Deep water wave-breaking in a High-Order Spectral model

B. R. Seiffert[1,2] and G. Ducrozet[1]

[1]Ecole Centrale Nantes, LHEEA Lab., UMR CNRS 6598, Nantes, France
[2]Institut de Recherche Technologique Jules Verne, Bouguenais, France
[*]betsy.seiffert@ec-nantes.fr

Highlights

- Presentation of an efficient and robust wave-breaking onset criteria implemented in two HOS models.
- Validation of the criteria for modulated, chirped and random waves in deep water.

Introduction

In nature, a steep wave may grow to surpass some limiting threshold leading to a collapse of the water surface as a broken wave. Energy from the wave is transferred through the generation of currents and turbulence, and changes occur in the spectral distribution of energy before and after a wave breaks.

The numerical models used in this study, HOS-ocean and HOS-NWT, are computationally efficient, open source codes that solve for highly nonlinear wave fields in the open ocean and a numerical wave tank, respectively, using the High-Order Spectral (HOS) method (Ducrozet et al. [4], Bonnefoy et al. [3], Ducrozet et al. [5]). Due to the assumptions of solving for potential flow, the HOS solvers assume a single-valued free surface and therefore cannot produce breaking waves. The goal of implementing a wave-breaking mechanism into the HOS models is to approximate the broken free surface as a single value. By determining some means of accounting for wave-breaking within the HOS models, we can increase the application range of the models including calculating more extreme sea states. An accurate description of the wave field is necessary for predicting dynamics of offshore vessels and marine renewable energy devices, predicting loads on marine structures and the general physics of ocean waves, for example.

To implement a wave-breaking mechanism into the HOS models, first a wave-breaking onset parameter that can be calculated within the models needs to be identified, and second, a strategy for dissipating and distributing the energy after the wave has broken needs to be determined.

Wave breaking onset

The nonlinear and irregular processes that occur just before a wave breaks make predicting the onset difficult. Wave-breaking onset criteria is traditionally classified into one of three categories: geometric, kinematic or dynamic. Perlin et al. [7] provides a recent review of each of the three wave breaking criteria and the strengths and limitations of each.

In their numerical calculations, Barthelemy et al. [2] proposed a wave breaking threshold parameter based on the ratio of local energy flux velocity to the local crest velocity. The local energy flux velocity is calculated as the ratio of the local energy flux vector to local energy density. Barthelemy et al. [2] defines this breaking criterion as $B_x = F_x/EC_x$ where F_x and C_x are the projections in the direction of wave propagation (in this case, the x-axis) of energy flux and crest velocity, respectively, and E is the local energy density. Barthelemy et al. [2] determined wave breaking occurs in the narrow range of $0.85 < B_x < 0.86$. Saket et al. [8] investigated the breaking criterion of Barthelemy et al. [2] experimentally and found B_x to be 0.84 ± 0.016, which is in good agreement with Barthelemy et al. [2]. Additionally, Saket et al. [8] found this threshold parameter to be robust for different classes of wave groups and wave steepness.

It is noted at the free surface, the energy flux velocity reduces to the water particle velocity, reducing this dynamic breaking criterion to a kinematic criterion. This is different from the traditional kinematic breaking criterion which states $U/C \geq 1$, which would under predict wave-breaking events according to Perlin et al. [7], Barthelemy et al. [2], and Saket et al. [8], for example. It is advantageous to use the breaking criterion of Barthelemy et al. [2] as is straightforward to calculate in the HOS solver, applicable to 2D and 3D, and has a narrow threshold. However, since the wave is not of permanent form, the calculation of the wave crest velocity is non-trivial and must be calculated carefully.

Barthelemy et al. [2] measured a forward and backward leaning cycle of a wave crest as it moves through a wave train, and it is at the point close to when the crest is moving at its slowest speed that the wave breaks. Therefore a method of calculating the crest velocity that can accurately capture this slow-down is essential to accurately predict breaking onset.

Calculating wave-breaking onset in the HOS model

To calculate wave breaking onset in the HOS solvers, the criterion described by Barthelemy et al. [2] is tested. In 2D, the breaking criterion reduces to the ratio of water particle velocity to crest velocity in the direction of wave propagation, which can be written in terms of velocity potential as follows:

$$B_x = \frac{F_x/E}{C_x} = \frac{U_x}{C_x} = \frac{\partial \phi}{\partial x}\frac{1}{C_x} < threshold \tag{1}$$

As mentioned above, the calculation of wave crest velocity is non-trivial and must be able to capture the crest slow-down that occurs just before the wave breaks. A method of calculating the instantaneous crest velocity $C_x(x,t)$ using the partial Hilbert transform is used to overcome this problem. Following Kurnia and van Groesen [6], the instantaneous, or local crest speed and local wave number are defined by

$$C_x(x,t) = \sqrt{\frac{g \cdot tanh(k(x,t) \cdot h)}{k(x,t)}}, \qquad k(x,t) = \frac{1}{\eta^2 + \mathbb{H}^2[\eta]}\left(\eta \frac{\partial}{\partial x}\mathbb{H}[\eta] - \mathbb{H}[\eta]\frac{\partial}{\partial x}\eta\right) \tag{2}$$

and is $\mathbb{H}[\eta(x,t)]$ is the partial Hilbert transform with respect to x.

Figure 1 shows a "chirped" wave packet, generated using the HOS-NWT solver following the method outlined in Song and Banner [9], which is a widely used method of generated controlled breaking waves in the laboratory. The two columns on the left show the surface elevation and free surface modal amplitude for a wave with a maximum breaking parameter $B_x = 0.79825$ at time $t = 24.85$ s, which would be classified as an "unbroken" wave following Barthelemy et al. [2] and Saket et al. [8]. The two columns on the right show the surface elevation and free surface modal amplitude of a slightly larger wave packet where the breaking parameter is calculated as $B_x = 0.85091$ at the time $t = 28.50$ s, which would be just above the threshold of Barthelemy et al. [2] and Saket et al. [8] to be classified as a "broken" wave. At $t = 10.00$ s, 24.85 s and 28.50 s, the surface elevation and modal amplitudes are comparable between the "unbroken" and the "broken" wave. However, at $t = 38.39$ s, the "unbroken" wave returns to its original amplitude and modal distribution, whereas the "broken" wave shows some high frequency components

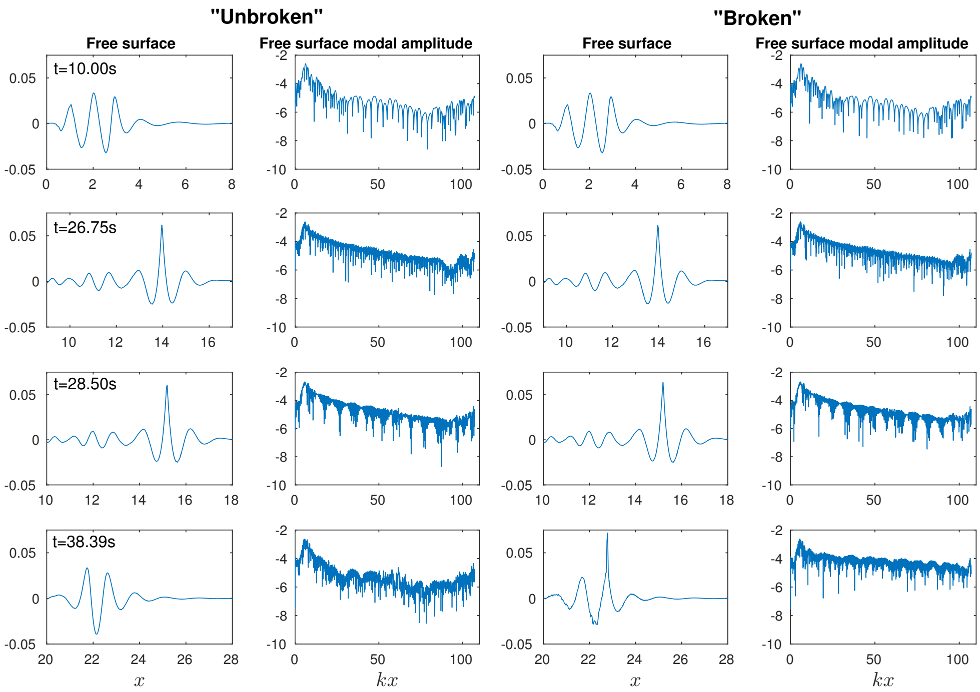

Figure 1: Free surface and free surface modal amplitude for an “unbroken” and “broken” wave at t =10.00 s, 26.75 s, 28.50 s and 38.39 s. The “unbroken” wave reaches a maximum $B_x = 0.79825$ at time $t = 26.75$ s before returning to its original amplitude and modal distribution. The “broken” wave reaches a maximum of $B_x = 0.85091$ at the time $t = 28.50$ s then shows high-frequency components in the free surface and in its modal distribution at $t = 38.39$ s.

in the free surface and in its modal distribution. This is likely an artifact of a wave which in nature would overturn, but since the HOS model solves for a single-valued free surface, these high frequencies develop instead. This suggests that a threshold of $B_x \approx 0.84-0.86$ as suggested by Barthelemy et al. [2] and Saket et al. [8] would distinguish between a broken and an unbroken wave in the HOS solvers.

A comparison is made between the experimental measurements of Saket et al. [8] and those calculated using HOS-NWT for local wave steepness (S_c) as defined in [8], and breaking parameter (B_x) for maximally recurring and marginally breaking waves in Table 1. In these experiments, breaking waves were generated using “chirped” wave packets, as in Fig. 1. In the HOS-NWT calculations, B_x is calculated at at each location x and time step. If the calculated B_x surpasses the threshold of 0.84, then the wave is classified as “broken”, the simulation is stopped, and the calculated B_x and S_c are reported at this time. If B_x did not surpass 0.84, then the maximum B_x and corresponding S_c are reported and the wave is classified as “unbroken”. Although HOS-NWT calculates slightly larger wave steepness when breaking than Saket et al. [8], the results are still in reasonable agreement.

The numerical calculations of Barthelemy et al. [1] also used chirped wave packets to produce breaking waves. Results for calculated wave breaking parameter and corresponding local wave steepness are shown in Fig. 2 for the numerical calculations of Barthelemy et al. [2] and Saket et al. [8] along with a set of calculations using chirped wave packets in HOS-NWT, as well as modulated and random waves, calculated using HOS-ocean. Classification of “broken” and “unbroken” waves are the same as those above with a threshold of 0.84. The hollow markers represent unbroken waves and the solid markers are broken waves. The breaking parameters and corresponding local wave steepness calculated using HOS-NWT and HOS-ocean are in good agreement with the Barthelemy et al. [2] calculations and Saket et al. [8] experiments, giving us confidence in the applicability of this breaking parameter in the HOS models.

Conclusion

Due to the assumptions of calculating for potential flow, the HOS solvers require a single-valued free surface and therefore cannot calculate breaking waves. The goal of this study is to be able to approximate a broken free surface as a single value. To do this, we need to determine a measurable wave breaking onset criteria, and determine a strategy for wave energy dissipation and distribution after the wave has broken. So far, we have identified a robust wave breaking onset criteria which has been successful in identifying the threshold at which a wave is "broken" in the HOS solvers, evidenced by the manifestation of high frequency components in the surface elevation and modal distribution in a "broken" wave. Further research is needed to determine the wave energy dissipation and distribution.

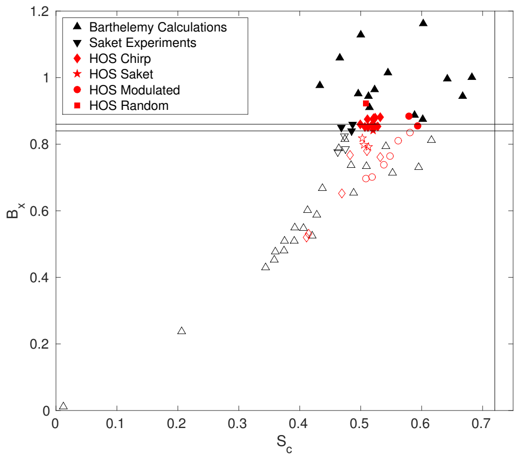

Figure 2: Breaking parameter B_x versus crest steepness S_c. Hollow points represent unbroken waves and solid points represent broken waves. The zone $0.84 < B_x < 0.86$ is the breaking threshold, and $S_c > 0.72$ is the deep water Stokes limit.

	Maximum Recurrence			
	Saket		HOS	
Wave	Sc	Bx	Sc	Bx
C3N5	0.473	0.825	0.505	0.798
C3N7	0.462	0.777	0.512	0.633
C3N9	0.475	0.786	0.502	0.818
	Marginal Breaking			
	Saket		HOS	
Wave	Sc	Bx	Sc	Bx
C3N5	0.487	0.859	0.519	0.851
C3N7	0.468	0.850	0.525	0.852
C3N9	0.485	0.840	0.520	0.842

Table 1: Wave steepness S_c and breaking parameter B_x measured for maximum recurring and marginally breaking waves measured in the experiments of Saket et al. [8] and calculated using using HOS-NWT.

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