Cloaking by a Floating Thin Plate

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1 Introduction

Cloak of invisibility was first introduced in optical community (Pendry *et al.*, 2006; Leonhardt, 2006) in an attempt to make objects invisible from sight. An invisibility cloak covers an object and bends the light rays around in such a way that the incident light bypasses the object. If light does not hit and interact with the object, then the object does not block nor reflect nor scatter any light, and hence becomes invisible from outside observer. The object inside the cloak also does not feel the wave at all. As a result, a cloak other than invisibility will shield the object from incoming waves.

The concept of cloaking has also been investigated for other types of waves, such as: acoustic waves (Cummer & Schurig, 2007), seismic waves (Brûlé *et al.*, 2014), elastodynamic waves (Farhat *et al.*, 2009), and water waves (Zareei & Alam, 2015). Specifically, using alternating sea-bed, cloaking in water waves is achieved. This cloak shields an offshore structure from destructive and damaging waves. Drawback of this approach is that changing the seabed topography is fixed, costly and difficult and also faces several major issues such as erosion, sedimentation, and biofouling that together soon make the cloak ineffective.

An alternative way of cloaking is to use an engineered elastic buoyant carpet placed on water (see figure 1b) around the object. The carpet will effectively bend the wave rays around the object and shield the object from impinging waves. The method can potentially open up a new avenue in the protection of ocean objects, particularly offshore structures, from the action of oceanic waves hence reducing load on such structures. Many offshore systems (particularly the so-called FPSO units) rely on the energy-intensive Dynamic Positioning to compensate their displacement that mostly comes from wave actions. A surface cloak thrown about the unit can substantially save energy for such systems. A cloak can lessen the force on the pile, hence keeping the tower more steady in an upright position, as well as lowering design requirements that will subsequently lower the cost of the operation

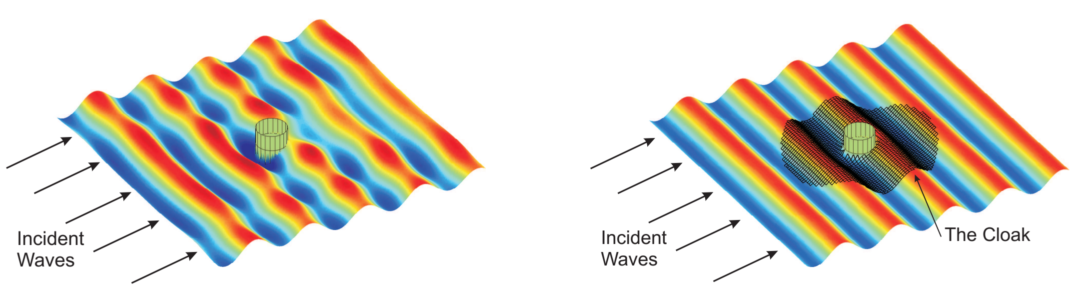

Figure 1: Figure (a) shows the scattering pattern of a cylinder under the action of water waves without a cloak, and figure b. shows the same cylinder protected by a surface wave cloak. Clearly in figure (b) there is no scattering and the cylinder is ***cloaked***.

2 Approach

In this section we briefly discuss the governing equations and also design procedure of the floating elastic carpet. We assume that the carpet is always in contact with the water underneath that is homogeneous, irrotational, inviscid and incompressible with density ρ_w and depth h. We define a Cartesian coordinate system with the x, y-axes on the mean free surface of the fluid and the z-axis positive upward. If we denote the pressure exerted from the elastic plate on the water surface by P, then the governing equations and boundary conditions read

$$\nabla.\mathbf{u} = 0, \qquad -h \le z \le \eta \tag{2.1a}$$

$$\mathbf{u}_{,t} + \mathbf{u}.\nabla\mathbf{u} = -\nabla P/\rho_w - g\hat{\mathbf{z}}, \qquad -h \le z \le \eta \tag{2.1b}$$

$$w = \eta_{,t} + u\eta_{,x} + v\eta_{,y}, \qquad z = \eta \tag{2.1c}$$

$$w = -uh_{,x} - vh_{,y}, \qquad z = -h \tag{2.1d}$$

$$P = D\nabla_\perp^4\eta + R\eta_{,tt} \qquad z = \eta \tag{2.1e}$$

where, $\mathbf{u} = (u, v, w)$ is velocity; η is surface elevation; $D = Et^3/12(1-\nu^2)$ is the flexural rigidity of the plate in which E is the Young module of elasticity, ν is the Poisson ratio, and $R = \sqcup\rho_s$ is the mass per unit area with ρ_s the plate density and t the plate thickness. Note that in equation (2.1), $\nabla_\perp$ is the horizontal gradient operator while ∇ is the complete gradient operator. In the set (2.1), equation (2.1a) is the continuity equation, (2.1b) is the Navier-Stokes equation, (2.1c) and (2.1d) are respectively surface and bottom kinematic boundary condition and (2.1e) is the flexural thin plate's equation.

Assuming that the vertical length scale H is much smaller than the horizontal length scale L, where $H/L = \mathscr{O}(\mu) \ll 1$, from the continuity equation, we obtain that vertical velocity is much smaller than horizontal velocities. Then using z component of the Navier-Stokes equation (2.1b), we find that the pressure is linearly dependent on z as $P_{,z} = -\rho_w g$ and then considering the boundary condition of (2.1e), the pressure is fully obtained. Using this pressure and the linear horizontal part of equation (2.1b) the governing equation for the surface elevation η is obtained as

$$\nabla_\perp.\left\{h\nabla_\perp\left(\frac{D}{\rho_w}\nabla_\perp^4\eta\right)\right\} + \nabla_\perp.\left(gh\nabla_\perp\eta\right) + \nabla_\perp.\left(h\nabla_\perp\frac{R}{\rho_w}\eta_{,tt}\right) - \eta_{,tt} = 0. \tag{2.2}$$

We can ignore the third term in the above equation, since comparing the last two terms, we observe that the ratio is $(\rho_s/rho_w)(H/\lambda)(t/\lambda)$ where all of the terms are much smaller than 1. In the limit where $(E/\rho_w g\lambda)(t/\lambda)^3 \gg 1$, where only flexural rigidity is deriving the surface elevation, we obtain

$$\nabla_\perp.\left\{h\nabla_\perp\left(\frac{D}{\rho_w}\nabla_\perp^4\eta\right)\right\} - \eta_{,tt} = 0. \tag{2.3}$$

This equation is not form invariant, we propose a form invariant approximate to this equation, where in the limit of a homogeneous and isotropic material the exact equation is recovered. We have

$$\nabla_\perp.\left\{\mathscr{D}^{1/3}\nabla_\perp\left\{\nabla_\perp.\left\{\mathscr{D}^{1/3}\nabla_\perp\left[\nabla_\perp.\left(\mathscr{D}^{1/3}\nabla_\perp\eta\right)\right]\right\}\right\}\right\} - \frac{\partial^2\eta}{\partial t^2} = 0, \tag{2.4}$$

In order to cloak a cylinder of radius a with a annular region around this cylinder with outer radius of b, we use the nonlinear transformation in (Zareei & Alam, 2015). The rigidities are found as

$$\mathscr{D}_r = \left(1 - \frac{a^2}{b^2}\right)^3\left(\frac{r^2 - a^2}{r^2}\right)^3\mathscr{D} \qquad \mathscr{D}_\theta = \left(1 - \frac{a^2}{b^2}\right)^3\left(\frac{r^2}{r^2 - a^2}\right)^3\mathscr{D} \tag{2.5}$$

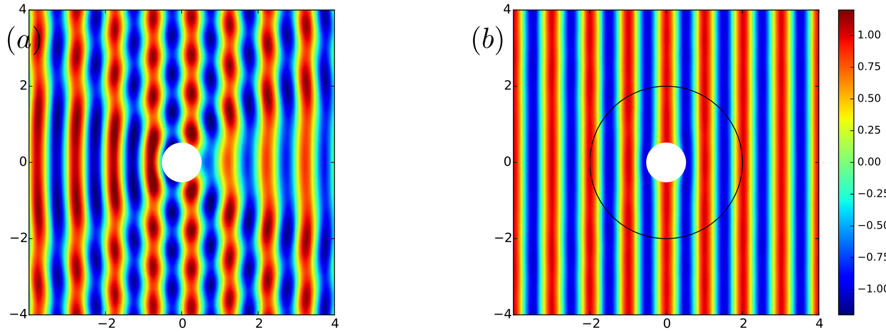

Figure 2: Scattered field of waves in the presence of a cylinder without a cloak (a), and with the cloak (b). The black circle in figure (b) shows the boundary of the cloak.

These flexural rigidities for a buoyant carpet could cloak a cylindrical structure in the middle from incoming waves.

3 Results

Equation (2.4) with rigidities obtained from (2.5) can be solved analytically. Basically we separate the solution in the region of inside and outside of the cloak. Assuming an incoming wave and then implementing the boundary conditions and continuity equations on the edge of the carpet, analytical solution is found. Boundary conditions are no shear and no moment at the edge of the buoyant carpet and also no flux condition on the cylinder. Comparison between scattered waves and cloaked version is shown in figure 2. It shows that the scattered waves are suppressed and cloaking is achieved.

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