Implementation of the far-field method for calculation of added resistance using a high order finite-difference approximation on overlapping grids

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HIGHLIGHT

The far-field method for calculation of the wave drift force is implemented in the high order finite-difference seakeeping solver. The implementation is based on the Maruo formulation which employes the Kochin function to obtain the complex amplitude of the velocity potential in the far-field. The results are shown both for zero and forward speed for the floating hemisphere and two ship geometries. Comparisons with WAMIT and near-field calculations are also presented.

1 INTRODUCTION

At the Technical University of Denmark, our group will soon release an open-source seakeeping solver (*OceanWave3D-Seakeeping*), which has been developed over the past five years (Amini Afshar, 2015). The code is a linearized potential flow solver, and can be used to solve for the *wave resistance, radiation, diffraction* and also *generalized flexible modes* problems. 4th order finite-difference approximation on overlapping grids are used to solve the Laplace equation in the whole flow domain. The free-surface conditions are integrated in time using 4th order Runge-Kutta scheme to update the solution at the free surface. For a more detailed explanation about the numerical method and the stability of the scheme please refer to (Amini Afshar, 2015). The solver has been written in C++ inside an open-source library called **Overture** (Brown et al., 1999). The library also provides the *hyperbolic grid generation* facilities, which can be used to build the overset grids for ship geometries. The near-field and far-field methods for calculation of the wave drift forces have been implemented. This paper presents the results and also discusses some challenges of the far-field calculations based on the Kochin function integral.

2 FORMULATION

Although the double-body flow linearization is implemented in the code, we outline here only the Neumann-Kelvin linearization of the problem for simplicity. The continuity equation $\nabla^2\phi = 0$, is solved in the fully discretized flow domain, subjected to the dynamic and kinematic free-surface conditions at $z = 0$ as:

$$\frac{\partial \phi}{\partial t} = -g\zeta + U\frac{\partial \phi}{\partial x} \quad \text{and} \quad \frac{\partial \zeta}{\partial t} = \frac{\partial \phi}{\partial z} + U\frac{\partial \zeta}{\partial x}\,.$$

The forward speed of the body is denoted by U. The surface elevation and velocity potential are denoted ζ and ϕ respectively. The boundary conditions applied at the undisturbed body surface S_B for the motion of the body in kth direction ξ_k (radiation problem ϕ_k), and for the diffraction problem (ϕ_s) are:

$$\frac{\partial \phi_k}{\partial n} = \dot{\xi}_k \cdot n_k + \xi_k \cdot m_k \quad \text{and} \quad \frac{\partial \phi_s}{\partial n} = -\mathbf{n}\cdot\nabla\phi_0(\mathbf{r},t)\,.$$

The time differentiation is denoted by the over-dot. The position vector is shown by $\mathbf{r}$, and the incident wave velocity potential is ϕ_0. The generalized outward normal vector to the body surface $\mathbf{n} = (n_1, n_2, n_3)$ and $(n_4, n_5, n_6) = (\mathbf{r} \times \mathbf{n})$. The *m-terms* are defined by $m_k = (0, 0, 0, 0, Un_3, -Un_2)$. All velocity potentials are also subjected to the no-flux condition at the bottom of the domain and at the far-field truncation boundary. The applied forces are obtained by integration of the first-order pressure over the body surface integration. The velocity potentials, the radiation and the wave excitation forces in the time domain are obtained by the solution to the above initial boundary value problem. After Fourier transforming the time-domain solutions, the equations of motion are solved in the frequency domain to obtain the first-order displacement of the body in the kth direction $\hat{\xi}_k$. By this all first-order quantities required for calculation of the wave drift force in the frequency domain will be ready.

3 WAVE DRIFT FORCE USING FAR-FIELD APPROACH

Using the principle of conservation of the momentum, the Reynolds transport theorem and also the Bernoulli equation, one can obtain the following relation for the mean wave drift force F_1 as an integral over a far-field control surface as:

$$F_1 = \overline{-\iint_{S_\infty} (pn_1 + \rho V_1 V_n)\, ds}\,, \tag{1}$$

in which the overline signifies a time average quantity. The normal component of the velocity vector at the control surface is V_n, and the velocity vector along the length of the ship (x direction) is given by V_1. Using the Kochin function one can calculate the complex amplitude of the velocity potential at the infinity from an integral over the surface of the body. By expressing V_1 and V_n in the above far-field integral, as a function of these potentials at infinity, one can transform the wave drift force to an integral over the surface of the body. This procedure is shown by (Newman, 1967) for the zero speed problem. For the forward-speed problems the derivation of the far-field method was given by (Maruo, 1960) who used the Kochin function to obtained the following relationship for the added resistance R_W:

$$R_w = \frac{\rho}{8\pi}\left\{\int_{-\pi/2}^{-\theta_0} + \int_{\theta_0}^{\pi/2} - \int_{\pi/2}^{3\pi/2} |H_1(\kappa_1, \theta)|^2 \frac{\kappa_1(\kappa_1\cos\theta - k\cos\beta)}{\sqrt{1-4\Omega\cos\theta}} d\theta + \int_{\theta_0}^{2\pi-\theta_0} |H_1(\kappa_2, \theta)|^2 \frac{\kappa_2(\kappa_2\cos\theta - k\cos\beta)}{\sqrt{1-4\Omega\cos\theta}} d\theta \right\}. \tag{2}$$

The Kochin function which gives the far-field complex amplitude of the velocity potential at an azimuthal angle θ is defined as:

$$H_1(\kappa_j, \theta) = \iint_{S_B} \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n}\right) \exp\left[\kappa_j z + \mathrm{i}\kappa_j (x\cos\theta + y\sin\theta)\right] ds\,, \tag{3}$$

where ϕ_B includes both the radiation and scattering velocity potentials. Corresponding to each θ a local wave number κ_j is given by: $\kappa_{1,2} = \kappa_0(1 - 2\Omega\cos\theta \pm \sqrt{1-4\Omega\cos\theta})/(2\cos^2\theta)$. In which $\kappa_0 = g/U^2$, $k = 2\pi/\lambda$ (λ is the wave number), $\Omega = U\omega/g$, $\omega = k(c - U\cos\beta)$, $c^2 = g/k$ where g is acceleration due to gravity, and the heading angle is shown by β with $\beta = \pi$ corresponds to head seas. The integration bound parameter $\theta_0 = 0$ for $\Omega \leq 1/4$, and otherwise its value is given by $\theta_0 = \cos^{-1}(1/4\Omega)$.

4 ZERO SPEED RESULTS (FLOATING HEMISPHERE AND KVLCC2 SHIP HULL)

For the zero-speed case where $\Omega = 0$, the value of κ_1 becomes infinity, and κ_2 approaches the wave number k. It can be also seen that the terms related to the $H_1(\kappa_1, \theta)$ from Eq. 2 are all proportional to $\kappa_1^2 \exp(\kappa_1 z)$ which in the limit as $\kappa_1 \to \infty$ approaches to zero. Thus the only contribution to the wave drift force comes from the last line integral with the bounds from 0 to θ. The surge and heave problems together with the scattering problem for a floating hemisphere are solved. The results for the drift force are shown in (Figure 1(a)), where D denotes the diameter and a is the radius of the hemisphere. The incident wave amplitude is also given as A. The computation based on the near-field formulation is also given together with WAMIT results. In (Figure 1(b)), results are shown for the added resistance of the KVLCC2 hull in head seas which is free to heave, pitch and surge. The ship length is L, the breadth is B and the far-field and near-field methods are compared.

5 FORWARD-SPEED RESULTS (WIGLEY HULL)

When $U \neq 0$ all four line integrals from Eq. 2 are relevant for the wave drift calculation. As has also been shown by (Liu et al., 2011), $\kappa_1 \gg \kappa_2$ which implies that at the corresponding θ angles, the κ_1 waves are much shorter than the κ_2 waves. This could justify neglecting the contributions from the first three integral which are related to κ_1. For the Wigley hull the heave and pitch problems together with the scattering problem for the head seas have been solved. The Froude number $Fn = U/\sqrt{gL}$ is 0.3. The added resistance by the near-field and the far-field formulations are plotted together with the experimental data in (Figure 2(a)). In the calculation of the integral from Eq. 2, close to the integration bounds (θ_0 and $2\pi - \theta_0$) we have faced some difficulties. First it can be seen that the term $\kappa_2 (\kappa_2 \cos\theta - k \cos\beta) / (\sqrt{1 - 4\Omega \cos\theta}) = Q(\kappa_2, \theta)$ goes to infinity at these bounds. We expect however, the product of the Kochin function with this term at the limits to be zero, thus we have set the whole integrand to zero at the bounds. However, the above-mentioned term is also very large close to the bounds, see (Figure 2(b)) which is given for $\omega = 3.997$. Therefore it is necessary for the H_1 function to be very close to zero at the limits in order to ensure a finite integrand. Our original calculation of H_1 is shown in (Figure 2(d)), and this fails to give accurate results near the θ bounds. This results in an unreasonably large integrand as shown in (Figure 2(c), (d)) and an inaccurate drift force. We attribute this behavior to the errors associated with numerical integration of short wavelength components (large κ_2) on a relatively coarse discretization of the ship hull. As a first step to overcome this issue, a linear extrapolation has been used to evaluate the integrand between θ_0 and a selected value of θ corresponding to a highest resolvable κ_2. Work is underway to understand

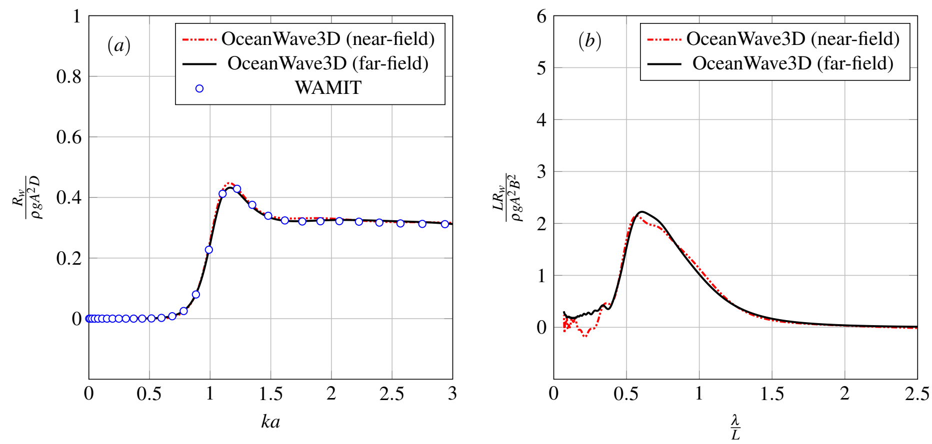

Figure 1: The wave drift force for: (a) a floating hemisphere and (b) the KVLCC2 ship. $U = 0$

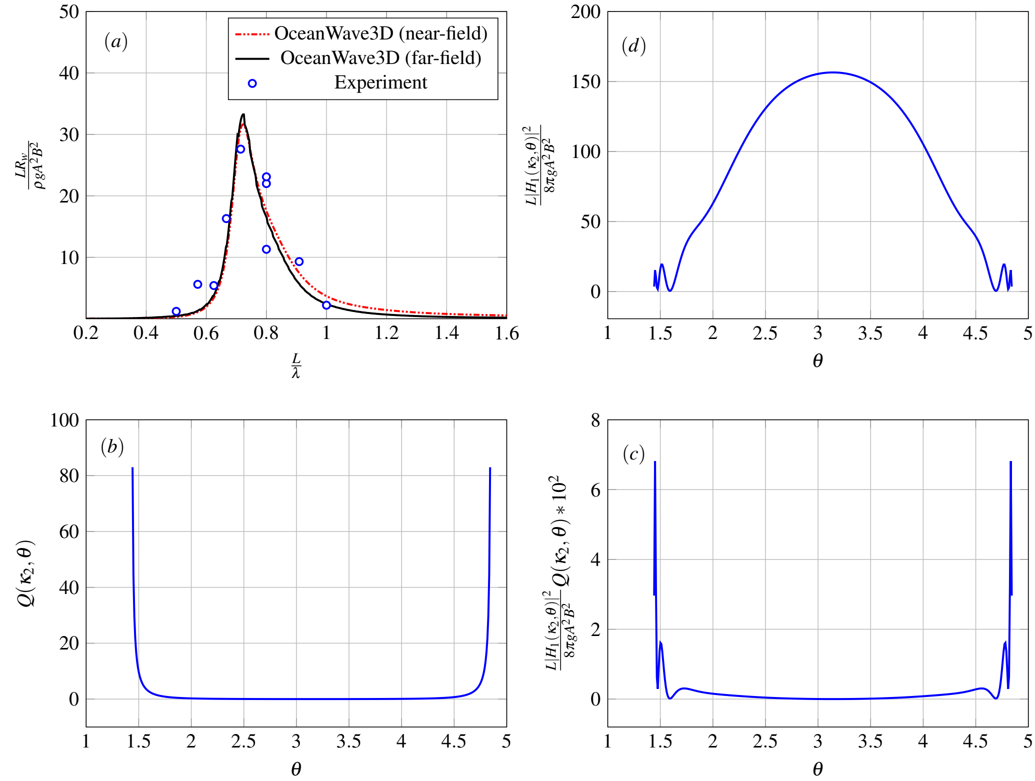

Figure 2: (a) The added resistance, (b) the Q function, (c) the integrand in Eq. 2, and (d) $|H_1|^2$ function

the problem better and to calculate the Kochin function accurately out to the integration bounds.

6 CONCLUSIONS

The Kochin function far-field approach for calculation of the added resistance has been implemented in the finite-difference seakeeping code. For the forward-speed problems the Kochin function requires to be evaluated appropriately close to the integration bounds.

7 ACKNOWLEDGEMENT

The authors of this paper would like to acknowledge The Danish Maritime Fund (Den Danske Maritime Fond) for their financial support (No. 2014-088).

REFERENCES

Amini Afshar, M. *Towards Predicting the Added Resistance of Slow Ships in Waves*. PhD thesis, DTU Mechanical Engineering, 2015.

Brown, D. L., W. D. Henshaw, and D. J. Quinlan. Overture: An object-oriented framework for solving partial differential equations on overlapping grids. *Object Oriented Methods for Interoperable Scientific and Engineering Computing, SIAM*, pages 245–255, 1999.

Liu, S., A. Papanikolaou, and G. Zaraphonitis. Prediction of added resistance of ships in waves. *Ocean Engineering*, 38 (4):641–650, 2011.

Maruo, H. Wave resistance of a ship in regular head seas. *Bulletin of the Faculty of Engineering, Yokohama National University*, vol. 9, 1960.

Newman, J. t. The drift force and moment on ships in waves. *Journal of Ship Research*, 11(1):51–60, 1967.