Nonlinear harmonics in the roll motion of a moored barge coupled to sloshing in partially filled spherical tanks

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Highlights

- For partially filled spherical tanks, internal sloshing can be driven by 2x and 3x frequency harmonics of the main linear roll response of a floating barge;
- The sloshing drives the resulting high-frequency roll of the barge linearly, although the internal sloshing is excited nonlinearly;
- A combination of NewWave theory and the 4-phase combination method is applied to combined and strongly coupled fluid-structure responses in a broad-banded random sea.

1. Introduction

The roll response of LNG carriers is important for side-by-side offloading, when two floaters remain in very close proximity for considerable periods. Of concern are the largest roll events at around 3^{o} ~ 3.5^{o} which may limit the operational window for offloading[1] and could also be associated with significant sloshing of the contents of partially filled tanks within the vessels. Does the roll couple to the sloshing? Sloshing often occurs at higher frequencies than that of the main roll motion, so could there be excitation of sloshing at or close to integer harmonics of the linear roll frequency, and does this sloshing feedback to the overall roll motion?

To understand these phenomena, a series of experiments are conducted for a single barge-like model with two identical internal spherical tanks. The roll responses and internal sloshing surface elevations have been measured for broadside wave excitation. NewWave theory [2], generally used to represent the large events of wave surface elevations, is introduced to represent the average properties of the large roll profiles in time, looking at both linear and nonlinear effects. The phase-combination method due to Fitzgerald et al. [3] is used to separate components in the measured signals in a random sea, clarifying contributions from higher-order wave harmonics to the overall roll response via the excitation of internal sloshing.

2. Theoretical preliminaries

2.1 NewWave theoretical representation

The shape of each individual roll response peak in a random sea is unique. However, it is useful to examine the average shape associated with large roll events, by analogy with large wave profiles [2]. The averaged roll shape is obtained from a given number of the largest such events, by creating shorter time series with the extreme roll point located at the relative time of zero and then averaging across the short records.

According to the classic Stokes perturbation expansion, a regular incident wave of linear wave amplitude A and frequency ω will induce a response of offshore structures (up to 2nd-order):

$$\varphi(\tau) = Af_1\cos(\omega\tau) + A^2 f_2\cos(2\omega\tau) + \ldots, \tag{1}$$

The phase-inversion method [3] can be used to separate the linearized profile and the second-order harmonics by pairing the averaged largest crest and troughs, assuming that higher-order contributions can be neglected. So

$$\varphi_L \approx \varphi_{odd} = \frac{1}{2}(\bar{\varphi}_+ - \bar{\varphi}_-), \qquad \varphi_S \approx \varphi_{even} = \frac{1}{2}(\bar{\varphi}_+ + \bar{\varphi}_-), \tag{2,3}$$

where, $\bar{\varphi}_+$ and $\bar{\varphi}_-$ are the averaged large rolls positive downstream or upstream of any point above roll-centre.

To examine the averaged profiles for the measured roll motions, the NewWave theoretical representation is calculated. Lindgren [4] showed the average shape of a large event in a linear random Gaussian process tends to be the autocorrelation function, which has been described as NewWave [2, 5]. By analogy, the linear NewWave profile of the large roll peak is given:

$$\varphi(\tau) = A \cdot R(\tau) = A \cdot \sigma^{-2} \int_0^\infty S(\omega)\cos(\omega\tau)d\omega, \quad (4)$$

where A is the linear crest amplitude, σ the standard deviation of the roll time history, $S(\omega)$ the power spectrum density for roll, $\tau = t - t_0$ time relative to the moment when the large value occurs. To examine how well the averaged profiles of the large roll events fit the NewWave auto-correlation function, error bars are estimated statistically.

$$\acute{\sigma}_{Lg}{}^2 = \sigma^2(1 - R(\tau)^2 - \dot{R}(\tau)^2\lambda^{-2}), \quad (5)$$

The Lindgren variance (5) is used to express the variation in shape of individual crests around the expected shape of the NewWave autocorrelation function [4,6]; Here $\acute{\sigma}_{Lg}$ is the Lindgren standard deviation, $R(\tau)$ is given in eqn.(4), $\dot{R}(\tau)$ is the auto-correlation function for roll velocity and λ is a weighting of spectral moments.

2.2 Decomposition of the high-order harmonics

To clarify whether there is coupling into sloshing from higher-order harmonics of the main linear roll peak, the measured roll time series have been band pass filtered, forming low-pass and high-pass components. The high-pass signals are then averaged after conditioning on the magnitude and phase of the low-pass signals (assumed linear), so that this analysis picks out any parts of the high frequency component that are locked to the linear components.

To extract high-order harmonics, the four-phase method has been used to separate the conditioned signals. The corresponding equations for the harmonic separation are given as follows [3]:

Linearized 1st: $${}_l\varphi_h^{11} A\cos(\theta) + {}_l\varphi_h^{31} A^3\cos(\theta) = \frac{1}{4}\left({}_l\varphi_{h(0)} - {}_l\varphi_{h(90)}^H - {}_l\varphi_{h(180)} + {}_l\varphi_{h(270)}^H \right), \quad (6)$$

2nd: $${}_l\varphi_h^{22} A^2\cos(2\theta) + {}_l\varphi_h^{42} A^4\cos(2\theta) = \frac{1}{4}\left({}_l\varphi_{h(0)} - {}_l\varphi_{h(90)} + {}_l\varphi_{h(180)} - {}_l\varphi_{h(270)} \right), \quad (7)$$

3rd: $${}_l\varphi_h^{33} A^3\cos(3\theta) = \frac{1}{4}\left({}_l\varphi_{h(0)} + {}_l\varphi_{h(90)}^H - {}_l\varphi_{h(180)} - {}_l\varphi_{h(270)}^H \right), \quad (8)$$

4th: $${}_l\varphi_h^{44} A^4\cos(4\theta) = \frac{1}{4}\left({}_l\varphi_{h(0)} + {}_l\varphi_{h(90)} + {}_l\varphi_{h(180)} + {}_l\varphi_{h(270)} \right), \quad (9)$$

where superscript H refers to the Hilbert transform, ${}_l\varphi_h$ the high-pass signals conditioned on low-pass signals, and the subscripts (0, 90, 180 and 270°) represent the shifted phase of the conditioned signals, respectively. As the low-frequency roll response is rather narrow-banded, the identification of the largest crests and troughs and the steepest zero up- and down- crossings is relatively straightforward; and the occurrence of maxima of these in the low frequency part of the roll signal provides the conditioning points, the times at which to extract short segments of high frequency signal, to look for phase-locked harmonic components in the high-frequency roll.

3. Experimental method

Experiments were conducted at the Deepwater Wave Basin in Shanghai Jiao Tong University, which is 50 m long by 40 m wide with the water depth of 10 m set with an artificial bottom. Waves were generated by flap-hinged wave makers along one side of the basin and wave absorbers are installed on the opposite side to minimize reflection. A 3.33 m long and 0.77 m wide barge-like model was designed with two identical spherical tanks, as shown in Fig.1. The model was soft moored with a horizontal mooring system. The natural frequency of the mooring restrained surge motion is approximately 0.05 Hz, which is far below the fundamental roll frequency of 0.5 Hz. A white noise wave spectrum is adopted with frequencies ranging from 0.27 Hz to 1.94 Hz. The significant wave height is chosen to be small at 5 cm, so the waves remain close to linear and the excited large roll responses are 3° ~ 3.5°, which is a critical design criterion for the side-by-side offloading operation.

Fig. 1 The horizontally moored model in the wave basin.

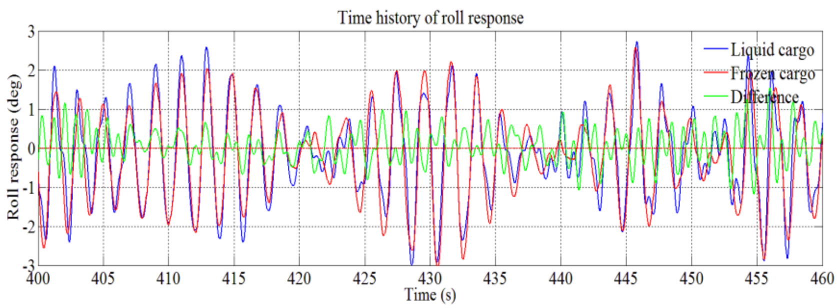


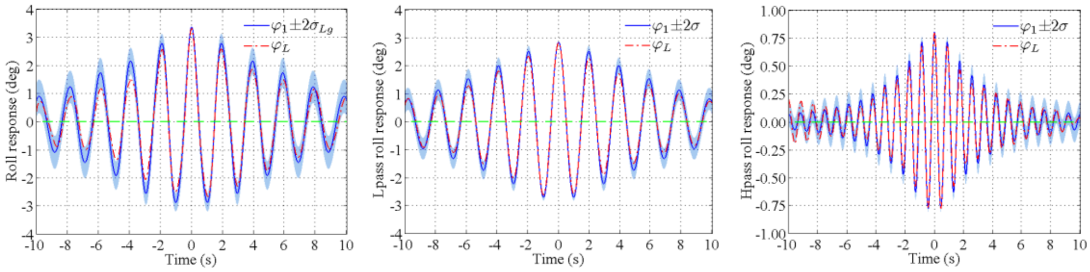
Fig. 2 Representative time series of the roll responses in liquid cargo and frozen cargo cases.

4. Results

In the model tests, the irregular waves were generated for 1500 sec and the corresponding responses were measured. Two types of experiments were performed in identical realisations of random waves: firstly, the spherical tanks were filled with liquid cargo (water) and, secondly, the liquid-filled tanks were replaced by a 'frozen load' - steel weights with equivalent mass and inertia properties. Typical roll response time series are shown in Fig. 2: the (liquid-frozen) difference of the two measured signals shows higher-frequency wiggles. To investigate these further, the total roll signals have been filtered into two components, low-pass and high-pass, with the cut-off at 1.5x linear roll frequency.

4.1 NewWave theoretical representation

NewWave theory is used to represent the average properties of the largest 30 roll events, as shown in Fig. 3. We find that for all signals the NewWave autocorrelation function fits the measured average profile to within $\pm 2\sigma_{Lg}$, indicating the ability of the NewWave theory to fit large roll events in a random sea.

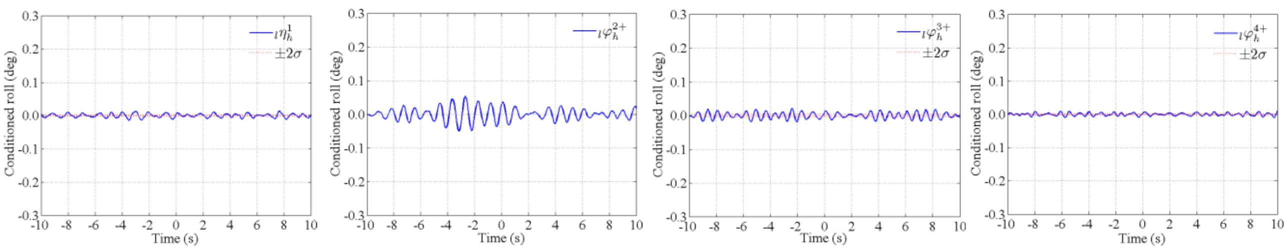


(a) (b) (c)

Fig. 3 Ability of NewWave profile φ_1 to represent the average of the total measured large profiles φ_L for the roll response in the 50% load condition with liquid cargo: (a) total signal, (b) low-pass component, (c) high-pass component.

4.2 High-order harmonics

The high-pass signals are then analyzed: conditioned on the low-pass linear signals for the roll responses in frozen cargo and liquid cargo cases. Four-phase separation is applied to the conditioned roll signals. Fig. 4 shows the first four harmonics *for the frozen cargo case*, whereby the 1st-order, 3rd-order and 4th-order components are very small and negligible compared with the 2nd-order harmonics. As shown in Fig. 4 (b), the maximum 2nd-order harmonic contribution in the roll signal is 0.05^{o}, whereas the total high-frequency roll signal is 0.25^{o}. As a consequence, we may roughly estimate the broad-banded linear excitation contribution as the remaining 0.20^{o} of roll motions in the high frequency range. These high-frequency roll responses are very small, but this provides some information on relative contributions of the linear excitation and the higher-order harmonic effects associated with the low-frequency linear roll resonance *in the absence of internal sloshing.*



(a) (b) (c) (d)

Fig. 4 Harmonic separation of the conditioned roll signal ($_{I}\varphi_{h}$) for the 50% load condition with frozen cargo in tanks: (a) 1st-order harmonic, (b) 2nd-order harmonic, (3) 3rd-order harmonic, (4) 4th-order harmonic.

For comparison, the results *for the liquid cargo case* are shown in Fig. 5. The harmonic separation results have notable double frequency harmonics as the dominant coupled nonlinearity and visible triple frequency harmonics. One can see in Fig. 5 (b) that the largest roll signal contribution from the 2nd-order harmonic is 0.22^{o} (the largest total roll signal is 0.75^{o} over the higher frequency range). Based on the results for the frozen cargo case, linear wave excitation over the high frequency range produces around 0.20^{o} roll before the effect of sloshing is included. As a consequence, we may estimate the relative contributions at the secondary response peak as being roughly equally split: the linear excitation of the barge and frozen cargo, the extra barge motion induced by sloshing in response to the linear high frequency excitation and the double-frequency harmonics coupled to the internal sloshing all contributing close to equally.

If the connection between the high-frequency roll response and internal sloshing is linear, we would expect reciprocity to hold: similar shapes between the conditioned roll motions and conditioned sloshing signals. To identifiy the coupling phenomenon, the high-pass sloshing elevation (along the side wall of the tank) conditioned on the high-pass roll signals is decomposed up to the 4th-order harmonics, shown in Fig. 6. Correspondingly, the high-pass roll signals conditioned on high-pass sloshing are obtained and will be presented at the Workshop. Only a 1st-order harmonic is found in the conditioned signals. The 1st-order harmonic of the conditioned high-pass sloshing signals is very similar in shape to the conditioned high-pass roll motions, and the signal shape is symmetric with respect to $t = 0\,s$. To demonstrate this, these signals are plotted together in Fig. 7, with the conditioned high-pass roll signals scaled by a transfer coefficient of 9 (consistent with high-frequency linear roll response). The two signals are virtually identical in shape, confirming that the coupling between high-pass sloshing and high-pass roll motion is a linear phenomenon.

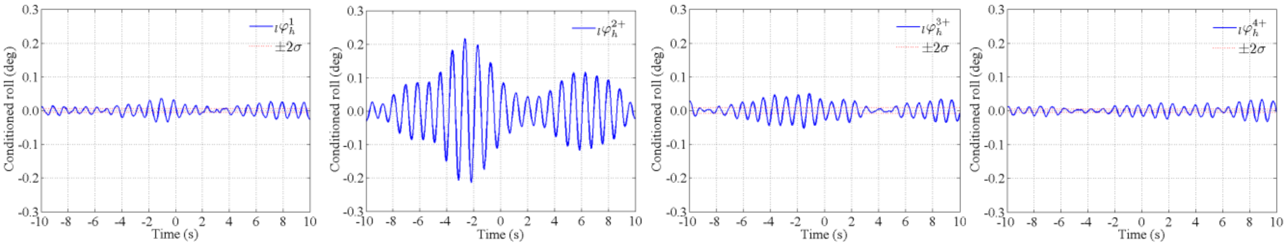


Fig. 5 Harmonic separation of the conditioned roll signal ($_l\varphi_h$) for the 50% load condition with liquid cargo.

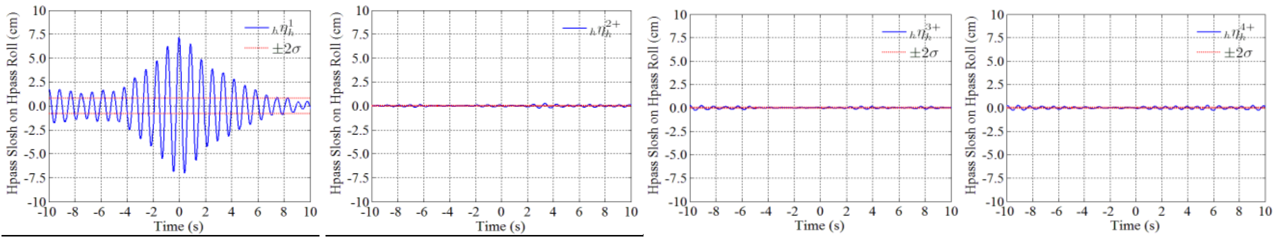


Fig. 6 The first four harmonics of the high-pass sloshing signal ($_h\eta_h$) conditioned on the high-pass roll motions.

5. Conclusions

This paper has presented experimental results on the roll response of a moored barge with partially filled spherical tanks. There is a large roll component at the linear roll frequency. Under broadband wave excitation, there are also high-frequency contributions; one is simply the linear high-frequency response. There is also significant excitation of internal sloshing within the partially filled tanks. When the lowest slosh mode frequency is ~2 or 3x that of the linear roll response of the barge, nonlinear excitation of sloshing occurs. This slosh response is large enough that it back-reacts on the barge and produces significant extra high-frequency roll, this back-reaction between the sloshing and the barge motion being linear.

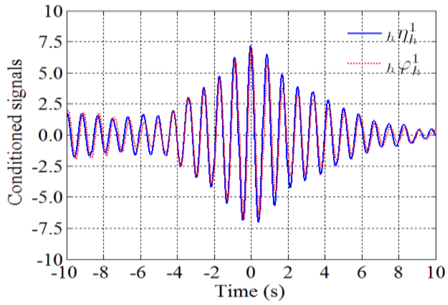


Fig. 7 High-pass sloshing signal conditioned on high-pass rolls vs high-pass roll signal conditioned on high-pass sloshing.

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