Wave forces on porous geometries with linear and quadratic pressure-velocity relations

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1 HIGHLIGHTS

The wave diffraction-radiation problem of a porous geometry of arbitrary shape is obtained by a set of integral equations. The effects of quadratic and linear resistance laws are compared. The linear forces and the mean drift force are obtained mathematically and visualized by computations.

2 INTRODUCTION

The motivation for the present analysis of wave-interaction including porous structures is generally fundamental. The structures are proposed for industrial applications. In the fish farming industry porous or slotted geometries may serve as fish cages. Wave interaction with porous screens where studied by Molin (2011) from an analytical view-point assuming a quadratic relation between the pressure drop over the surface and the flow-through velocity. The theory was applied to model tuned liquid dampers used for protection of offshore structures and for damping of eigenfrequency-behaviour of slender structures such as skyscrapers. The model was used for experimental comparison in Molin & Remy (2013). A linear pressure-velocity relation was investigated by Chwang (1983) with application to porous wavemakers as model of landslides and leakage in wavemakers. Zhao et al. (2011) developed various hydrodynamic identities with a linear porous pressure-velocity law. They also investigated the drift-force due to diffraction.

The goal of our study has been a thorough comparison of the linear and quadratic porosity laws in the wave and arbitrary body case as this, to our knowledge has not been investigated before.

3 DESCRIPTION OF THE SPECIFIC PROBLEM

We consider a porous geometry located at the surface of the fluid. Periodic motion with frequency ω is assumed. Incoming plane progressive waves have amplitude A, wavenumber $k = \omega^2/g$ and wave angle β where g is the acceleration of gravity. A coordinate system is defined where x, y spans the horisontal plane and z is the vertical direction, with the fluid surface at $z = 0$. The water depth is infinite. We assume that the body moves in the six rigid modes of motion, with amplitudes denoted by $\xi_j, j = 1, \dots, 6$. The motion is decomposed into two parts, one for the fluid inside the porous body and one for the surrounding fluid. The exterior/inner velocity potential reads

$$\Phi^{E/I} = \mathrm{Re}\left\{ \left(i\phi_D^{E/I} gA/\omega + i\omega \sum\nolimits_{j=1}^{6} \xi_j \phi_j^{E/I} \right) e^{i\omega t} \right\}, \tag{1}$$

where $\phi_D^{E/I} = \phi_0 + \phi_7^{E/I}$ is the sum of the incoming velocity potential ϕ_0 given by $e^{kz - ikR\cos(\beta-\theta)}$, where $x = R\cos\theta, y = R\sin\theta$. The scattering potential is denoted by $\phi_7^{E/I}$, and $\phi_j^{E/I}$ denotes the radiation potentials.

From now on we will use the notation $\Phi_j^{E/I}$ for the radiation part, while $\Phi_D^{E/I}$ is the diffraction part of Eq.(1). The linearized free surface condition reads $\partial\phi_j^{E/I}/\partial z - k\phi_j^{E/I} = 0, \quad z = 0, \quad j = 0, \dots, 7$. In the far field the exterior potentials must satisfy the radiation condtion $\phi_j^E \propto R^{-\frac{1}{2}} e^{-ikR}, \quad R \to \infty, \quad j = 1, \dots, 7$. For non-porous bodies the normal velocity of the fluid is equal to the normal velocity of the body $U_j = \mathrm{Re}\left(i\omega\xi_j e^{i\omega t}\right)$. In the porous case we have an extra term, which is the flow-through velocity, giving

$$\frac{\partial\Phi_j^E}{\partial n} = \frac{\partial\Phi_j^I}{\partial n} = U_j + W_{j,n}, \quad j = 1, \dots, 6, \quad \frac{\partial\Phi_D^E}{\partial n} = \frac{\partial\Phi_D^I}{\partial n} = W_{D,n}, \tag{2}$$

where $W_{j,n}$ and $W_{D,n}$ are specified in the next section.

4 THE RESISTANCE LAW AT THE POROUS GEOMETRY

We model a porous geometry by representing it as a thin structure which allows fluid flow through its surface. It is common to model the flow-through velocity (W_n) as a function of the pressure drop (Δp) over the porous material; $W_n = W_n(\Delta p)$. Taylor (1956) described linear and quadratic laws which coupled the flow-through velocity and the pressure difference. Chwang (1983) described a porous wavemaker with a linear resistance law. The quadratic law presented by Molin (2011) includes separation effects. The equation is on the form

$$\Delta p = p^E - p^I = \frac{1-\tau}{2\mu_0\tau^2}\rho W_n|W_n| = \frac{1}{2}\rho C_0 W_n|W_n|, \tag{3}$$

where τ is the open area ratio in the range from 0 to 1, μ_0 is a discharge coefficient of order 1 and $C_0 = (1-\tau)/(\mu_0\tau^2)$. For reference we shall compare to calculations using also a linear resistance law. This law is only suitable for structures with very fine pores and does not account for separation of flow through the pores. The model applies Darcy's law and reads

$$W_n = (b/\mu)\Delta p, \tag{4}$$

where μ is the viscosity and b is a porosity parameter with dimension length.

4.1 Diffraction Problem

Having assumed a periodic motion we have, in the diffraction problem

$$W_{D,n} = \mathrm{Re}(i\omega A w_{D,n} e^{i\omega t}) = A\omega|w_{D,n}|\cos\tau, \tag{5}$$

where $w_{D,n} = |w_{D,n}|e^{i\delta}$ is a dimensionless flow-through velocity and $\tau = \omega t + \delta$. By use of equivalent linearization we have

$$|W_{D,n}|W_{D,n} = A^2\omega^2|w_{D,n}||\cos\tau|\cos\tau \cong (8/3\pi)A\omega|w_{D,n}|W_{D,n}, \tag{6}$$

where $|\cos\tau|\cos\tau| = (8/3\pi)\cos\tau + (8/15\pi)\cos(3\tau) + \ldots \cong (8/3\pi)\cos\tau$ is a truncation of the Fourier series such that the performed work or energy loss is conserved.

By applying the linearized Euler equation for the pressure jump we obtain

$$\Delta p = -\rho\partial/\partial t(\Phi_D^E - \Phi_D^I) = -\rho\,\mathrm{Re}(igA(\phi_D^E - \phi_D^I)e^{i\omega t}). \tag{7}$$

This gives the following condition for the quadratic resistance law

$$w_{D,n} = -i\frac{3\pi}{4}\frac{1}{C_0}\frac{1}{kA|w_{D,n}|}(\phi_D^E - \phi_D^I). \tag{8}$$

For the linear law using the linearized Euler equation, we find

$$w_{D,n} = -\frac{\rho i\omega b}{\mu}(\phi_D^E - \phi_D^I). \tag{9}$$

A similar procedure can be applied to the radiation problem obtaining an expression for the flow-through velocity.

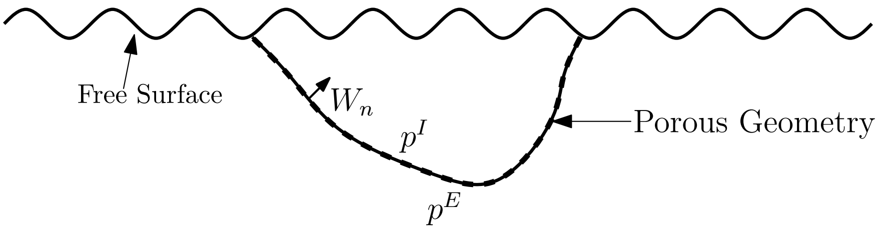

Fig. 1: Sketch of porous geometry. Symbols defined in the text.

5 INTEGRAL EQUATIONS FOR THE COUPLED SYSTEM

By using the Green's theorem a coupled set of equations for the inner and outer velocity potential is obtained. As Green function we use

$$G(x,y,z;\xi,\eta,\zeta) = \frac{1}{r} + \frac{1}{r'} + 2k\int_0^\infty \frac{e^{k(z+\zeta)}}{l-k}J_0(lR')\mathrm{d}l, \tag{10}$$

where J_0 is the Bessel function of the first kind and order zero, the integration is above the pole, $r^2 = (x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2$, $\quad r'^2 = (x-\xi)^2 + (y-\eta)^2 + (z+\zeta)^2$ and $R'^2 = (x-\xi)^2 + (y-\eta)^2$. By subtraction and addition of the equation for the inner and outer potentials, we find for the field point on the porous geometry S_b

$$2\pi\phi_D + \iint_{S_b} \psi_D G_n \mathrm{d}S = 4\pi\phi_0, \quad 2\pi\psi_D + \iint_{S_b} \phi_D G_n \mathrm{d}S - 2i\alpha_D \iint_{S_b} \psi_D G \mathrm{d}S = 4\pi\phi_0, \tag{11}$$

where

$$\alpha_D = \begin{cases} -\frac{3\pi}{4C_0 A|w_{D,n}|} & \text{for a quadratic resistance law} \\ -\frac{\rho b\omega}{\mu} & \text{for a linear resistance law,} \end{cases} \tag{12}$$

$$\phi_D = 2\phi_0 + \phi_7^E + \phi_7^I, \quad \psi_D = \phi_7^E - \phi_7^I. \tag{13}$$

In Eq.(12) in the quadratic case the variable $|w_{D,n}|$ is unknown. We obtain numerical solution by using Picard iteration with a relaxation factor of 0.5, where $w_{D,n}$ is expressed in Eq.(8). Both the linear and quadratic case have been solved with software described in the next section. The solution of the integral equations for a solid geometry is the special case when $\alpha_D = 0$, where there is no coupling effect between the inner and outer velocity potentials.

6 IMPLEMENTATION, VERIFICATION AND RESULTS

To solve the coupled integral equations, we have used WAMIT 5.3 modified for wave-current-body interaction by the University of Oslo. There has been done some major modifications to this software, due to the porous resistance laws. One of these modifications is to migrate the solver to Python. This was done due to instabilities in the original solver when obtaining the diffraction−radiation problem with a quadratic resistance law. The calculations of coefficients for added mass and damping, Haskind-relations, energy-dissipation and drift force have been modified to include the porous boundary condition. Convergence of the nonlinear solver is obtained when the maximum difference of the flow velocity on all of the panels are less than 10^{-5} from one iteration to another.

To verify our set of equations we started by computing added mass and damping coefficients for solid geometries and compared the results with analytical solutions. For verification of the nonlinear porous case we computed the added mass and damping of a sphere in unbounded fluid with excellent comparison to Molin (2011) (Dokken 2016, Fig 6.4).

The linear exciting force is expressed by $F_j^{ex} = \mathrm{Re}(\rho g A X_j e^{i\omega t})$. For verifying the relation between the radiation and diffraction problem we reexamined the Haskind relation, which is slighty modified by adding porous effects, where

$$X_j = \iint_{S_b} (\phi_D^E - \phi_D^I) n_j \mathrm{d}S = H_j(\beta + \pi) - \iint_{S_b} i(\phi_j^E - \phi_j^I)(\phi_7^E - \phi_7^I)(\alpha_j - \alpha_D)\mathrm{d}S, \tag{14}$$

where $\alpha_j = -3\pi/(4C_0\xi_j|w_{j,n}|)$ and α_D is given in Eq. (12) and H_j is the Kochin function with porous boundary conditions. This relation is different in the linear case, where $\alpha_j = \alpha_D$ and the only difference between linear porosity and the usual Haskind relation ($\alpha_D = \alpha_j = 0$) is the porous boundary condition included in the Kochin function. For further verification we compared the far field energy flux with the energy dissipation at the body. All derivations and verifications of these identities are available in Dokken (2016). In Zhao et al. (2011) the mean horisontal drift force for the diffraction problem including linear porous effects was derived. This was rexamined in Dokken et al. (2017) including linear porous effects, while in Dokken (2016) derivations of the drift force including quadratic effects appear. The mean drift force in the diffraction problem in x-direction can be expressed as

$$\bar{F}_1/(\rho g A^2) = -\frac{K}{8\pi}\int_0^{2\pi} |H_7(\theta)|^2(\cos\theta + \cos\beta)\mathrm{d}\theta + \frac{1}{2}\,\mathrm{Im}\left\{\iint_{S_b}\left(\phi_D^E - \phi_D^I\right)\frac{\partial \phi_D^*}{\partial n}\right\}\cos\beta\Bigg\}$$
$$+1/(2k)\,\mathrm{Im}\left\{\iint_{S_b}\alpha_D\left(\phi_D^E - \phi_D^I\right)\left(\frac{\partial\phi_D^{E*}}{\partial x} - \frac{\partial\phi_D^{I*}}{\partial x}\right)\right\}. \tag{15}$$

for both the linear and quadratic resistance law. The α_D is different for the two laws.

Computations for a hemisphere located at the free surface obtain the linear exciting forces X_1 and the mean drift force $\bar{F}_1$ as a function of the porosity, see Figure 2. Here the dots is computations of the the body integral of X_j and the lines are the RHS of Eq. (14). We observe the expected matching of these two computations. Observe that even the scaled exciting force on the solid body is larger than the exciting force for both the linear and quadratic resistance law for all wavenumbers.

We observe that $\bar{F}_1$ rapidly decreases with the shorter waves in the case of the quadratic law as compared to the linear law. This can be explained by the porosity factor α_D. By keeping the wave steepnes kA constant, an increase in kR would mean a proportional decrease in A, and a similar increase in α_D. This means that shorter waves have the same effect as increasing the porosity. We also observe that for short waves the drift force in the solid case is more than 4 times as large as in the porous cases. We observe that for long waves, the mean drift force in the case of the linear resistance law is significantly larger than in the solid body case. This is not the case with the quadratic law.

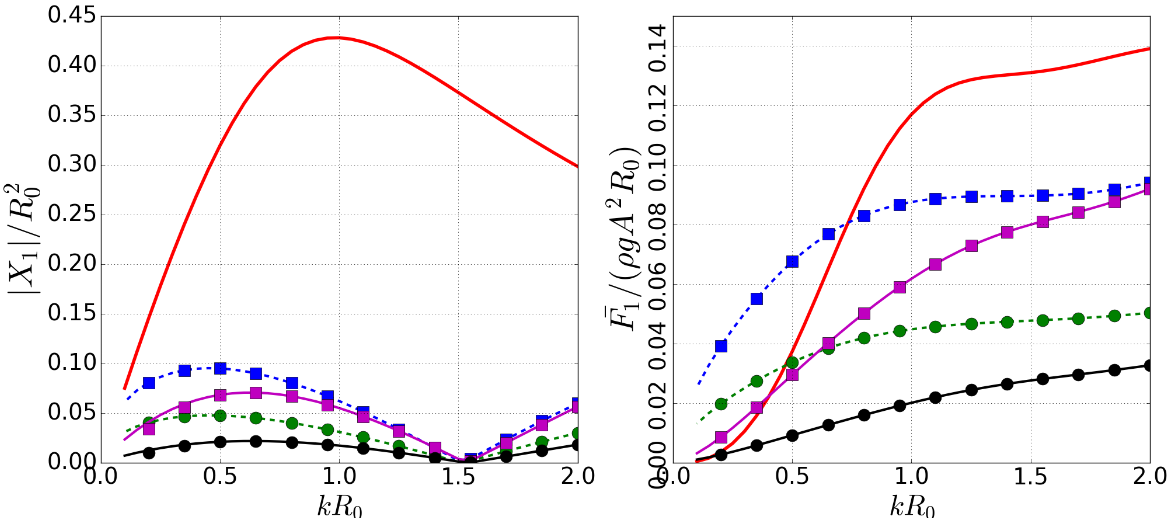

Fig. 2: a) Exiting force and b) Mean horisontal drift force as function of wavenumber. Quadratic resistance law: $\tau = 0.5$ (—■—) and $\tau = 0.7$ (—●—). Linear resistance law: $\sqrt{\mathbf{gR_0}}\rho\mathbf{b}/\mu = 9.7$ (--■--) and $\sqrt{\mathbf{gR_0}}\rho\mathbf{b}/\mu = 19.4$ (--●--). Force $\times 0.25$ on solid geometry (——).

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