Oblique wave scattering by a system of floating and submerged porous elastic plates

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Highlights

- An analytic model is developed for oblique wave scattering by floating and submerged porous elastic plates of different structural parameters.
- The nature of the roots of the complex dispersion relation is analyzed using the counting argument and contour plot.
- Eigenfunction expansion method is used to handle the physical problem. The reflection, transmission and dissipation coefficients, wave force and plate deflections are computed to examine the effects of various wave and structural parameters in a creating tranquility zone.
- It is observed that a significant amount of wave energy is dissipated due to the presence of the floating and submerged porous plates, thus resulting in less wave reflection and transmission.

1 Introduction

To reduce wave reflection and transmission, floating and/or submerged horizontal porous structure can be used as an effective breakwater or wave absorber. In comparison with traditional structures, horizontal structures are less dependent on local seabed geological conditions and are more economical in the use of construction materials. Since the early studies of Heins (1950) on wave interaction with a submerged horizontal solid plate, many researchers have developed various mathematical techniques to investigate the hydrodynamic performance of floating/submerged horizontal plates. Hassan et al. (2009) developed an analytic solution using the eigenfunction method for wave scattering by a submerged elastic plate. Recently, Meylan et al. (2016) used the eigenfunction, boundary-element and finite-element methods for wave interaction with a floating porous elastic plate. In the present study, matching conditions and the orthogonal property of the open water region vertical eigenfunctions along with free, simply-supported and clamped edge conditions are used to handle the the mathematical boundary value problem as in Hassan et al. (2009).

2 Mathematical formulation

The present physical problem is considered in a three-dimensional Cartesian co-ordinate system with the x- and y-axes being in horizontal directions and the z-axis in the vertical (positive upward) direction under the assumption of linearized water wave theory and small amplitude structural response. A finite thin porous elastic plate of length B is floating at the mean free surface $z = 0$ and another submerged elastic plate having the same length B of different structural parameters is kept horizontally at $z = -s$ in water of finite depth h. The fluid domain is divided into four regions as shown in Fig. 1. Assuming that the fluid is inviscid, incompressible and irrotational, the surface gravity waves interact with the floating flexible plate making an oblique angle θ with the x-axis and the wave motion is simple harmonic in time with an angular frequency ω. The velocity potentials are written in the form $\Phi_j(x, y, z, t) = \mathrm{Re}\{\phi_j(x, z)\mathrm{e}^{-\mathrm{i}(k_y y-\omega t)}\}$ for $j = 1, 2, 3, 4$ which satisfy the Helmholtz equation with $k_y = k_0 \sin\theta$. Further, $\zeta_j(x, y, t) = \mathrm{Re}\{\eta_j(x)e^{-\mathrm{i}(k_y y-\omega t)}\}$ for $j = 1, 2$ being the deflection of the floating and submerged plates respectively. It is assumed that the bottom bed is rigid, thus, $\partial\Phi_j/\partial z = 0$ on $z = -h$ for $j = 1, 3, 4$. The linearized kinematic conditions on the floating and submerged elastic

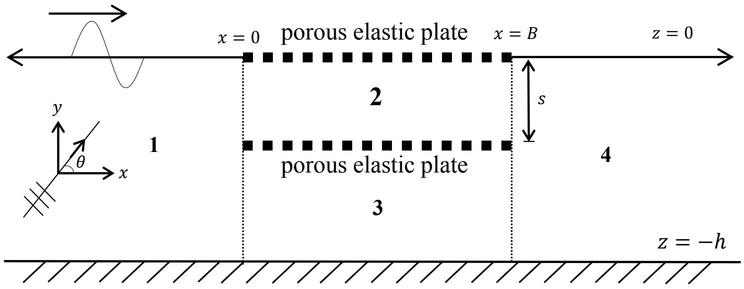


Figure 1: Schematic diagram of wave scattering by floating and submerged porous elastic plates.

porous plates are given by

$$\frac{\partial \Phi_2}{\partial z} = \frac{\partial \zeta_1}{\partial t} + \mathrm{i}k_0 G_1 \Phi_2 \quad \text{and} \quad \frac{\partial \Phi_2}{\partial z} = \frac{\partial \Phi_3}{\partial z} = \frac{\partial \zeta_2}{\partial t} + \mathrm{i}k_0 G_2(\Phi_2 - \Phi_3), \tag{1}$$

respectively, at $z = \{0, -s\}$ with G_j for $j = 1, 2$ are the porous-effect parameters of floating and submerged plates respectively, and k_0 is the wave number of the plane gravity wave. The plate deflection ζ_j in the presence of compressive force satisfies (as in Behera and Sahoo, 2015)

$$\Big(E_j I_j \nabla^4_{xy} + Q_j \nabla^2_{xy} + m_{pj}\frac{\partial^2}{\partial t^2}\Big)\zeta_j = P_{j+1}(x,y,z,t)\big|_{z=\beta-} - P_j(x,y,z,t)\big|_{z=\beta+}, \tag{2}$$

where $\beta = \{0, -s\}$, $\nabla^2_{xy} = \partial^2/\partial x^2 + \partial^2/\partial y^2$, the linearized hydrodynamic pressure $P_j(x, y, z, t)$ in the $j-$th region and other boundary conditions being same as given in Hassan et al. (2009). Moreover, $P_1(x, y, z, t)$ at $z = 0+$ is the constant atmospheric pressure. Further, E_j is the Young's modulus, Q_j is the compressive force acting on the $j-$th plate, $m_{pj} = \rho_{pj} d_j$ is the mass per unit length, ρ_{pj} is the density of the flexible plate, d_j is the thickness, $I_j = d_j^3/[12(1-\nu^2)]$ associated with the j-th plate and ν being the Poisson's ratio of each of the plate. Eliminating P_j and ζ_j, from Eqs. (1) and(2), the conditions on the floating and submerged plates are given by

$$\Big(E_1 I_1 \nabla^2_{xy} + Q_1 \nabla^2_{xy} + m_{p1}\frac{\partial^2}{\partial t^2}\Big)\Big(\frac{\partial \Phi_2}{\partial z} + \mathrm{i}k_0 G_1 \Phi_2\Big) = \rho\Big\{\frac{\partial^2 \Phi_2}{\partial t^2} - g\frac{\partial \Phi_2}{\partial z}\Big\}, \text{ on } z = 0, \tag{3}$$

$$\bigg(E_2 I_2 \nabla^4_{xy} + Q_2 \nabla^2_{xy} + m_{p2}\frac{\partial^2}{\partial t^2}\bigg)\bigg\{\frac{\partial \Phi_3}{\partial z} + \mathrm{i}k_0 G_2(\Phi_3 - \Phi_2)\bigg\} = \rho\bigg(\frac{\partial^2 \Phi_2}{\partial t^2} - \frac{\partial^2 \Phi_3}{\partial t^2}\bigg), \text{ on } z = -s. \tag{4}$$

The continuity of velocity and pressure at $x = 0$ and $x = B$ yield

$$\frac{\partial \Phi_j}{\partial x} = \frac{\partial \Phi_2}{\partial x}, \quad \frac{\partial \Phi_j}{\partial x} = \frac{\partial \Phi_3}{\partial x}, \quad \Phi_j = \Phi_2, \quad \Phi_j = \Phi_3, \tag{5}$$

where $j = 1$ at $x = 0$ and $j = 4$ at $x = B$. Assuming that both the floating and submerged plates are clamped at both ends, the vanishing of plate deflection and slope of the plate deflection yield

$$\frac{\partial \Phi_2}{\partial z} = 0, \quad \frac{\partial^2 \Phi_2}{\partial x \partial z} = 0 \quad \text{at} \quad (\alpha, \beta), \tag{6}$$

where $\alpha = \{0, B\}$ and $\beta = \{0, -s\}$. On the other hand, the conditions for simple-supported and free edges are same as described in Behera and Sahoo (2015)

3 Method of Solution

The form of the spatial velocity potentials $\phi(x, z)$ in regions 1, 2, 3 and 4 are given by

$$\phi_1 = I_0 e^{-\mathrm{i}\mu_0 x} f_{10}(k_0, z) + \sum_{n=0}^{\infty} R_n e^{\mathrm{i}\mu_n x} f_{1n}(k_n, z), \quad \text{for} \quad x < 0,\ -h < z < 0, \tag{7}$$

$$\phi_2 = \sum_{n=-4}^{\infty} \Big\{A_n e^{-\mathrm{i}\vartheta_n x} + B_n e^{\mathrm{i}\vartheta_n (x-B)}\Big\} f_{2n}(p_n, z), \quad \text{for} \quad 0 < x < B,\ -s < z < 0, \tag{8}$$

$$\phi_3 = \sum_{n=-4}^{\infty} \left\{ A_n e^{-\mathrm{i}\vartheta_n x} + B_n e^{\mathrm{i}\vartheta_n (x-B)} \right\} f_{3n}(p_n, z), \quad \text{for} \quad 0 < x < B,\ -h < z < -s, \tag{9}$$

$$\phi_4 = \sum_{n=0}^{\infty} T_n e^{-\mathrm{i}\mu_n (x-B)} f_{1n}(k_n, z), \quad \text{for} \quad x > B,\ -h < z < 0, \tag{10}$$

where I_0 is the incident wave amplitude, and R_n, A_n, B_n and T_n for $n = 0, 1, 2, \ldots$ are the unknown expansion coefficients with $\mu_n = \sqrt{k_n^2 - k_y^2}$ and $\vartheta_n = \sqrt{p_n^2 - k_y^2}$. The eigenfunctions $f_{1n}(k_n, z)$, $f_{2n}(p_n, z)$ and $f_{3n}(p_n, z)$ for $n = 0, 1, 2, \ldots$ are given by

$$f_{1n}(k_n, z) = \frac{\cosh k_n(z+h)}{\cosh k_n h}, \quad f_{2n}(p_n, z) = \frac{\cosh p_n(z+h) - F_n \sinh p_n(z+h)}{\cosh p_n h - F_n \sinh p_n h}, \tag{11}$$

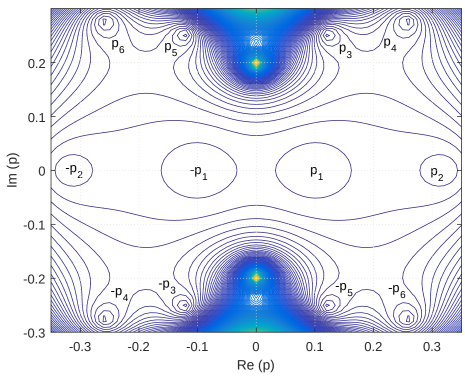
$$f_{3n}(p_n, z) = \frac{\tanh p_n(h-s) - F_n}{\tanh p_n(h-s)} \frac{\cosh p_n(z+h)}{\cosh p_n h - F_n \sinh p_n h}, \tag{12}$$

$$F_n = \frac{U_n p_n \tanh^2 p_n(h-s)}{U_n p_n \tanh p_n(h-s) - (K - \mathrm{i}k_0 G_2 U_n)\{1 - \tanh^2 p_n(h-s)\}}, \tag{13}$$

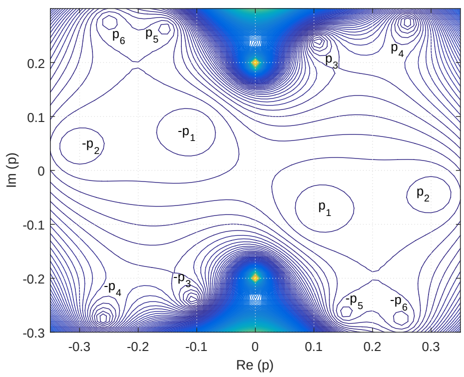
with $U_n = D_2 p_n^4 - N_2 p_n^2 - M_2$ and $M_j = m_{pj}\omega^2/(\rho g)$ for $j = 1, 2$. The eigenvalues k_n satisfy the dispersion relation $\omega^2 = g k_n \tanh k_n h$, where k_0 is positive real and $k_n = \mathrm{i}\kappa_n$ for $n = 1, 2, 3, \ldots$ with κ_n being real. The eigenvalues p_n satisfy the dispersion relation

$$K - V_n p_n \tanh p_n h + \mathrm{i}k_0 G_1 V_n = F_n(K \tanh p_n h - V_n p_n + \mathrm{i}k_0 G_1 V_n \tanh p_n h), \tag{14}$$

where $V_n = D_1 p_n^4 - N_1 p_n^2 - M_1 + 1$. It may be noted that for $G_j = 0$ for $j = 1, 2$, the dispersion relation in Eq. (14) reduces to the dispersion relation in case of wave interaction with floating and submerged impermeable elastic plates, which has two distinct positive real roots, eight complex roots p_n of the form $a \pm \mathrm{i}b$ and $c \pm \mathrm{i}d$, and infinite number of imaginary roots as shown in Fig. 2(a), and, in case of porous elastic plates, all the roots of the dispersion relation in Eq. (14) are complex in nature as shown in Fig. 2(b), and the position of the roots are close to the roots for $G_j = 0$. In the present study, the roots lying in the first and fourth quadrants of the complex plane are considered for keeping the boundedness property of the velocity potentials. Using the velocity potentials as in Eqs.



(a)



(b)

Figure 2: Contour plots of roots of the dispersion relation for flexible and submerged horizontal (a) non-porous elastic plate and (b) porous elastic plate with $H = 5$ m, $h/H = 0.5$, $T = 8$ sec., $g = 9.81$ m/sec^2, $\nu = 0.3$, $E_j = 5$ GPa, $d_j = 0.1$, $Q_j = 0$ and $G_j = 1$ for $j = 1, 2$.

(7)–(10), matching conditions Eq. (5) and the orthogonality of the eigenfunctions f_{1n} (as in Hassan et al., 2009), a system of $4N$ equations is obtained. Further, for the determination of the unknowns R_n, T_n, A_n and B_n, edge conditions as in Eq. (6) are used to find another eight additional equations.

4 Results and discussion

In the present study, various physical parameters are kept fixed as $h/H = 0.5$, $\nu = 0.3$, $E_j = 5$ GPa, $d_j = 0.1$, $Q_j = \sqrt{E_j I_j \rho g}$, $\theta = 30^\circ$ and $G_j = 1$ unless it is stated otherwise. The reflection, transmission and dissipation coefficients are computed using the formulae $K_r = |R_0/I_0|$, $K_t = |T_0/I_0|$ and $K_d = 1-(K_r^2+K_t^2)$. Figs. 3(a) and (b) depict that in case of porous elastic plates, very low wave reflection

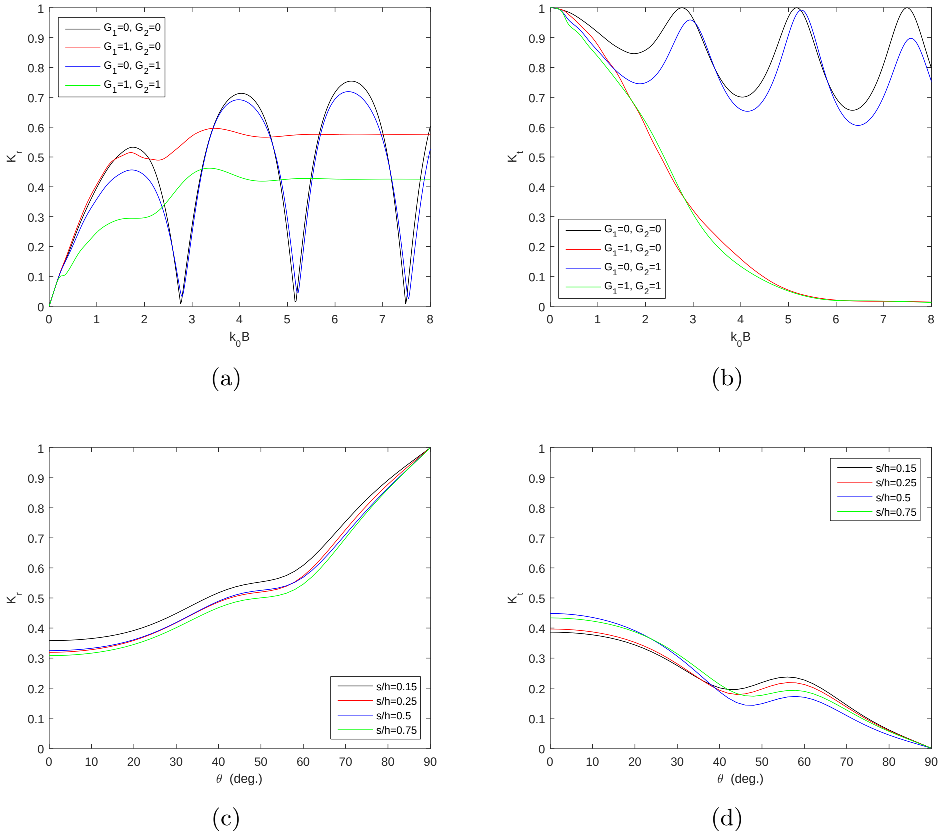


Figure 3: Effect of G_j on (a) K_r and (b) K_t as a function of $k_0 b$. Effect of s/h on (c) K_r and (d) K_t as a function of θ for $b/h = 2$, $G_1 = 0$ and $G_2 = 1$.

and transmission will occur as a result of the dissipation of wave energy. Further, the amplitude of the oscillatory pattern of the reflection and transmission coefficients decreases with an increase in $k_0 B$ in case of $G_j \neq 0$. Moreover, in case of impermeable plates, nearly zero reflection and full transmission is observed periodically and satisfies $K_r^2 + K_t^2 = 1$. On the other hand, from Figs. 3(c) and (d), it is observed that the reflection coefficient increases and transmission coefficient decreases when the submerged plate is nearer the floating plate. Further results will be presented in the workshop.

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