Excitation of Ship Waves by a Submerged Object: New Solution to the Classical Problem

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HIGHLIGHT

We have proposed a new method for solving the problem of ship waves excited on the surface of a nonviscous liquid by a submerged object that moves at a variable speed. A new solution of the classic problem of ship waves generated by moving a submerged ball with constant velocity parallel to the equilibrium surface of the liquid has been described. As a second example of usage our method, we have considered vertical oscillations of the submerged ball.

1 INTRODUCTION

In this paper we describe a new method for solving the problem of ship waves excited by the motion of a submerged object which has been proposed by us in [1].

In contrast to Havelock's papers [3,4], we do not introduce artificial viscosity to ensure the convergence of the integrals. We begin with a solution of the non-stationary problem, suggesting that once in the past a submerged object had been in the state of rest. Transforming our solution to the limit of motion at a constant speed, we automatically arrive at a rule handling the singularity in the integrand, which is completely analogous to the Landau bypass rule in plasma physics [6]. The very same singularity in the integrand corresponds to the Cherenkov resonance $V\cos\theta = \omega/k$, which generates gravity waves with frequency $\omega = \sqrt{gk}$ and wave vector $\mathbf{k}$, which forms an angle θ with the direction of the velocity. In contrast to the Cherenkov radiation of electromagnetic waves in the optics [2, 7], where phase velocity ω/k has a predetermined value (equal to the speed of light in the medium), due to the dispersion of phase velocity $\omega/k = \sqrt{g/k}$, the gravity waves are emitted in the entire range of angles θ from 0 (forward in the direction of the body motion) to π (against the direction of motion). Each value of the angle θ corresponds to certain value of the wave number

$$k(\theta) = \frac{g}{V^2\cos^2\theta}. \tag{1}$$

As a consequence, the smallest wave number (i.e., the highest wavelength) that is compatible with the Cherenkov resonance condition is

$$k_g = g/V^2. \tag{2}$$

It should be explained that the Cherenkov resonance concept was not mentioned earlier in the theory of ship waves. Instead, various authors refer to a so called "steady-state condition" or to a "radiation condition".

As the second example, we considered vertical oscillatory motion of the ball with a small amplitude. In this case, the solution is expressed in terms of a single integral and describes a radial wave on the liquid surface, diverging from the epicenter over the ball.

2 EXCITATION OF SHIP WAVE BY A SUBMERGED BALL

We first construct a solution of the Laplace equation

$$\nabla^2 \phi(x,y,z,t) = 0 \tag{3}$$

with the boundary conditions

$$\frac{\partial}{\partial z}\phi(x,y,0,t) = \frac{\partial}{\partial t}\zeta(x,y,t), \tag{4}$$

$$\frac{\partial}{\partial t}\phi(x,y,0,t) + g\zeta(x,y,t) = -\frac{1}{\rho}\delta p(x,y,t) \tag{5}$$

at the plane $z = 0$ of unperturbed liquid surface, where the pressure is given by the external field $\delta p(x,y,t)$. Assuming that in the distant past there was no external pressure source and, respectively, the liquid surface was quite flat, we arrived at the following expression for the water surface elevation

$$\zeta(x,y,t) = \iint \frac{\mathrm{d}k_x}{2\pi}\frac{\mathrm{d}k_y}{2\pi}\zeta_k(t)\,\mathrm{e}^{ik_x x + ik_y y}, \tag{6}$$

where

$$\zeta_k(t) = \frac{\sqrt{kg}}{\rho g}\int_{-\infty}^{t} \sin\left[\sqrt{kg}(\tau - t)\right]\delta p_k(\tau)\,\mathrm{d}\tau, \tag{7}$$

$$\delta p_k(t) = \iint \mathrm{d}x\,\mathrm{d}y\,\delta p(x,y,t)\,\mathrm{e}^{-ik_x x - ik_y y}. \tag{8}$$

We then extend this solution to the case of a submerged body and obtain the following expression

$$\zeta_k(t) = -2k\int_{-\infty}^{t} \cos\left[\sqrt{kg}(\tau - t)\right]\phi_k^{(0)}(\tau)\,\mathrm{d}\tau, \tag{9}$$

where

$$\phi_k^{(0)}(t) = -\frac{\pi a^3}{k}\frac{\partial}{\partial t}\mathrm{e}^{kZ(t) - ik_x X(t) - ik_y Y(t)} \tag{10}$$

in case of submerged ball of radius a moving along a given trajectory $\{X(t), Y(t), Z(t)\}$. Having found the Fourier-amplitude $\zeta_k(t)$, one can restore function $\zeta(x,y,t)$ using Eq. (6), although computation of the involved integrals represents a challenge task. Convergence in the integral (9) for more or less realistic functions $X(t)$, $Y(t)$, $Z(t)$ is guaranteed by the fact that their time derivatives tend to zero as $t \to -\infty$.

3 UNIFORM MOTION OF THE BALL

As a first application of our method we treated the limit of a ball that moves at fixed depth $Z(t) = -h =$ const with constant velocity $V =$ const. The result of computation is represented as a sum of two terms, ζ_0 and ζ_1, each of which is a single integral. We managed to calculate these integrals and obtained relatively simple asymptotic expressions for ζ in case of either small or large Froude number $F = V/\sqrt{gh}$. The study of these asymptotics showed that the first term can be interpreted as describing the "Bernoulli hump", and the second term stands for what is called "Kelvin wedge". We note that Havelock's formula [3, 5] also contains two terms but they have no so simple physical meaning.

Two-dimensional maps of the liquid surface elevation are shown in Fig. 1 for a set of the Froude numbers.

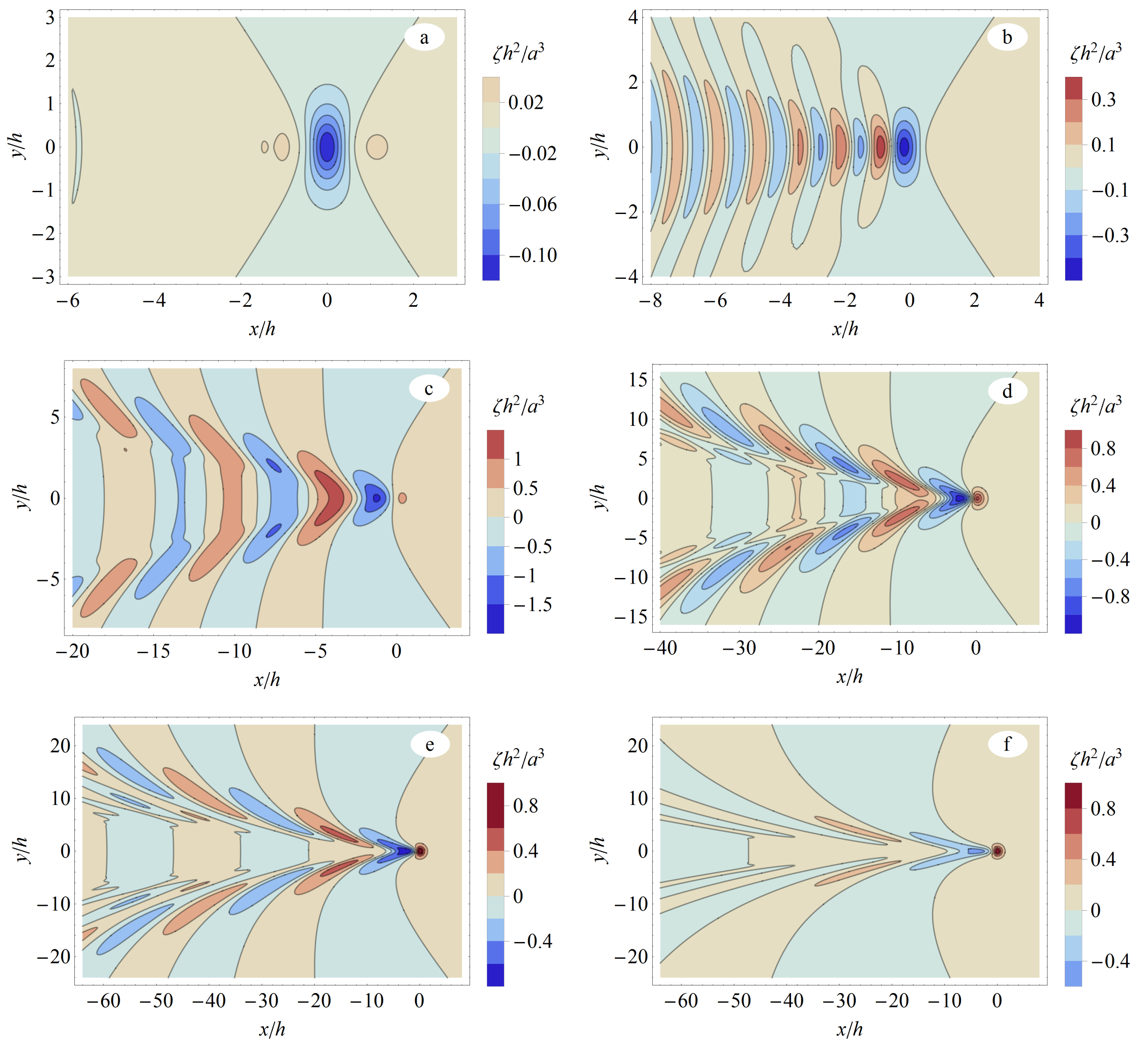

Figure 1. (Color online) Profile of the ship wave at different values of the Froude number: a) $F = 0.3$, $\lambda_g/h = 0.57$; b) $F = 0.45$, $\lambda_g/h = 1.27$; c) $F = 1.0$, $\lambda_g/h = 6.28$; d) $F = 1.5$, $\lambda_g/h = 14.13$; e) $F = 2.0$, $\lambda_g/h = 25.13$; f) $F = 3.0$, $\lambda_g/h = 56.55$.

4 VERTICAL OSCILLATION OF A SUBMERGED BALL

As a second example we treated vertical harmonic oscillations of the submerged ball with a small amplitude δZ and given frequency ω, so that

$$X(t) = Y(t) = 0, \qquad Z(t) = -h + \delta Z \cos(\omega t).$$

In this case the liquid surface elevation is given by the expression

$$\zeta(r,t) = \mathrm{Re}[\bar{\zeta}(r)\,\mathrm{e}^{-i\omega t}], \tag{11}$$

where

$$\bar{\zeta}(r) = -\frac{a^3\omega^2}{g}\,\delta Z \int_0^\infty \frac{k^2\,\mathrm{J}_0(kr)\,\mathrm{e}^{-hk}}{k - (\omega + i0)^2/g}\,\mathrm{d}k, \tag{12}$$

is the complex amplitude, J_0 denotes the Bessel function of zero order, and $r = \sqrt{x^2 + y^2}$.

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