ (12)

and

$$\int_{-H}^{-d_2}(\phi_i^{(2)}\frac{\partial\psi_m^{(2)}}{\partial x} - \frac{\partial\phi_i^{(2)}}{\partial x}\psi_m^{(2)})\,\mathrm{d}z + \frac{L_2}{\rho\omega^2}(\frac{\partial^4\phi_i^{(2)}}{\partial x^3\partial z}\frac{\partial\psi_m^{(2)}}{\partial z} - \frac{\partial^3\phi_i^{(2)}}{\partial x^2\partial z}\frac{\partial^2\psi_m^{(2)}}{\partial z\partial x} + \frac{\partial^3\psi_m^{(2)}}{\partial x^2\partial z}\frac{\partial^2\phi_i^{(2)}}{\partial z\partial x} - \frac{\partial^4\psi_m^{(2)}}{\partial x^3\partial z}\frac{\partial\phi_i^{(2)}}{\partial z})_{z=-d_2} = 0$$

at $x = x_2$ (13)

The last terms in the right hand side of Eqs. (12) and (13) will depend on the condition at the ice edge. Without loss of generality, we may assume the free edge which gives

$$\frac{\partial^2}{\partial x^2}(\frac{\partial\phi_i}{\partial z}) = \frac{\partial^3}{\partial x^3}(\frac{\partial\phi_i}{\partial z}) = 0\,,\ (x = x_j\,,\ z = -d_j) \tag{14}$$

3.2 Boundary integral equation in region 3

In the closed region Ω_3, using the Green's second identity, we have

$$\alpha(p)\phi_i^{(3)}(p) = \oint_S [G(p,q)\frac{\partial\phi_i^{(3)}(q)}{\partial n_q} - \frac{\partial G(p,q)}{\partial n_q}\phi_i^{(3)}(q)]\,\mathrm{d}S_q \tag{15}$$

where $G(p,q) = \ln(1/r_1) + \ln(1/r_2)$, $r_1 = \sqrt{(x-\xi)^2 + (z-\zeta)^2}$ and $r_2 = \sqrt{(x-\xi)^2 + (z+\zeta+2H)^2}$ are the distance between the field and the source point, and the distance between the field point and the mirror of the source point with respect to seabed respectively. The continuity of the potential and its normal derivative are then imposed on the interface of sub-domains.

4 NUMERICAL RESULTS

We shall use dimensionless variables based on a, ρ and g. It should be noted that the numerical results are obtained by taking $m = M-2$ in the summation Eq. (10) as the up limit, and N_0, N_F segments on the body surface and free surface respectively. We first consider the problem of an elliptic cylinder submerged in polynya. This case was studied by Sturova (2015) through the source distribution method together with the Green function satisfying the boundary conditions on the free surface and ice sheet. Fig. 2 shows the radiation force against dimensionless wave number σ. We can see that the results agree well with those of Sturova (2015). Computations are then carried out for a rectangular body floating on polynya. The semi-analytical solutions for this problem was given by Ren et al. (2016) via the matched eigen function expansion method. Fig. 3 presents the corresponding wave exciting force, and good agreement with the results of Ren et al. (2016) is obtained. More detailed results will be presented at the workshop.

ACKNOWLEDGEMENTS

This work is supported by Lloyd's Register Foundation through the joint centre involving University College London, Shanghai Jiaotong University and Harbin Engineering University, to which the authors are most grateful. Lloyd's Register Foundation helps to protect life and property by supporting engineering-related education, public engagement, and the application of research. This work is also supported by the National Natural Science Foundation of China (Grant No. 11472088)

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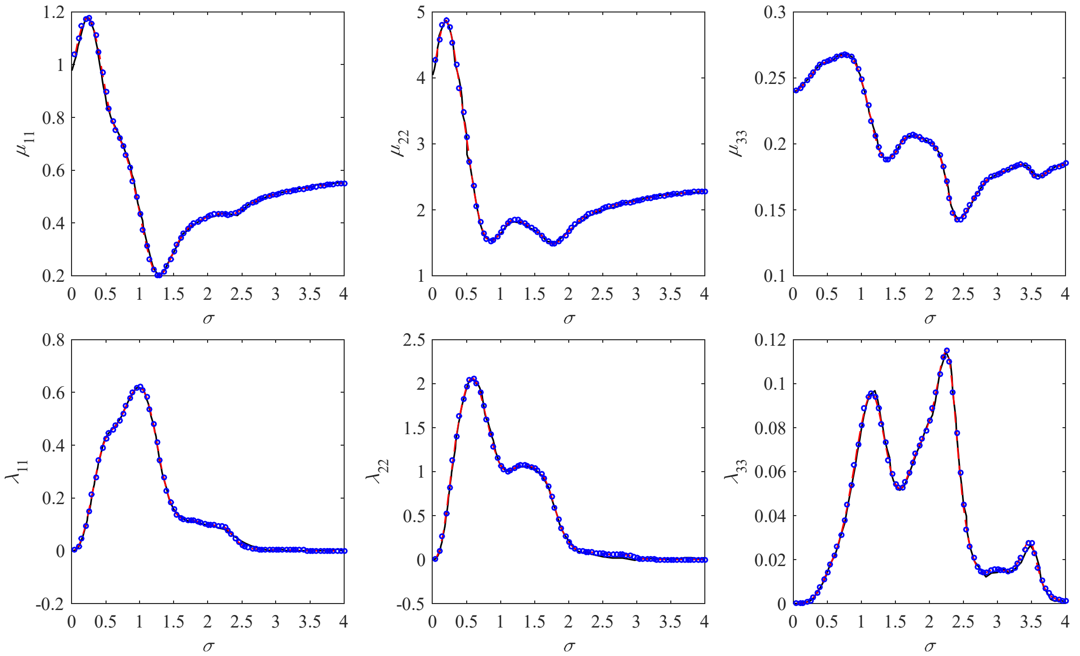

Fig. 2. Radiation force with different M, N_0 and N_F. Solid lines: results in Figs. 4 and 5 of Sturova (2015); dashed lines: $M=70$, $N_0=90$ and $N_F=180$; open circles: $M=100$, $N_0=180$ and $N_F=360$. ($a=1$, $b=0.5$, $(x',z')=(0,-1)$, $H=25$, $x_1=-x_2=-2.5$, $h_1=0.025$ and $h_2=0.1$, $d_1=0$ and $d_2=0$, $m_1=0.0225$ and $m_2=0.09$, $L_1=0.0356$ and $L_2=2.2791$)

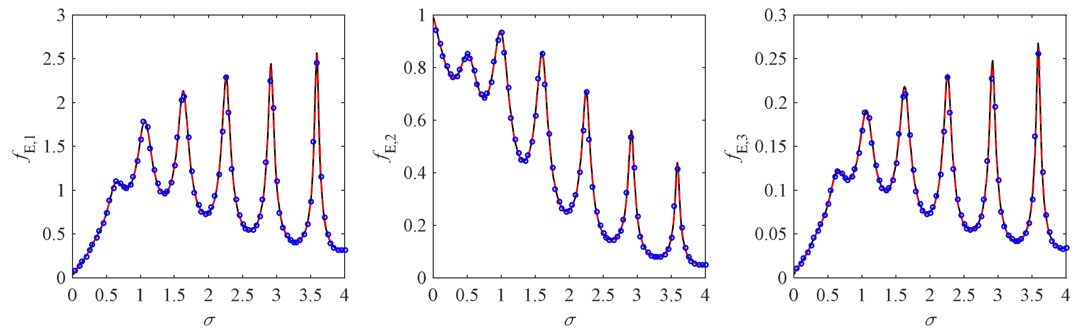

Fig. 3. Wave exciting force with different M, N_0 and N_F. Solid lines: results in Fig. 6 of Ren et al. (2016); dashed lines: $M=70$, $N_0=90$ and $N_F=180$; open circles: $M=100$, $N_0=180$ and $N_F=360$. ($a=1$, $b=0.5$, $(x',z')=(0,-b/2)$, $H=10$, $x_1=-x_2=-5$, $h_1=h_2=0.1$, $d_1=d_2=0.09$, $m_1=m_2=0.09$, $L_1=L_2=4.5582$)