Enhanced Unified Theory with Forward-Speed Effect in the Inner Free-Surface Condition

Masashi Kashiwagi*

Dept of Naval Architecture & Ocean Engineering, Osaka University, Osaka, Japan

E-mail: kashi@naoe.eng.osaka-u.ac.jp

1 INTRODUCTION

As a practical calculation method which is fast and accurate enough for the estimation of seakeeping performance of a ship, we can use the unified slender-ship theory developed by Newman (1978), which was enhanced by Kashiwagi (1995) to analyze in the same fashion the surge radiation problem and also the wave-scattering problem near the ship's bow. Consequently the added resistance can be computed with reasonable accuracy using this enhanced unified theory (EUT). Notwithstanding relatively good agreement with measured values, it is known that forward-speed effects in cross-coupling radiation forces (particularly between heave and pitch) are not properly accounted for in EUT.

Regarding this deficiency, Ogilvie and Tuck (1969) developed a rational strip theory in which the free-surface boundary condition for the inner solution retains a speed-dependent term proportional to $\tau = U\omega/g$, where U and ω are the forward speed and oscillation frequency, respectively. After comprehensive analyses, it was proven that the inner solution representing the forward-speed effect in the free-surface condition contributes only to the cress-coupling hydrodynamic forces.

Recalling these results, we realize that the analysis in the rational strip theory must be adopted for the particular solution in the unified theory, and then 3D effects in the low-frequency range must be incorporated through the homogeneous solution as in the original unified theory. With this idea, the present paper proposes a new slender-ship theory while keeping the framework of EUT, and its validity is confirmed by comparison with experiments for cross-coupling radiation forces, resulting ship motions, and added resistance in waves.

2 THEORY

2.1 Formulation, Outer Solution and Its Expansion

We consider a ship with constant forward speed U and harmonic oscillation of circular frequency ω in deep water. The right-handed Cartesian coordinate system moving with the ship is chosen, with the x-axis pointing in the direction of forward motion and the z-axis downward. Only the radiation problem is considered here. Under the assumption of inviscid fluid with irrotational motion, the velocity potential is introduced and expressed as

$$\Phi(x,y,z;t) = U\Phi_B(x,y,z) + \mathrm{Re}\sum_{j=1}^{6} i\omega X_j\,\phi_j(x,y,z)\,e^{i\omega t} \tag{1}$$

Here $\Phi_B = -x + \phi_S(x,y,z)$ denotes the double-body flow due to steady forward motion at U (thus ϕ_S is the disturbance potential), and ϕ_j the radiation potential due to j-th mode of motion with complex amplitude X_j, where in particular $j = 1$ for surge, $j = 3$ for heave, and $j = 5$ for pitch.

The velocity potential ϕ_j must be sought to satisfy the body boundary condition, the free-surface boundary condition, and the radiation condition at infinity. For subsequent citation, the linearized free-surface boundary condition to be satisfied by ϕ_j is written below:

$$\begin{aligned} &-g\frac{\partial\phi_j}{\partial z} + (i\omega)^2\phi_j + 2i\omega\,U\nabla\Phi_B\cdot\nabla\phi_j + \big(U\nabla^2\Phi_B + \mu\big)(i\omega + U\nabla\Phi_B\cdot\nabla)\,\phi_j \\ &+U^2\nabla\Phi_B\cdot\nabla\left(\nabla\Phi_B\cdot\nabla\phi_j\right) + \frac{1}{2}U^2\nabla\left(\nabla\Phi_B\cdot\nabla\Phi_B\right)\cdot\nabla\phi_j = 0 \quad \text{on } z=0 \end{aligned} \tag{2}$$

where g is the gravitational acceleration, ∇ denotes the gradient operator only in the horizontal plane (x, y), and μ is Rayleigh's artificial friction coefficient ensuring the radiation condition to be satisfied.

In order to obtain a solution of the problem mentioned above, we apply the concept of slender-ship theory, by dividing the fluid region into the outer and inner regions. In the outer region far from the ship, the free-surface boundary condition, Eq. 2, reduces to the uniform-flow version because of $\phi_S \to 0$ and hence $\Phi_B \simeq -x$, and the disturbance by the ship can be expressed by a line distribution of 3D sources along the x-axis in the form

$$\phi_j^{(o)}(x,y,z) = \int_{-\infty}^{\infty} Q_j(\xi) G_{3D}(x-\xi,y,z)\, d\xi = \frac{1}{2\pi} \int_{-\infty}^{\infty} Q_j^*(k) G_{3D}^*(k;y,z)\, e^{-ikx}\, dk \tag{3}$$

where Q_j is the source strength along the x-axis which is unknown at this stage, G_{3D} is the 3D Green function equal to the velocity potential of 3D source with unit strength, and the asterisk stands for the Fourier transform with respect to x.

The Fourier transform of the 3D Green function has been well studied. Referring to the results in Kashiwagi (1997), its expansions at higher and lower frequencies may be given as follows:

$$G_{3D}^*(k;y,z) \sim i\, e^{-K(z+i|y|)} \left\{ 1 - 2\tau k (z + i|y|) \right\} \quad \text{for } KR \gg 1 \tag{4}$$

$$G_{3D}^*(k;y,z) \sim G_{2D}(y,z) - \frac{1}{\pi}(1 - Kz) f^*(k,\kappa) \quad \text{for } KR \ll 1 \tag{5}$$

where

$$\kappa = \frac{1}{g}(\omega + kU)^2 = K + 2\tau k + \frac{k^2}{K_0}, \quad K = \frac{\omega^2}{g},\ \tau = \frac{U\omega}{g},\ K_0 = \frac{g}{U^2}, \tag{6}$$

and $R = \sqrt{y^2 + z^2}$. Here we note that Eq. 4 retains leading two different orders with assumption of $\omega = O(\epsilon^{-1/2})$ and $U = O(1)$ according to the rational strip theory of Ogilvie and Tuck (1969), and also note that $G_{2D}(y,z) = G_{3D}^*(0;y,z)$ and the other 3D and forward-speed effects in the low frequency range are accounted for in function $f^*(k,\kappa)$.

Substituting these results in Eq. 3, we can obtain the expansion of the outer solution necessary for matching with the inner solution, in the form

$$\phi_j^{(o)}(x,y,z) \simeq i\, e^{-K(z+i|y|)} \left\{ Q_j(x) - i2\tau (z + i|y|) Q_j'(x) \right\} \quad \text{for } KR \gg 1 \tag{7}$$

$$\phi_j^{(o)}(x,y,z) \simeq Q_j(x) G_{2D}(y,z) - \frac{1}{\pi}(1 - Kz) \int_{-\infty}^{\infty} Q_j(\xi) f(x-\xi)\, d\xi \quad \text{for } KR \ll 1 \tag{8}$$

2.2 Inner Solution and Its Expansion

In the inner problem, the governing equation for the velocity potential becomes the 2D Laplace equation due to slenderness assumption. Furthermore, the order of ϕ_S for the steady disturbance is $O(\epsilon^2)$. Then, according to the rational strip theory assuming $\omega = O(\epsilon^{-1/2})$ and $U = O(1)$, the body and free-surface boundary conditions are given as follows:

$$[H] \quad \frac{\partial \phi_j}{\partial n} = N_j + \frac{U}{i\omega} M_j \quad \text{on } S_H(x) \tag{9}$$

$$[F] \quad \frac{\partial \phi_j}{\partial z} + K\phi_j = -\frac{i\omega U}{g} \left\{ 2\frac{\partial \phi_j}{\partial x} - 2\frac{\partial \phi_S}{\partial y}\frac{\partial \phi_j}{\partial y} - \frac{\partial^2 \phi_S}{\partial y^2}\phi_j \right\} \quad \text{on } z = 0 \tag{10}$$

where N_j and M_j are slender-body approximations of the normal vector and the so-called m-term. It should be noted that the speed-dependent terms proportional to U in both Eq. 9 and Eq. 10 are smaller than the zero-speed leading terms with relative order of $O(\sqrt{\epsilon})$.

By taking account of this fact and the knowledge learned from the unified theory; that is, a homogeneous solution can be added because of absence of the radiation condition, we can construct the inner solution in the following form:

$$\phi_j^{(i)}(x;y,z) = \varphi_j(y,z) + C_j(x)\varphi_H(y,z) + \frac{U}{i\omega}\left\{ \widehat{\varphi}_j(y,z) + (i\omega)^2 \psi_j(y,z) \right\} \tag{11}$$

where φ_j and $\widehat{\varphi}_j$ are the particular solutions satisfying the first and second terms respectively on the right-hand side of Eq. 9 and the free-surface condition with right-hand side of Eq. 10 set equal to zero.

The homogeneous solution is obtained by $\varphi_H(y,z) = \varphi_3(y,z) - \overline{\varphi_3(y,z)}$, where the overbar denotes the complex conjugate, and $C_j(x)$ in Eq. 11 is the coefficient of homogeneous solution which is unknown at this stage and to be determined from matching. $\psi_j(y,z)$ is the additional term to account for the forward-speed effect on the right-hand side of the free-surface condition Eq. 10.

Taking the same order of terms after substituting Eq. 11 into Eq. 9 and Eq. 10, the body and free-surface boundary conditions for ψ_j can be of the following form:

$$\left.\begin{aligned} \frac{\partial \psi_j}{\partial n} &= 0 \quad \text{on } S_H(x) \\ \frac{\partial \psi_j}{\partial z} + K\psi_j &= -\frac{1}{g}\left\{ 2\frac{\partial \varphi_j}{\partial x} - 2\frac{\partial \phi_S}{\partial y}\frac{\partial \varphi_j}{\partial y} - \frac{\partial^2 \phi_S}{\partial y^2}\varphi_j \right\} \quad \text{on } z=0 \end{aligned}\right\} \tag{12}$$

Since this is a problem equivalent to a pressure distribution applied on the free surface, its expansion valid for $KR \gg 1$ can be obtained in the same manner as in the rational strip theory. The expansion for $KR \ll 1$ can be the same as that in the unified theory. Thus the results of inner solution expansion can be written as follows:

$$\phi_j^{(i)}(x;y,z) \simeq i\, e^{-K(z+i|y|)} \left\{ \sigma_j(x) + \frac{U}{i\omega}\widehat{\sigma}_j(x) - i2\tau(z+i|y|)\sigma_j'(x) \right\} \quad \text{for } KR \gg 1 \tag{13}$$

$$\begin{aligned} \phi_j^{(i)}(x;y,z) \simeq \Big[\sigma_j(x) + \frac{U}{i\omega}\widehat{\sigma}_j(x) + C_j(x)\big\{\sigma_3(x) - \overline{\sigma_3(x)}\big\} \Big] G_{2D}(y,z) \\ + i\,2\,C_j(x)\,\overline{\sigma_3(x)}\,e^{-Kz}\cos Ky \quad \text{for } KR \ll 1 \end{aligned} \tag{14}$$

where $\sigma_j(x)$ and $\widehat{\sigma}_j(x)$ are the 2-D Kochin functions computed from φ_j and $\widehat{\varphi}_j$, respectively.

By comparing these results with Eq. 7 and Eq. 8, we can realize that the matching for determining two unknowns, $Q_j(x)$ and $C_j(x)$, is possible with essentially the same results as those in the unified theory. The only difference is that the inner solution contains a new component $\psi_j(y,z)$ which represents the contribution from speed-dependent term in the inner free-surface condition and is physically very important as a correction to the unified theory.

3 HYDRODYNAMIC FORCES

3.1 Added mass and damping coefficients

Once the inner solution has been determined, the analyses for computing the hydrodynamic force can be the same as those in the rational strip theory and in the unified theory. For the radiation problem, the result can be expressed with the added mass (A_{ij}) and damping coefficient (B_{ij}) in the i-th direction due to the j-th mode of motion, in the following form:

$$\begin{aligned} A_{ij} + \frac{B_{ij}}{i\omega} &= -\rho \int_L dx \int_{S_H(x)} \Big(N_i - \frac{U}{i\omega}M_i\Big)\Big\{\varphi_j(y,z) + \frac{U}{i\omega}\widehat{\varphi}_j(y,z)\Big\}\, d\ell \\ &\quad -\rho \int_L dx\, C_j(x) \int_{S_H(x)} \Big(N_i - \frac{U}{i\omega}M_i\Big)\varphi_H(y,z)\, d\ell - \rho\, i2\tau\, \mathcal{Z}_{ij} \end{aligned} \tag{15}$$

Here $\mathcal{Z}_{ij}$ represents the additional term of forward-speed effect in the free-surface condition, to be computed from the new term $\psi_j(y,z)$ in the inner solution. Following the analysis in the rational strip theory, we can show that $\mathcal{Z}_{ij} = 0$ for the case of $i = j$ and hence the forward-speed effect in the free-surface condition contributes only to the cross-coupling terms for $i \neq j$. Specifically the result for the case of $i = 3$ and $j = 5$ (or $i = 5$ and $j = 3$) can be expressed as

$$\mathcal{Z}_{35} = -\mathcal{Z}_{53} = \int_L dx \left[\int_{y_0(x)}^{\infty} \Big\{ \varphi_3^2(y,0) - \big(i\sigma_3\, e^{-iKy}\big)^2 \Big\}\, dy + \frac{i}{2K}\sigma_3^2\, e^{-i2Ky_0(x)} \right] \tag{16}$$

It is noteworthy that the relation of $\mathcal{Z}_{35} = -\mathcal{Z}_{53}$ implies that the Timman-Newman relation for the forward-speed effect is also satisfied in this additional term.

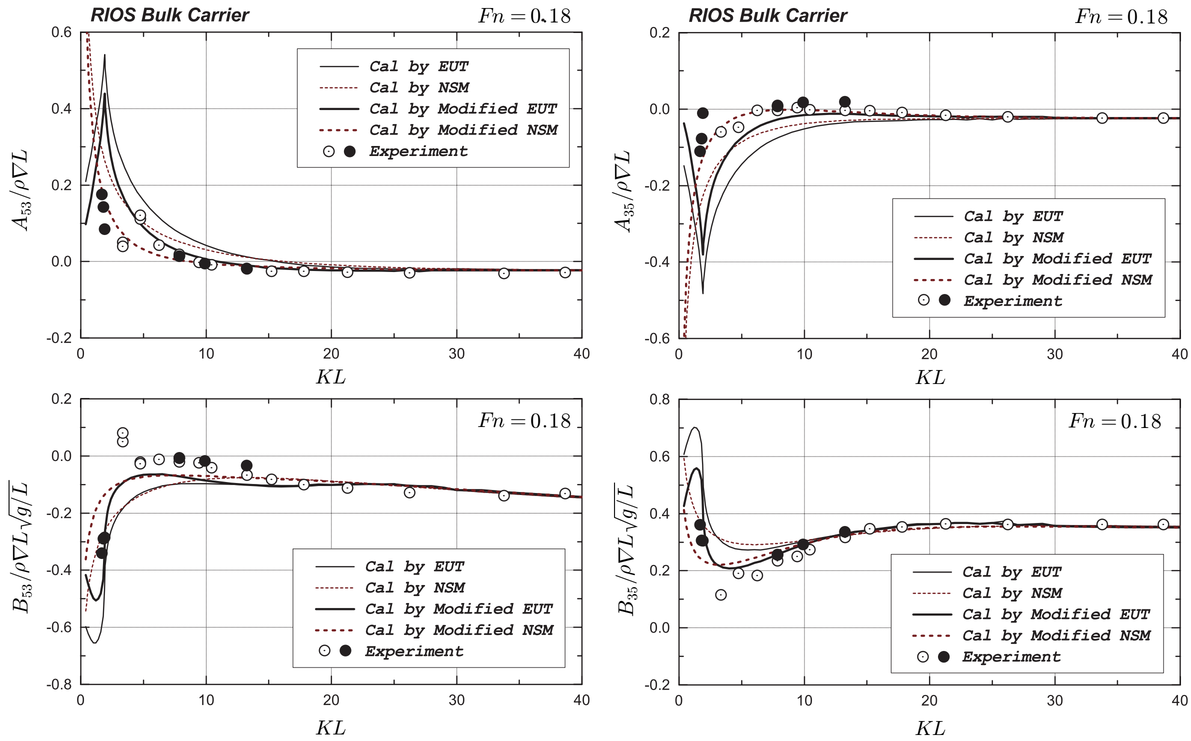

Fig. 1 Cross-coupling added mass and damping coefficients between heave and pitch for RIOS Bulk carrier at $Fn = 0.18$

4 RESULTS

Due to shortage of space, only one example is shown of the results for the cross-coupling added-mass and damping coefficients between heave and pitch. Shown in Fig. 1 are the results for a bulk-carrier ($C_B = 0.8$) tested by the Research Initiative on Oceangoing Ships (RIOS) at Osaka University.

The values to be computed from the first term on the right-hand side of Eq. 15 is the same as the strip method (which is referred to as NSM), the second term is the contribution from the homogeneous solution in the unified theory (thus the sum of the first and second terms is referred to as EUT), and the third term is a newly added correction accounting for the forward-speed effect in the inner free-surface condition. Thus the results including this correction term in NSM and EUT are denoted as modified NSM (essentially the same as the rational strip theory) and modified EUT, respectively.

Clearly the degree of agreement is improved by including the correction term originating from the forward-speed effect in the free-surface condition. Noticeable improvement is also confirmed in ship motions, especially around the peak of heave motion, by including the correction in the cross-coupling terms; which will be shown in the Workshop together with other results for different ship models.

5 CONCLUSION

The solution of Ogilvie and Tuck's rational strip theory has been used in EUT as the high-frequency solution to account for the forward-speed effect in the inner free-surface condition. Good agreement was found in comparisons between experiments and numerical results on the cross-coupling radiation forces and wave-induced ship motions. These results indicate that the forward-speed effect in the inner free-surface condition is of crucial importance for improvement in seakeeping computations.

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