Large Wave Groups - Their Probability, Profiles, and Mean Offsets

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Introduction

The definition and classification of wave groups is a strong area of current research, particularly due to the influence of wave groups on extreme dynamic events. The classical wave group definition constitutes of some combination of threshold crossings (envelope or distinct peaks), narrow band or broad band spectral assumptions, Markov chain processes for large successive wave heights, or Boccotti's 'quasi-deterministic' theory [1]. Bassler et al. [2] and Kim and Troesch [3] suggested relaxing the successive threshold crossing requirement to allow for the more probable wave group sequence which may include some minor down crossings. Such groups can still cause extreme system response through resonant excitation such as parametric roll [3]. Seyffert, Kim, and Troesch [4] offered a mathematical formulation of wave groups, which will be used here to examine a singular time series taken from the Pt. Reyes Buoy [5]. We consider the spectral statistics of this 30-minute time series and compare it with the mathematical wave group formulation, and then calculate peak-to-trough statistics for comparison with a second-order Stokes wave.

Mathematical Formulation

As in [4], a mathematical formulation of a wave group is used in this work to compare with physical ocean data. Specifically for this paper, we consider a single time series and delve deeper into intriguing issues raised by that pervious work. The full derivation of the mathematical wave group is developed in [4], and a summary is given here. We start with the definition of a Gaussian derived process, as first defined by [3], with $\eta(t)$ as wave elevation at a specific spatial coordinate. Here, τ is a defined period of interest and k is the wave group index.

$$z_k(t) = \sum_{p=1}^{k} \eta(t + (p-1)\tau) \tag{1}$$

In order to relate large (i.e. extreme) values of $z_k(t)$, defined as $\hat{z}_k$, to wave groups, it was necessary to consider the conditional expected value of a normal, zero mean, function, $y(t)$, that is, $E[y(t)\,|\,y(t_o) = \hat{y}, \dot{y}(t_o) = 0]$. For a stationary process, t_o is arbitrary and can be shifted to $t_o = 0$ without loss of generality. Since $y(t)$ is Gaussian with covariance $r(\tau)$, it has been shown (e.g [1, 6, 7, 8, 9]) that as $\hat{y} \to \infty$,

$$E[y(t)|y(0) = \hat{y}, \dot{y}(0) = 0] \to \hat{y}\frac{r(t)}{r(0)} \tag{2}$$

Using Fourier Transform theory, along with the derived process and the Wiener-Khinchine relations, the mathematical formulation for the expected shape of the wave group conditioned on large values of $\hat{z}_k$ has been derived in [4]. In that work, we show that the wave elevation expected value, conditioned on a maximum of the derived process at t_o, forms a wave group of k waves, proportional to the sum of k autocorrelation functions of the wave elevation, separated in time by $(p-1)\tau$, $p = 1 \ldots k$. The constant of proportionality is the value of the maximum of the derived process, with group index k, divided by its variance, ${\sigma^2}_{z_k}$. The final result is given in Eq. 3 below:

$$E[\eta(t)\,|\,z_k(0) = \hat{z}_k, \dot{z}_k(0) = 0] \;=\; \frac{\hat{z}_k}{r_{z_k z_k}(0)} \sum_{p=1}^{k} r_{\eta\eta}(t_p) \;=\; \frac{\hat{z}_k}{{\sigma^2}_{z_k}} \sum_{p=1}^{k} r_{\eta\eta}(t_p) \tag{3}$$

As shown in [4], the scaling factors can be estimated (Eq. 4 - 5). The result is a good match between theory, Monte Carlo simulations, and the physical data. Spectral moments are given from the derived process spectral density function, $S_{z_k z_k}(\omega)$ as below. The most probable extreme maxima, $\hat{z}_k$, for a given exposure time, T, in seconds, is then given (e.g. [10]) using the zero-crossing period, T_o.

$$m_{z_{k_n}} = \int_{-\infty}^{\infty} d\omega \; \omega^n \; S_{z_k z_k}(\omega) \tag{4}$$

$$\hat{z}_k = \sqrt{m_{z_{k_o}}} \; \left[2 \; ln \left(\frac{T}{2\pi} \sqrt{m_{z_{k_2}} / m_{z_{k_o}}} \right) \right]^{\frac{1}{2}} \tag{5}$$

Identification of Wave Groups in Time Series Data

As in earlier works by the authors [4], [11], data is taken from the Pt. Reyes Buoy operated by the Coastal Data Information Program, UC San Diego [5]. For this workshop, a single 30-minute wave record from January 20, 2010 15:09:00 is considered. The selected time series is one ensemble sample that is part of a larger "bin" of seventy 30-minute time series [4], and has the largest derived process value for a wave group of 8 waves of all seventy time series in the bin. We denote the maximum of the derived process of this particular time series $\widehat{z}_8(TS)$, to later compare with the mean of the derived process maxima of all seventy time series, $\widehat{z}_8(M)$. Here, we will also compare our single sample time series with ensemble statistics for this bin ($H_s = 7.1m$ and $T_{modal} = 14.88s$). Then, by forming the ratio of the value of $\widehat{z}_8(TS)$ to the mean $\widehat{z}_8(M)$, we can state that the individual time series of the bottom insert of Fig. 1 is representative of a rare wave group with Probability of Nonexceedence PNE = 0.980 when selected from a sample set of like wave groups taken from 30 minute records.

In the top insert of Fig. 1, wave groups of 8 waves for all 70 time series, identified by a maximum in the derived process $z_8(t)$, are shifted without loss of generality to 200 seconds. The ensemble average of all time series is overlaid. The middle inset shows the temporal ensemble average overlaid with the ensemble average of the 70 scaled, shifted autocorrelation functions based on Eq.(3). The bottom inset shows the time series from January 20, 2010 15:09:00, overlaid with the scaled, shifted autocorrelation function (using Eqs. 3, 4 and 5) of that same time series.

As shown in Tab. 1, theory overestimates the empirical data for both the January 20, 2010 15:09:00 time series and the ensemble average by 3 to 15%. The 15% difference in $(\bullet)_{TH}/(\bullet)_{TS}$ may not be significant since individual samples from an ensemble will be part of a distribution with non-zero variance.

Table 1: Nondimensional scale factors based on theory (TH) (i.e. Eq. 4 and 5) and empirical time series (TS) for k = 8. Separation period $\tau = T_{modal} = 14.88$s. Comparing Time Series January 20, 2010 15:09:00 and Ensemble Wave Group

Wave Group ($k = 8$)	$\sqrt{m_{z_{k_o}}}/k$	$(\hat{z}_k/\sqrt{m_{z_{k_o}}})_{TH}$	$(\hat{z}_k/\sqrt{m_{z_{k_o}}})_{TS}$	$\frac{(\hat{z}_k/\sqrt{m_{z_{k_o}}})_{TH}}{(\hat{z}_k/\sqrt{m_{z_{k_o}}})_{TS}}$
January 20, 2010 15:09:00	0.253	4.2982	3.7151	1.1569
Ensemble Average	0.253	3.1587	3.0565	1.0334

Peak to Trough Variation and Mean Offset

We now are in a position to examine the single wave group time series for specific water wave characteristics. The wave group with wave index $k = 8$ is defined in Fig.1 bottom insert, as starting at $t = 200$s and continuing for $7.5 \times T_{modal}$ seconds. The wave group duration is selected such that 8 successive peak-trough pairs can be identified. Here we look at only the average wave group crest maximum (crest height), the average wave group minimum (trough), the ratio of crest to height, and the mean offset for $200s \leq t \leq 309.8s$. The results are shown in Table 2.

Comparisons can now be made with the average offset as predicted by second order Stokes waves versus the average offsets predicted by Eq. 3 and the average offset of the smaller ensemble average. The second order Stokes offset is defined in Eq. 6. Deep water approximations are assumed for simplification (the Pt. Reyes buoy is located 24.7 miles west of Pt. Reyes, CA in 1804 feet of water [5]). For the Stokes

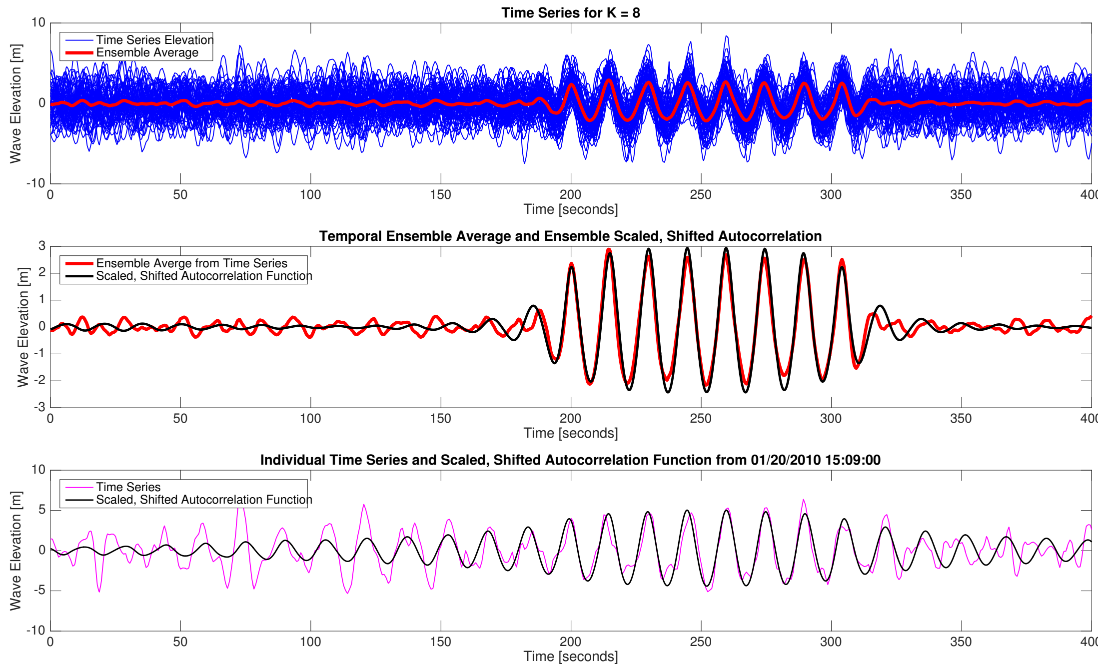

Figure 1: Comparison of temporal average and theoretical wave group formulation from Eq. 3 with scale factors from Eq. 4 and 5. Ensemble Average based on 70 thirty minute wave records, $H_s = 7.1m$, $T_{modal} = 14.88s$.

offset, we use the mean wave amplitude, H/2, from the physical wave record as the wave amplitude and the peak modal period (T_{peak}) to define the wave number κ, where for the linear deep water dispersion relation $\kappa = 4\pi^2/(gT^2)$. For a mean wave amplitude of $H/2 = (4.714 + 3.758)/2 = 4.23$m, the Stokes offset would be approximately 0.164m. This is approximately 1/3 of the offset measured in the time series or 1/3 of the offset predicted by the theory (Eq. 3, 4, and 5) for a similar mean wave height. It is interesting that the mathematical wave group formulation captures the mean offset inherent in natural water waves much more closely than a Stokes second order wave, at least for this times series and wave group. Also of significance is that the mathematical wave group formulation [4] is based on linear operator theory but still manages to capture the mean offset, which is often thought to be a second order effect.

$$\text{Stokes crest/trough} \approx \pm\frac{H}{2} + \frac{kH^2}{8} = \pm\frac{H}{2} + \frac{H^2\pi^2}{2gT^2} \tag{6}$$

Table 2: Comparison of (absolute value) Mean Peak and Trough Values for January 20, 2010 15:09:00, Autocorrelation Function Wave Group ($k = 8$) from January 20, 2010 15:09:00, and Ensemble Average of 70 Samples.
Fig. 1, $200s \leq t \leq 309.8s$.

	Mean Peak Value [m]	Mean Trough Value [m]	Peak-Trough Ratio	Mean Amplitude [m]	Mean Offset [m]
January 20, 2010 15:09:00	4.714	3.758	1.254	4.23	0.481
Eq. 3, 4, 5	2.708	2.170	1.248	4.23*	0.478*
Stokes Wave	-	-	-	4.23	0.164

*Note, the mean amplitude and offset, based on Eq. 3, 4, and 5, and ensemble average of the seventy records are scaled up to match the mean amplitude of the January 20, 2010 15:09:00 record.

Conclusions

In this paper, a mathematical formulation of wave groups developed by Seyffert, Kim, and Troesch [4] was used to investigate properties of a single physical time series record from the Pt. Reyes Buoy. The mathematical wave group formulation was compared to the individual time series and compared with an ensemble average of seventy representative 30-minute time series. Average peak and trough values were compared with a Stokes second order wave. The mathematical wave group formulation captured the mean offset seen in nature more closely than the Stokes second order model. More research is being conducted to better understand the phenomenon. Most interesting is the ability of the wave group formulation based on the Gaussian derived process to capture the mean offset present in natural waves, often thought to be a second order effect. Research is being done is answer the questions identified by this, and is already yielding promising results. In summary, this wave group formulation has shown close agreement with oceanographic measurements and is a promising new method of considering large wave group properties.

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