

Implementation of Generalized Modes in a 3D Finite Difference Based Seakeeping Model

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1 INTRODUCTION

This work is an extension of the finite difference potential flow solver *OceanWave3D-Seakeeping* developed by Afshar (2014) to include generalized modes. The continuity equation is solved using a fourth-order centered finite difference scheme which requires that the entire fluid domain is discretized as opposed to the more popular panel method where only the body surface - and sometimes the free surface and sea bottom - are discretized. The advantage for the finite difference solver is thought to be found for complex or high-resolution problems where the computational time will scale better due to the sparse nature of the coefficient matrix. The solver is built using the open source framework **Overture** which consists of C++ libraries for solving partial differential equations on overlapping grids and has a built-in overlapping grid generator **Ogen**.

2 MATHEMATICAL FORMULATION

A flexible floating or submerged body of length L with a constant forward speed, U , is considered in a semi-infinite fluid of depth h with a free surface. The linearized equation of motion can be expressed in the frequency domain as:

$$\sum_{j=1}^{6+N} \left[-\omega^2 (M_{ij} + a_{ij}(\omega)) + i\omega B_{ij}(\omega) + (K_{ij} + C_{ij}) \right] \xi_j = X_i \quad i = [1 : 6 + N] \quad (1)$$

With N being the number of flexible modes. Here M_{ij} is the structural mass matrix, a_{ij} is the hydrodynamic added mass, B_{ij} is the hydrodynamic damping, K_{ij} is the structural stiffness, C_{ij} is the hydrostatic restoring stiffness, X_i is the wave exciting force and ξ_j is the motion response phasor. How the coefficients for the matrices are obtained are described in the subsections.

2.1 Hydrodynamic Problem

The governing equation is the continuity equation:

$$\Delta\Phi = 0 \quad (2)$$

The solution is decomposed into a steady and an unsteady solution. The steady solution, $\phi_b = -Ux + \phi_{db}$, is found using the Double-Body linearization by solving the boundary value problem:

$$\begin{aligned} \Delta\phi_{db} &= 0 \\ \frac{\partial\phi_{db}}{\partial n} &= \mathbf{U} \cdot \mathbf{n} \quad \text{on } S_0 \\ \frac{\partial\phi_{db}}{\partial z} &= 0 \quad \text{on } z = 0 \\ \nabla\phi_{db} &\rightarrow 0 \quad \text{in the far field} \end{aligned} \quad (3)$$

With $\mathbf{U} = (-U, 0, 0)$ and \mathbf{n} the unit normal vector to the body surface S_0 . The unsteady solution, ϕ_u , consists of the velocity potential due to the incoming waves, the scattered waves and the radiated waves for all degrees of freedom respectively. It is found by subjecting (2) to the following linearized boundary conditions. The dynamic and kinematic free surface boundary conditions can be written (see for example Afshar (2014)):

$$\frac{\partial \phi_u}{\partial t} = -g\eta_u + U \frac{\partial \phi_u}{\partial x} - \frac{1}{2} \nabla \phi_{db} \cdot \nabla \phi_{db} + U \frac{\partial \phi_{db}}{\partial x} - g\eta_b - \nabla \phi_{db} \cdot \nabla \phi_u = 0 \quad \text{on } z = 0 \quad (4)$$

$$\frac{\partial \phi_u}{\partial t} = \frac{\partial \phi_u}{\partial z} + U \frac{\partial \eta_u}{\partial x} - \nabla \phi_{db} \cdot \nabla \eta_u + \eta_u \frac{\partial^2 \phi_{db}}{\partial z^2} - \nabla \phi_{db} \cdot \nabla \eta_b + U \frac{\partial \eta_b}{\partial x} + \eta_b \frac{\partial^2 \phi_{db}}{\partial z^2} \quad \text{on } z = 0 \quad (5)$$

Since the wave elevation is likewise split in a steady and an unsteady solution: $\eta = \eta_b + \eta_u$. The double-body elevation can be calculated using the dynamic free-surface boundary condition (4) dropping the unsteady terms. The body boundary condition is:

$$\frac{\partial \phi_j}{\partial \mathbf{n}} = \dot{\zeta}_j(t) n_j + \zeta_j(t) m_j \quad \text{on } S_0 \quad , \quad m_j = \left((W \cdot \nabla) \mathbf{S}_j - (\mathbf{S}_j \cdot \nabla) W \right) \cdot \mathbf{n} \quad (6)$$

Where $W = \nabla \phi_b$ and \mathbf{S}_j are the mode shapes as defined in the next section. And finally the bottom boundary condition:

$$\frac{\partial \phi}{\partial \mathbf{n}} = 0 \quad \text{on } z = -h \quad (7)$$

This initial boundary value problem is solved in the time domain using a pseudo-impulsive Gaussian impulse and its time derivative as the input displacement, $\zeta_j(t)$, and velocity $\dot{\zeta}_j(t)$ in the boundary condition. Since the floating body is regarded as a linear system the hydrodynamic coefficients in the frequency domain can subsequently be found by a Fourier transform of the radiation and diffraction forces F_{ij} and F_i

$$\omega^2 a_{ij}(\omega) - i\omega B_{ij}(\omega) = \frac{\mathcal{F}\{F_{ij}(t)\}}{\mathcal{F}\{\zeta(t)\}} \quad , \quad X_i(\omega) = \frac{\mathcal{F}\{F_i(t)\}}{\mathcal{F}\{\zeta(t)\}} \quad (8)$$

2.2 Structural Model

The flexibility of the ship is considered by extending the rigid mode analysis (first 6 modes) to several flexible modes as first described by Newman (1994) by introducing a vector shape function $\mathbf{S}_j = [u_j, v_j, w_j]$. This yields a new expression for the generalized normal vector as the normal component of the shape function on the body surface: $n_j = \mathbf{S}_j \cdot \mathbf{n}$. The total displacement of a point on the ship \mathbf{x} is then given as a sum over the products of modal amplitude, ξ_j , and the shape function:

$$\mathbf{H}(\mathbf{x}, t) = \sum_{j=1}^{6+N} \xi_j \mathbf{S}_j(\mathbf{x}) \quad (9)$$

The simplest approximation for the vertical deflections of the hull is the set of 1D free-free homogeneous beam modes which are given analytically by:

$$w_{2n+5} = \frac{1}{2} \left[\frac{\cos \kappa_{2n} q}{\cos \kappa_{2n}} + \frac{\cosh \kappa_{2n} q}{\cosh \kappa_{2n}} \right] \quad , \quad w_{2n+6} = \frac{1}{2} \left[\frac{\cos \kappa_{2n+1} q}{\cos \kappa_{2n+1}} + \frac{\cosh \kappa_{2n+1} q}{\cosh \kappa_{2n+1}} \right] \quad (10)$$

Where $n = 1, 2, \dots, N$, $q = 2x/L$ is the normalized x-coordinate and κ_m are the eigenvalues satisfying the eigenvalue equation:

$$(-1)^m \tan \kappa_m + \tanh \kappa_m = 0 \quad (11)$$

These modes are orthogonal leading to simple expressions for the flexible part of the mass matrix and the structural stiffness matrix:

$$M_{ij} = \frac{1}{4} M \delta_{ij} \quad K_{ij} = 4(EI/L^3) \kappa_i^4 \delta_{ij} \quad (12)$$

Here M is the total mass, EI is the bending stiffness comprised of the elasticity modulus and the moment of inertia of the cross section. The rigid part of the mass matrix is defined as usual and the rigid part of the structural stiffness matrix consists of zeroes.

2.3 Hydrostatic Problem

The expression for the hydrostatic restoring force is given in Malenica (2009):

$$C_{ij} = C_{ij}^H + C_{ij}^M = \rho g \int \int_{S_0} \mathbf{n}_j (w_i + z D_i) dS + g \int \int \int_V \rho_s (\mathbf{S}_i \nabla) w_j dV \quad (13)$$

3 FLEXIBLE RESPONSE OF PRISMATIC BARGE

The response amplitude operator for the first flexible mode in figure 1a is compared to experiments and numerical computations carried out by Malenica (2003) for a prismatic barge at zero speed in head seas. The characteristics of the prismatic barge are described in detail in Malenica (2003). Furthermore the vertical displacement amidships is compared to experiments by Malenica (2003) and numerical computations by Kashiwagi (2015) in figure 1b.

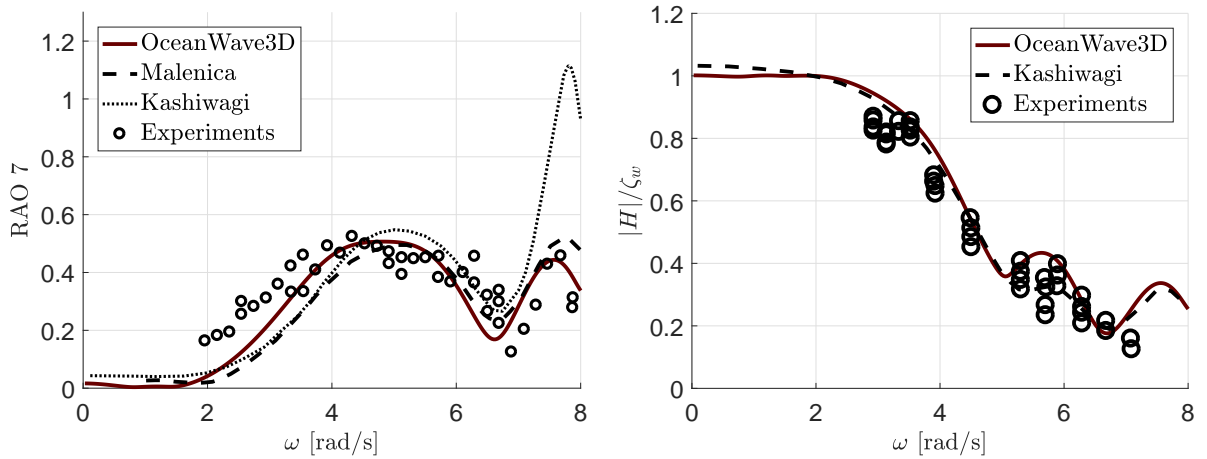


Fig. 1: Response amplitude operator for the first flexible hull and vertical displacement midship for the prismatic barge.

4 EFFECT OF FORWARD SPEED FOR THE MODIFIED WIGLEY HULL

Kashiwagi (2015) has presented forward speed results for a modified Wigley hull which is attempted replicated by the authors. It should be noted that the geometry of the ship hull for the *OceanWave3D-Seakeeping* computation is deviating due to the limitations of the finite

difference solver to only have rounded edges which in this case alters the stern and bow quite dramatically which is thought to be the main reason for the somewhat different results obtained here. The same conclusions as in Kashiwagi (2015) are drawn: The forward speed affects the radiation problem but not the diffraction problem.

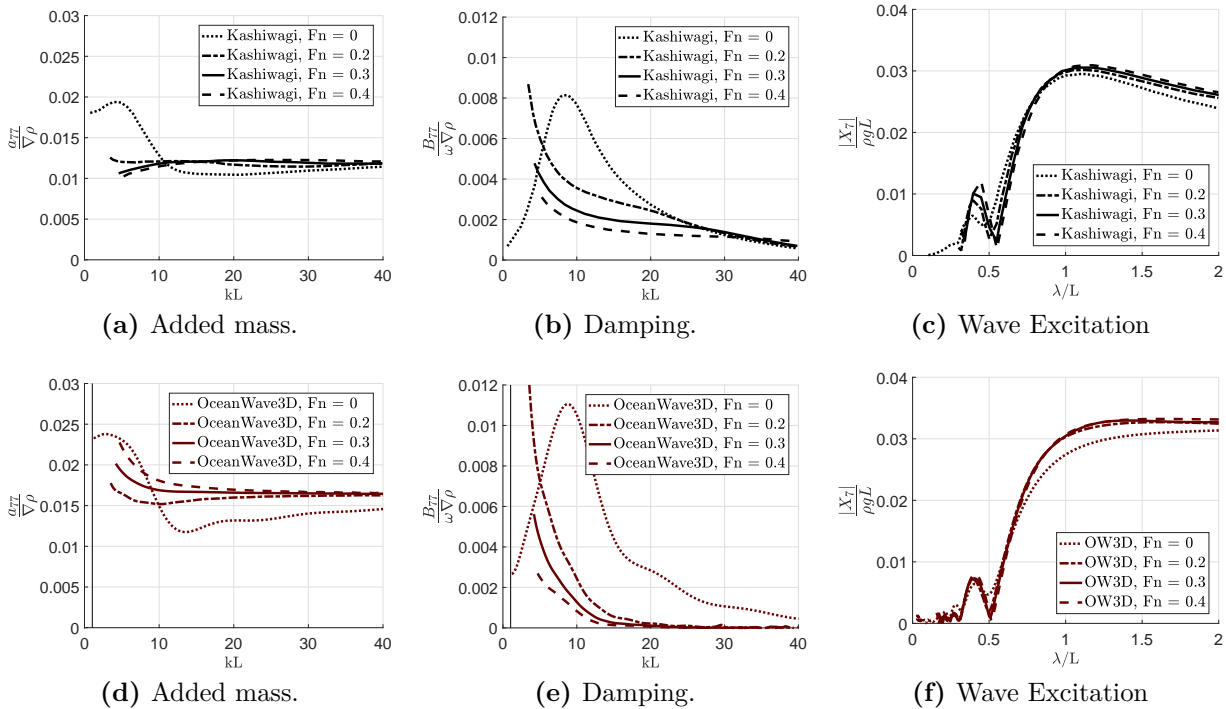


Fig. 2: Hydrodynamic coefficient dependency for first flexible mode on forward speed for the modified Wigley hull. Top row is Kashiwagi 2015.

5 CONCLUSIONS

Generalized modes have been implemented in the *OceanWave3D-Seakeeping* solver which has been validated against both numerical and experimental data for the zero speed case. For forward speed problems some preliminary results have been obtained which show the correct trends, but further validation is required. The next step will be to extend the solver to solve problems in quartering seas to validate the forward speed solver adequately.

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