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Hydroelastic Response of a Submerged Plate to Long Waves

Masoud Hayatdavoodi[a*], and R. Cengiz Ertekin[b]

a. Civil Eng. Disc., School of Science and Eng., Univ. of Dundee, Dundee DD1 4HN, UK
b. Guest Professor, College of Shipbuilding Engineering, Harbin Eng. Univ., Harbin, China

Email: mhayatdavoodi@dundee.ac.uk

Highlights

- Interaction of nonlinear shallow-water waves with an elastic submerged plate is studied.
- The nonlinear wave-structure problem is modeled by use of the coupled Level I Green-Naghdi equations and the thin, elastic-plate theory.
- A submerged elastic plate can function as a wave carpet for high-performance wave energy harvesting, or for mitigation of large waves.

Introduction

This study is concerned with the deformation of an elastic submerged plate in shallow water by nonlinear long waves. The submerged plate is fixed in its horizontal position, but may deform in the vertical direction due to time- and spatial-variant wave-induced pressure differential above and below the plate.

A submerged deformable plate, or elastic mat, can be applied to wave energy devices, namely as a wave carpet (see, *e.g.*, Alam (2012)), and for mitigation of large waves in coastal areas. The principle concept of a wave carpet energy device is similar to that of propagation of waves over a muddy seafloor, where significant amount of wave energy attenuates due to the strong interaction with the mud banks. Similarly, a wave carpet is a mud-resembling deformable plate that can potentially extract the entire wave energy. The plate deformation can be transferred into electricity generation by, for example, a direct-drive power take-off system connected to the plate.

The deformation of the plate can be controlled and optimized by a spring and damping system. The responding force of the power take-off system is linearly proportional to the vertical speed of the plate deformation, and hence it has damping effect on the plate motion. Here, we confine our attention to the hydroelastic response of the submerged plate to wave-induced loads alone. Our objective is to formulate the problem of interaction of nonlinear long-waves with the elastic submerged plate by use of the nonlinear Level I GN equations. Numerical results will be discussed during the workshop.

The Level I Green-Naghdi Equations

The Green-Naghdi theory of water waves, originally proposed by Green & Naghdi (1976), is an alternative method for modeling nonlinear wave propagation in shallow water. In the general form of the theory, incompressibility is the only assumption made about the medium. In this theory, a polynomial approximation of the vertical particle velocity is introduced, and this is the only assumption made about the kinematics of the fluid flow. In Level I GN equations, for example, a linear distribution for the vertical velocity is assumed, *i.e.*, $u_2(x_1, x_2, t) = C(x_1, t)x_2$,, where x_1 , x_2 are the spatial coordinates, and t is time. This assumption, along with the incompressibility condition, results in constant horizontal velocity along the water column, *i.e.*, $u_1(x_1, x_2, t) = u_1(x_1, t)$. This condition is applicable to propagation of long waves in shallow water. The resulting model postulates the integrated (over the water column) field equations, and nonlinear free surface boundary conditions are exactly satisfied.

The Level I GN equations are used here in their two-dimensional form. A right-handed Cartesian coordinate system, with x_1 pointing to the right and x_2 directed against gravity is considered, where the origin of the coordinate systems is on the still-water level (SWL). The fluid is inviscid and the

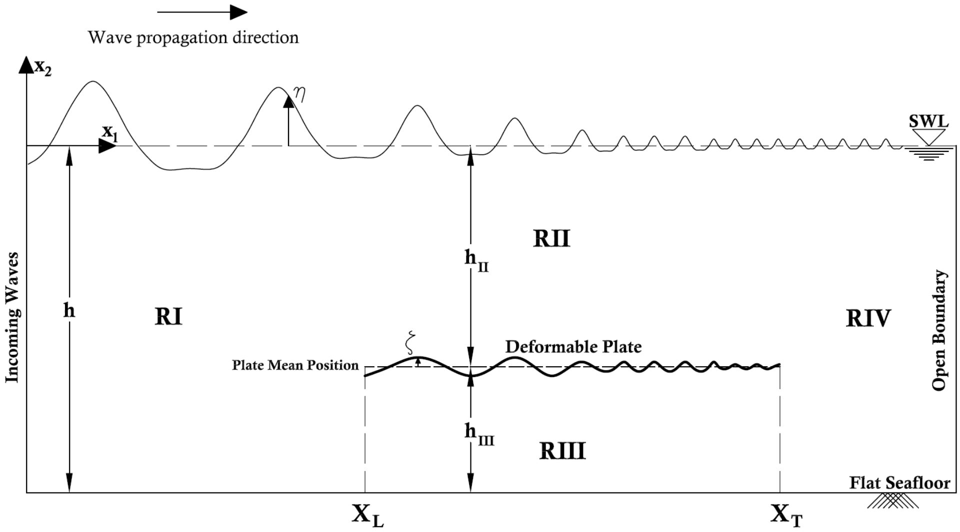

Figure 1: Schematic of the theoretical wave tank of wave interaction with an elastic submerged plate, showing the four regions discussed within the text. Figure not to scale.

flow is incompressible, but irrotationality is not required. The free surface, $\eta(x_1, t)$, is measured from the SWL. The Level I equations as used here are given by:

$$\eta_{,t} + \{(h + \eta - \alpha)u_1\}_{,x_1} = \alpha_{,t}\,, \tag{1a}$$

$$\dot{u}_1 + g\eta_{,x_1} + \frac{\hat{p}_{,x_1}}{\rho} = -\frac{1}{6}\{[2\eta + \alpha]_{,x_1}\ddot{\alpha} + [4\eta - \alpha]_{,x_1}\ddot{\eta} + (h + \eta - \alpha)[\ddot{\alpha} + 2\ddot{\eta}]_{,x_1}\}\,, \tag{1b}$$

$$u_2(x_1, x_2, t) = \dot{\alpha} + \frac{(x_2 + h - \alpha)}{(\eta + h - \alpha)}\left(\dot{\eta} - \dot{\alpha}\right)\,, \tag{1c}$$

$$P(x_1, t) = \left(\frac{\rho}{6}\right)(h + \eta - \alpha)^2\left(\ddot{\alpha} + 2\ddot{\eta} + 3g\right) + \hat{p}\left(h + \eta - \alpha\right)\,, \tag{1d}$$

$$\bar{p}(x_1, t) = \left(\frac{\rho}{2}\right)(h + \eta - \alpha)\left(\ddot{\alpha} + \ddot{\eta} + 2g\right) + \hat{p}\,, \tag{1e}$$

where $\alpha(x_1, t)$ is the elevation of the bottom of the fluid sheet, h is the water depth, ρ is the mass density of the fluid, g is the gravitational acceleration, and superposed dot is the two-dimensional material time derivative. P is the integrated (over the water column) pressure, $\bar{p}$ is the pressure on the bottom curve (α), and $\hat{p}(x_1, t)$ is the pressure on the top curve of the fluid sheet. The subscripts after comma indicate partial differentiation. For further information on the above GN equations see *e.g.*, Ertekin et al. (1986).

Hydroelastic Response of the Submerged Plate

A schematic of the theoretical domain of interaction of nonlinear waves with a deformable submerged plate is shown in Fig. 1. The plate is fixed in the horizontal direction between $x_1 = X_L$ and $x_1 = X_T$, and it is initially submerged at a depth h_{II} from the SWL. The plate is assumed thin, hence $h_{III} = h - h_{II}$ is the average depth from the plate position to the flat seafloor, *i.e.*, the plate can deform about its initial vertical position, but its average elevation from the seafloor is fixed. For wave-induced vertical oscillation of a horizontal, submerged plate see Hayatdavoodi et al. (2016). Upwave and downwave from the plate, the water depth is constant h.

To study the problem of interaction of water waves with the submerged elastic plate, we split the domain into four computational regions, shown in Fig. 1. Regions RI and RIV are upwave and downwave of the plate, respectively. Regions RII and RIII are above and below the deformable plate, respectively. Each of the regions is subject to specific boundary conditions. At the discontinuity curves, the leading and trailing edges of the plate, certain conditions are enforced to ensure satisfaction of the conservation laws and a continuous solution in the entire domain.

In Regions RI and RIV, $x_1 < X_L \,\&\, x_1 > X_T$, we are concerned with the propagation of nonlinear waves over a flat and stationary seafloor, *i.e.*, $\alpha(x_1, t) = 0$. The top surface, $\eta(x_1, t)$, is free and subject to the atmospheric pressure taken as $\hat{p} = 0$, without loss in generality. Hence, in RI and RIV regions, the GN mass and momentum equations, Eqs. (1a) and (1b), respectively, read

$$\eta_{,t} + \{(h+\eta)u_1\}_{,x_1} = 0\,, \tag{2a}$$

$$\dot{u}_1 + g\eta_{,x_1} = -\frac{1}{3}\{(2\eta_{,x_1}\ddot{\eta}) + (h+\eta)\,\ddot{\eta}_{,x_1}\}\,. \tag{2b}$$

The unknowns in RI and RIV regions are the surface elevation (η) and the horizontal velocity (u_1).

Region RII, $X_L < x_1 < X_T \,\&\, (\alpha_{II} - h_{II}) \le x_2 \le \eta$, above the plate, where $\alpha_{II}(x_1, t)$ is elevation of the bottom surface of RII, is subject to atmospheric pressure on the top surface, $\hat{p} = 0$, and constant water depth above the plate h_{II}. In this study, we assume that the fluid is directly in touch with the plate at all times, that is we do not allow air gaps and dry points above the plate. Hence, the bottom surface of RII is always the deformable plate. Therefore, in RII, $\alpha_{II}(x_1, t) = \zeta(x_1, t)$, where $\zeta(x_1, t)$ is the plate elevation from its initial position. The GN mass and momentum equations and the bottom pressure, Eqs. (1a), (1b) and (1e), respectively, in RII read as

$$\eta_{,t} + \{(h_{II} + \eta - \zeta)u_1\}_{,x_1} = \zeta_{,t}\,, \tag{3a}$$

$$\dot{u}_1 + g\eta_{,x_1} = -\frac{1}{6}\{[2\eta+\zeta]_{,x_1}\ddot{\zeta} + [4\eta-\zeta]_{,x_1}\ddot{\eta} + (h+\eta-\zeta)[\ddot{\zeta}+2\ddot{\eta}]_{,x_1}\}\,, \tag{3b}$$

$$\bar{p}_{II}(x_1, t) = \left(\frac{\rho}{2}\right)(h+\eta-\zeta)\left(\ddot{\zeta}+\ddot{\eta}+2g\right)\,, \tag{3c}$$

where $\bar{p}_{II}$ is the bottom pressure in RII, *i.e.*, plate top pressure. The unknowns in RII are η, u_1, ζ and $\bar{p}_{II}$.

In Region RIII, $X_L < x_1 < X_T \,\&\, (-h \le x_2 \le \zeta - h_{II})$, below the plate, the top surface is the deformable plate. The fluid and plate are always in contact, and hence, in RIII, $\eta(x_1, t) = \zeta(x_1, t)$. We assume horizontal and stationary seafloor, i.e, $\alpha(x_1, t) = 0$ in RIII, and water depth is constant $h = h_{III}$. The GN mass and momentum equations and the bottom pressure, Eqs. (1a), (1b) and (1e), respectively, for this region are given as

$$\zeta_{,t} + \{(h_{III}+\zeta)u_1\}_{,x_1} = 0\,, \tag{4a}$$

$$\dot{u}_1 + g\zeta_{,x_1} + \frac{(\hat{p}_{III})_{,x_1}}{\rho} = -\frac{1}{3}\{\left(2\zeta_{,x_1}\ddot{\zeta}\right) + (h_{III}+\zeta)\,\ddot{\zeta}_{,x_1}\}\,, \tag{4b}$$

$$\bar{p}(x_1, t) = \left(\frac{\rho}{2}\right)(h_{III}+\zeta)\left(\ddot{\zeta}+2g\right) + \hat{p}_{III}\,, \tag{4c}$$

where $\hat{p}_{III}$ is the top pressure in RIII, *i.e.*, plate bottom pressure. The unknowns in RIII are η, u_1, ζ and $\hat{p}_{III}$.

To obtain a continuous solution in the entire domain, jump and matching conditions are enforced at the discontinuity curves where the regions meet. The jump conditions, demanded by the theory, ensure conservation of mass and momentum across the discontinuities. A derivation of the GN jump conditions for wave propagation over a submerged plate is given in, *e.g.*, Hayatdavoodi & Ertekin (2015). The matching conditions, demanded by the physics of the problem, include continuous surface elevation and continuous bottom pressure across the discontinuity curves.

In this study, we consider the interaction of long waves with the submerged deformable plate. Under such framework, we use the one-dimensional version of the thin, elastic-plate theory (see, *e.g.*, Timoshenko & Woinowsky-Krieger (1959)) to study the structure deformation, *i.e.*,

$$m\zeta_{,tt} + D\zeta_{,x_1x_1x_1x_1} + mg = F_3\,, \tag{5}$$

where m is the two-dimensional mass (per unit width, into the page) of the plate, and $F_3(x_1, t)$ is the net two-dimensional pressure on the plate, *i.e.*, $F_3(x_1, t) = \hat{p}_{III} - \bar{p}_{II}$. Recall that we do not allow the top and bottom surfaces of the plate to become dry and, therefore, that $\bar{p}_{II}$ and $\hat{p}_{III}$ are always the water pressure. D is the flexural rigidity of the plate, and in two-dimensions it is defined by $D = Et_p^3/[12(1-\nu^2)]$, where E and ν are Young's modulus and Poisson's ratio of the plate, respectively, and t_p is the plate thickness.

On the left side of RI, a numerical wavemaker capable of generating cnoidal waves is installed. On the right side of RIV, the Orlanski's condition is used to minimise the wave reflection to the tank.

At the edges of the plate, the free-free end boundary condition requires vanishing of the bending moments and shear forces on the plate. Thus, $\zeta_{,x_1x_1} = \zeta_{,x_1x_1x_1} = 0$ at $x_1 = X_L$ and $x_1 = X_T$, see *e.g.*, Ertekin & Xia (2014). Since there is no gap between the fluid and the surface of the plate, the fluid at the edges of the plate should also satisfy this boundary condition at any time t. To enforce this boundary condition, we take second and third derivatives of the GN mass equation in RIII, Eq. (4a), and set $\zeta_{,x_1x_1} = \zeta_{,x_1x_1x_1} = \zeta_{,x_1x_1t} = \zeta_{,x_1x_1x_1t} = 0$. The fluid boundary conditions at the edges of the plate ($x_1 = X_L$ and $x_1 = X_T$) will then read

$$3\zeta_{,x_1} u_{1,x_1x_1} + (h_{III} + \zeta) u_{1,x_1x_1x_1} = 0\,, \tag{6a}$$

$$4\zeta_{,x_1} u_{1,x_1x_1x_1} + (h_{III} + \zeta) u_{1,x_1x_1x_1x_1} + \zeta_{,x_1x_1x_1x_1} u_1 = 0\,. \tag{6b}$$

We note that the condition (6) is only applied to Region III, below the plate. A similar condition can also be applied to RII, instead. The two regions, however, share the same ζ function, to which the condition (6) is applied, and hence it is sufficient to enforce the boundary condition on RII.

The governing equations in the four regions, subject to the boundary conditions, along with the jump and matching conditions are solved simultaneously for the unknowns. The system of equations is solved numerically by the central-difference method, second-order in space, and with the fourth-order Runge-Kutta method for time marching. A Gaussian elimination method is used to solve the system of equations.

Conclusions

Interaction of nonlinear long waves with an elastic submerged plate is formulated by use of the coupled Level I Green-Naghdi equations and the thin, elastic-plate theory. The waves propagate above and below the plate and the deformation of the plate results in energy attenuation and dissipation. In the limit of a very long plate, there will be no transmitted wave downwave, *i.e.*, the entire wave energy passing the leading edge of the deformable plate is absorbed. Hence, a submerged deformable plate, connected to direct drive power take-off system, can make a very attractive wave energy device. The submerged wave carpet will also perform as an efficient wave breaker. The proposed hydroelastic model can also be used to simulate the complicated nature of wave interaction with seafloor mud.

The numerical solution procedure and results of hydroelastic response of the plate will be given during the workshop.

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