WATER-ENTRY OF AN EXPANDING TWO-DIMENSIONAL SECTION

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1. INTRODUCTION

A general similarity solution for the water-entry problem of a two-dimensional arbitrarily shaped body undergoing expansion in a prescribed manner is obtained. The free surface is assumed to detach from the curved body surface in the mathematical formulation of the problem. The point of detachment is determined based on the Brillouin-Villat criterion, which states that the curvature of the body and the curvature of the free surface must be the same at the point of flow detachment. In this extended abstract, results are presented for water entry of an expanding circular cylinder. The integral hodograph method [1,2] is used to derive analytical expressions for the complex potential and for the complex velocity, both defined in a parametric plane, for which the first quadrant is chosen. It enables the original partial differential equation with nonlinear boundary conditions on the unknown free surface to be reduced to a system of integro-differential equations along straight lines in the parametric plane. For each given distance b in the parameter plane between the tip of the jet and the detachment point, the coupled equations are then solved through successive approximations. The obtained free surface shapes and streamlines for different b, which corresponds to different expansion speeds, are presented and discussed. As the expansion speed tends to zero, the result is found to tend to that corresponding to the steady Helmholtz flow.

The model of an expanding body, which may seem to be artificial in itself, may be useful in the analysis of the water entry of fixed-size bodies within the framework of the so-called 2D+t approach used for fast ships [3]. This enables one to predict the hydrodynamic forces and the free surface shapes in transverse planes, for example, in the case of high-speed planing crafts or ditching aircrafts. Vella and Li [4] showed at initial stage of a small cylinder moving down suddenly, the motion of the interface with surface tension could be modelled by the self similar flow, which corresponded to an expanding body in their rescaled coordinate system. The similarity solution for the oblique water entry of a 3D expanding paraboloid has been presented in [5]. However the flow there was assumed to be attached while flow detachment can be allowed in the present solution. The free surface due to an expanding cylinder can also be related to that due to the expansion by a large pressure near the free surface [6].

2. FORMULATION OF THE PROBLEM

Without of loss of generality, we consider a circular cylinder whose radius R expands in time at constant speed V_R during its entry into a liquid of infinite depth at constant vertical velocity V. The flow detaches at some point of the circular section. The liquid is assumed to be inviscid and incompressible, the flow is assumed to be irrotational, gravity and surface tension are ignored. The flow is studied with the velocity potential theory in the frame of reference with the origin at point A shown in figure 1a where water entry is modeled equivalently by a uniform flow with velocity V at infinity towards the cylinder. The self-similar problem in the physical plane $Z = X + iY$ can be written in terms of the self-similar variable $z = x + iy$ with $x = X/(Vt)$, $y = Y/(Vt)$. At the instant when the body meets the liquid surface, points A, O, and B are at the same location. Immediately after the impact, A stays at the same place, O has moved to become the point of detachment and B becomes the tip of a splash jet OB. μ in the figure is the tip angle at point B. The complex velocity potential $W(Z,t)$ for the self-similar flow can be written as $W(Z,t) = V^2 tw(z) = V^2 t\left[\phi(x,y) + i\psi(x,y)\right]$. We choose the first quadrant of the ζ-plane in figure 1b to formulate boundary-value problems for the nondimensional complex velocity, dw/dz, and for $dw/d\zeta$, both as functions of the variable ζ.

Then, these functions can be found by applying integral formulae similar to those used in [1] and [2], or

$$\frac{dw}{dz} = v_0 \left(\frac{\zeta - 1}{\zeta + 1}\right)^{\frac{1}{2}} \exp\left[\frac{1}{\pi}\int_0^1 \frac{d\beta}{d\xi} \ln\left(\frac{\xi - \zeta}{\xi + \zeta}\right) d\xi - \frac{i}{\pi}\int_0^\infty \frac{d\ln v}{d\eta} \ln\left(\frac{i\eta - \zeta}{i\eta + \zeta}\right) d\eta - i\left(\beta_0 + \frac{\pi}{2}\right)\right] \tag{1}$$

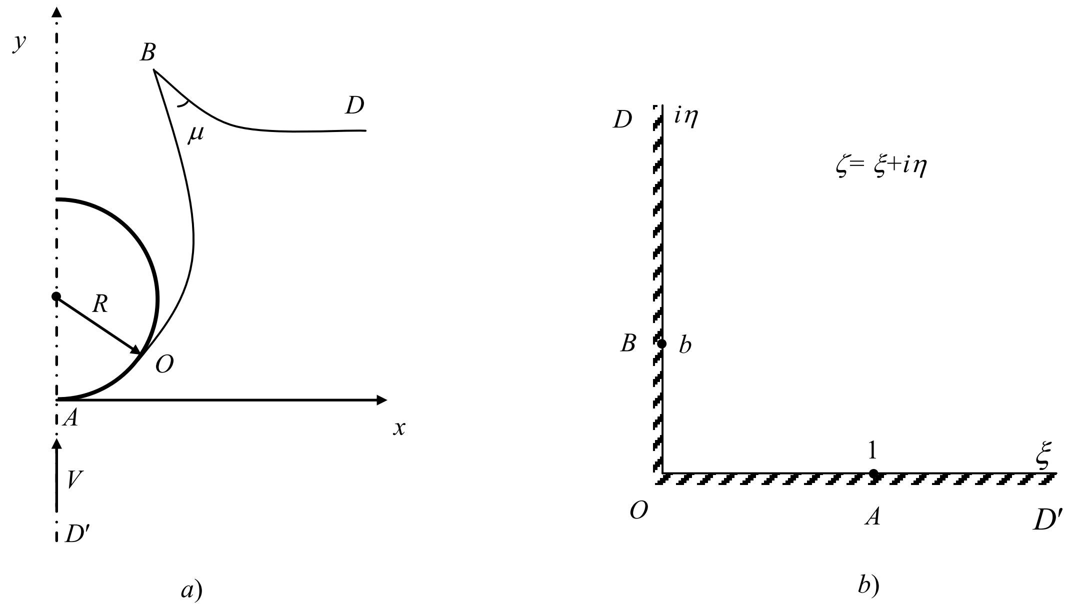

Figure 1. *a*) The similarity plane $z = x + iy$, *b*) the $\zeta -$plane of the parametric variable ζ.

$$\frac{dw}{d\zeta} = \frac{K\zeta}{\left(\zeta^2 + b^2\right)^{\frac{\mu}{\pi}-1}} \exp\left[\frac{1}{\pi}\int_0^1 \frac{d\gamma}{d\xi}\ln(\xi^2 - \zeta^2)d\xi + \frac{1}{\pi}\int_0^\infty \frac{d\theta}{d\eta}\ln(\eta^2 + \zeta^2)d\eta\right], \tag{2}$$

where $\beta(\xi)$ is the velocity direction along the wetted part of the cylinder, $v(\eta)$ is the magnitude of the velocity along the free surface, $\beta_0 = \beta(\xi)_{\xi=0}$ and $v_0 = v(\eta)_{\eta=0}$ are respectively their values at point O, K is a real scale factor, and b is the free parameter of the solution which governs the speed of expansion; $\theta(\eta) = \tan^{-1}(v_n / v_s)$ is the angle between the velocity vector and the free surface, and $\gamma(\xi) = \tan^{-1}(v_n / v_s)$ is the angle between the velocity vector and the wetted part of the cylinder surface. The velocity field and the relation between the $\zeta -$plane and the similarity plane, $z = z(\zeta)$, can be determined based on the above expressions.

The functions $v(\eta)$ and $\theta(\eta)$ are determined from the dynamic and the kinematic boundary conditions on the free surface, while the functions $\beta(\xi)$ and $\gamma(\xi)$ can be found from the given shape of the body through its slope $\delta[s(\xi)]$, where $s = s(\xi)$ is the arc length coordinate along the boundary and $s = 0$ corresponds to point B. From the similarity expansion of the body, the normal component of the velocity is obtained as $v_n = \mathrm{Im}(\bar{z}_b e^{i\delta})$, where $\bar{z}_b$ is the complex conjugate coordinate of a point on the body.

Flow detachment. According to the Brillouin–Villat criterion of flow detachment from a curved body, the curvature of the free streamline is equal to the curvature of the body at the point of detachment. This criterion has been used for steady flows. Here we use this criterion to determine detachment point for the present case. We introduce the function $\bar{\delta}(\eta)$ which is the slope of the free surface. The velocity direction $\bar{\beta}(\eta)$ on the free surface can be obtained from Eq.(1) at $\zeta = i\eta$ as

$$\bar{\beta}(\eta) = -\mathrm{Im}\left(\ln\left.\frac{dw}{dz}\right|_{\zeta=i\eta}\right) = \arctan\eta - \frac{1}{\pi}\int_0^1 \frac{d\beta}{d\xi}\left(-2\arctan\frac{\eta}{\xi}\right)d\xi + \frac{1}{\pi}\int_0^\infty \frac{d\ln v}{d\eta}\ln\left|\frac{\eta' - \eta}{\eta' + \eta}\right|d\eta' + \beta_0 . \tag{3}$$

Then, the curvature of the free surface can be determined as follows

$$\chi = \frac{d\bar{\delta}}{ds} = \frac{d\bar{\delta}}{d\eta} / \frac{ds}{d\eta}, \tag{4}$$

where $ds/d\eta = \left|dz/d\zeta\right|_{\zeta=i\eta} = \left|dw/d\zeta\right|_{\zeta=i\eta} / \left|dw/dz\right|_{\zeta=i\eta} = \left|dw/d\zeta\right|_{\zeta=i\eta} / v(\eta)$ and $\bar{\delta}(\eta) = \bar{\beta}(\eta) + \theta(\eta)$ are obtained using Eq.(1) and (2). At the detachment point O Eq.(4) becomes

$$\chi_O = \lim_{\substack{s\to s_O\\ \eta\to 0}} \left(\frac{d\bar{\beta}}{ds} + \frac{d\theta}{ds} \right), \tag{5}$$

where $s_O = s(\eta)_{\eta=0} = s(\xi)_{\xi=0}$ is the arc length coordinate of detachment point O.

From the definition of the function $\theta(\eta)$ we can write $dw/ds = ve^{i\theta}$. On the other hand, using the definition of the complex velocity and taking into account that $dz/ds = e^{i\bar{\delta}}$, we can obtain $dw/ds = e^{i\bar{\delta}} dw/dz$. Thus, we can write

$$ve^{i\theta} = \frac{dw}{dz} e^{i\bar{\delta}} . \tag{6}$$

By differentiating the left- and right-hand sides of the above equation with respect to s, we obtain

$$ve^{i\theta} \left(\frac{d\ln v}{ds} + i\frac{d\theta}{ds} \right) = e^{2i\bar{\delta}} \frac{d^2 w}{dz^2} + ie^{i\bar{\delta}} \frac{dw}{dz}\frac{d\bar{\delta}}{ds} . \tag{7}$$

The term d^2w/dz^2 on the right-hand side of the above equation is a bounded function because the complex potential w is an analytical function. From the above equation it follows that the function $\left|d\theta/ds\right| < \infty$ if the curvature of the free surface is also bounded, $\left|d\bar{\delta}/ds\right| < \infty$. In this case, from Eq.(5) it follows

$$\lim_{\substack{s\to s_O\\ \eta\to 0}} \left| \frac{d\bar{\beta}}{ds} \right| < \infty, \tag{8}$$

It was shown by Villat [7] that for the case of a bounded derivative, $\left|d\bar{\beta}/ds\right| < \infty$, its value at point O can only be equal to the value $\left|d\beta/ds\right|$ of the body surface at the same point. Similar discussion for the steady flow can also be found in Yoon & Semenov [8]. Taking the limit in Eq.(9) and using the relation $d\bar{\beta}/ds = (d\bar{\beta}/d\eta)/(ds/d\eta)$ and Eq.(4), we obtain

$$\lim_{s\to s_O} \frac{d\bar{\beta}}{ds} = \lim_{\eta\to 0} \left(\frac{d\bar{\beta}}{d\eta} / \frac{ds}{d\eta} \right) = \lim_{\eta\to 0} \left\{ \left(\frac{1}{1+\eta^2} + \frac{1}{\pi}\int_0^1 \frac{d\beta}{d\xi}\frac{2\xi}{\xi^2+\eta^2} d\xi - \frac{1}{\pi}\int_0^\infty \frac{d\ln v}{d\eta}\frac{2\eta' d\eta'}{\eta'^2-\eta^2} d\eta' \right) \frac{1}{\eta} \right\} < \infty , \tag{9}$$

from which it follows that the leading order of the above equation has to be equal to zero, or

$$\int_0^1 \frac{d\beta}{d\xi}\frac{d\xi}{\xi} - \int_0^\infty \frac{d\ln v}{d\eta}\frac{d\eta'}{\eta'} + \frac{\pi}{2} = 0 . \tag{10}$$

The streamline patterns at different values of b in Eq. (2), corresponding to the different expansion speeds V_R, are shown in figure 2. Here, the reference velocity $V = 1$. The cylinder surface is shown as a thick line. The slopes of the streamlines show the instant flow velocity direction, and their density shows the velocity magnitude because the flowrate between the streamlines is constant. For the larger expansion speed in fig.2*a*, one can see a smaller inclination of the splash jet to the $x-$axis and a larger velocity magnitude of the splash jet, which is equal to the distance between points O and A. For the flow patterns shown in fig. 2*b* – 2*e*, the expansion speed gradually decreases. For very small $V_R = 0.013$ in fig. $2f$, the flow has been found to approach to the steady Helmholtz flow past the cylinder, and the angle of flow detachment tends to $55°2'$ measured from the vertical axis.

Acknowledgements

This work is supported by Lloyd's Register Foundation (LRF) through the joint centre involving University College London, Shanghai Jiaotong University and Harbin Engineering University, to which the authors are most grateful. LRF supports the advancement of engineering-related education, and funds research and development that enhances safety of life at sea, on land and in the air.

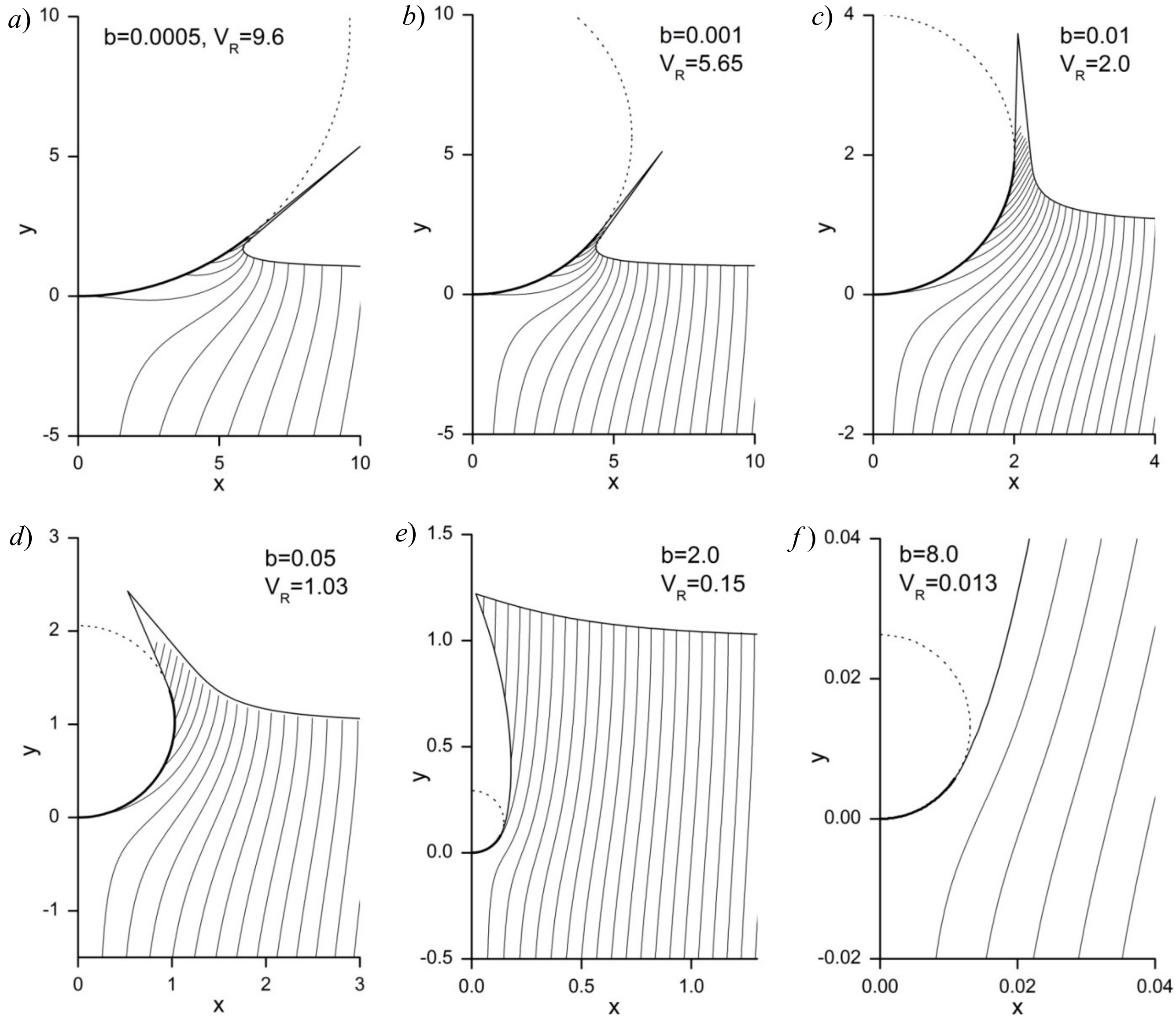

Figure 2. The free surface shape and streamlines for different expansion speeds of a circular cylinder.

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