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Some aspects of high kinematics in breaking waves due to sloshing.

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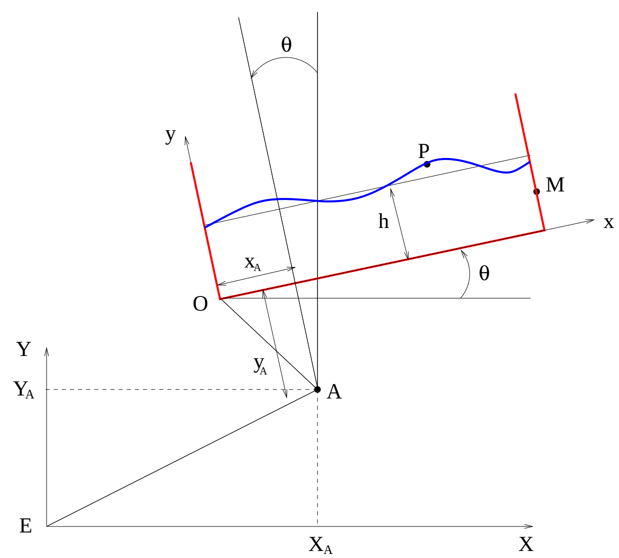
Highlights

- Free surface flows are computed in a tank in forced motion,
- Fully nonlinear free surface problems are solved,
- Extremely accelerated localized free surface motions are simulated.

1) Introduction

The literature is rather rich regarding the simulation of nonlinear free surface flow in a tank submitted to a forced motion as long as the response is weakly nonlinear. For example, the Finite Difference Method used by Frandsen (2004) among others shows the ability to catch the spectral content of the wave pattern. The abundant contributions by Faltinsen, Rognebakke and Timokha (2000 and later) also give access to weakly nonlinear of the free surface responses. When the free surface deformation becomes more pronounced, the literature is rather poor. We propose here some new insights into that problem formulated in potential theory. The physical configuration is sketched below



The total velocity potential Φ describes the flow in a coordinate system attached to earth. It verifies the following boundary value problem

$$\left\{\begin{array}{ll} \Delta\Phi = 0 & \text{in the fluid domain} \\ \Phi_{,t} + \frac{1}{2}\vec{\nabla}^2\Phi + g\left(\vec{Y}\cdot\vec{EP} - h\right) = 0 & P \text{ on the free surface} \\ \frac{d}{dt}\vec{EP} = \vec{\nabla}\Phi & P \text{ on the free surface} \\ \Phi_{,n} = \frac{d}{dt}\vec{EM}\cdot\vec{n} & M \text{ on the wall of the tank} \end{array}\right. \qquad (1)$$

where $\vec{n}$ is the unit normal vector on any considered surface. As it is formulated we could solve that problem directly with standard two-dimensional Panel Method. In practice we use here the desingularized technique as described in Cao *et al* (1991) or Tuck (1998). In order to optimize the numerical scheme, the total potential Φ is split into three components. The first two account for the forced motion. The third (noted φ) describes the flow motion in a coordinate system attached to the tank. It fulfills a homogeneous Neumann boundary condition on the tank walls. With the notations introduced in the figure above, a time differential system is written from a Lagrangian transport of φ on Lagrangian markers (x, y) attached to the free surface

$$\left\{\begin{array}{rcl}
\dfrac{d\varphi}{dt} & = & \frac{1}{2}\vec{\nabla}^2\varphi + \frac{1}{2}\vec{v}_A^2 - \dot{\theta}^2\left(\frac{1}{2}\vec{\nabla}^2\Omega + y\Omega_{,x} - x\Omega_{,y}\right) \\
 & & -\dot{\vec{v}}_A\vec{AP} - \ddot{\theta}\Omega \\
 & & +\dot{\theta}\left(\vec{k}(\vec{AP}\wedge(\vec{v}_A + \vec{\nabla}\varphi)) - \vec{\nabla}\varphi\vec{\nabla}\Omega\right) \\
 & & -g\left(Y_A - h + (x - x_A)\sin\theta + (y - y_A)\cos\theta\right) \\
\dfrac{dx}{dt} & = & \varphi_{,x} \\
\dfrac{dy}{dt} & = & \varphi_{,y}
\end{array}\right. \tag{2}$$

The velocity potential Ω is known as the Stokes-Joukowski potential (see Joukowski, 1885) and the corresponding Boundary Value Problem is purely of Neumann type

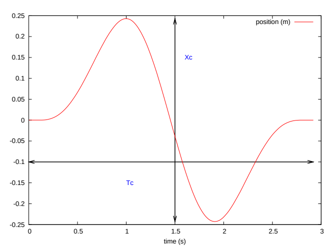
$$\left\{\begin{array}{ll}
\Delta\Omega = 0 & \text{in the fluid domain} \\
\Omega_{,n} = (\vec{k}\wedge\vec{AP})\cdot\vec{n} & \text{all over the fluid boundary}
\end{array}\right. \tag{3}$$

By using a desingularized technique, both φ and Ω are represented as a finite set of singularities (Green function of Rankine type). They are located outside the fluid domain and at some distance from the actual free surface. The convergence of the computation of Ω is checked with the exact linear solution for a rectangular tank. That solution is given in Faltinsen *et al* (2000) among others. Attention must be paid on the choice of the desingularizing distance. As explained in Scolan (2015), convergence criteria for linear cases (non moving boundary) differ from the criteria for nonlinear computations.

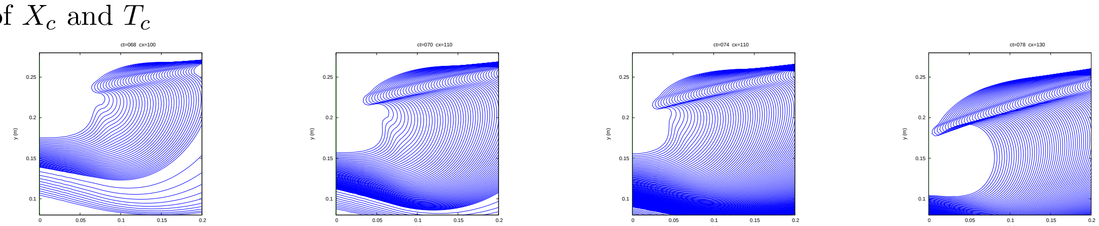
When the tank is shaked with stronger forced oscillations, the free surface is much more distorted and the problem becomes less standard. In fact due to either numerical difficulties or restrictions in terms of computational resources, few numerical simulations in potential theory are reported in the literature.

2) Breaking wave due to a simple forced motion

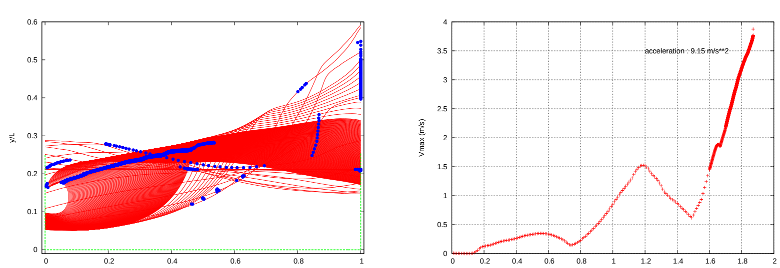
Based on the experiments by Karimi (see Karimi *et al*, 2015), we perform numerical simulations with a very simple tank kinematics. The figure below shows a typical horizontal motion for a tank which may have chamfers or not



By tuning the factors X_c and T_c, we can simulate a large range of overturning crest configurations. That is shown in the figures below where the successive profiles of the free surface are plotted for different choices of X_c and T_c

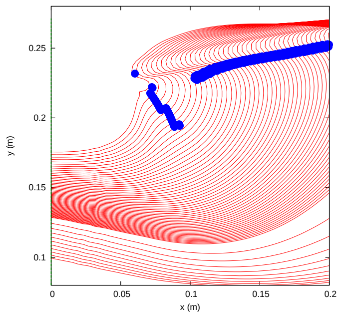


In the last left figure, nothing exotic occurs. The analysis of the kinematics along the free surface is illustrated in the figure below for this "soft" case

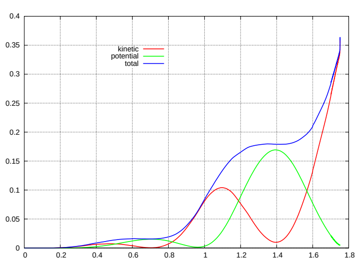


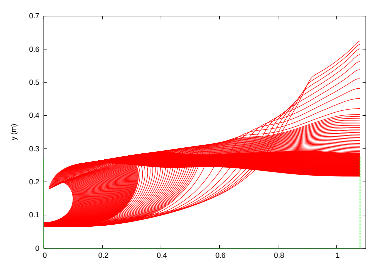
The left figure shows the whole free surface profiles and for each of them the location of the highest velocity amplitude $|\vec{\nabla}\varphi|$ in the coordinate system attached to the tank. The right figure collects those data and plot the time variation of that maximum velocity. It is worth noting that at the very end the velocity grows almost linearly. It is hence possible to extrapolate the velocity and the time at which the wave would impact the wall even if the present computations cannot reproduce that. By doing such parametrical study, much knowledge can be gained about the correlation between the tank kinematics and the resulting fluid kinematics. It is clear that linear or weakly nonlinear approaches cannot predict such results.

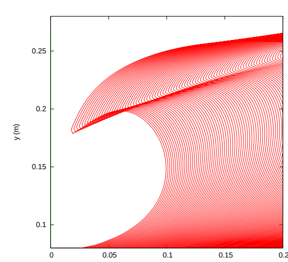
The most interesting cases are those for which a sudden localized free surface growth occurs. For the first three cases illustrated above, it is observed that when the overturning crest develops –thus leading to the possible entrapment of a gas pocket– a point of very strong kinematics grows along the free surface at some intermediate distance between the crest and the wall. The analysis of the kinematics along the free surface is illustrated in the figure below for one of these "exotic" cases



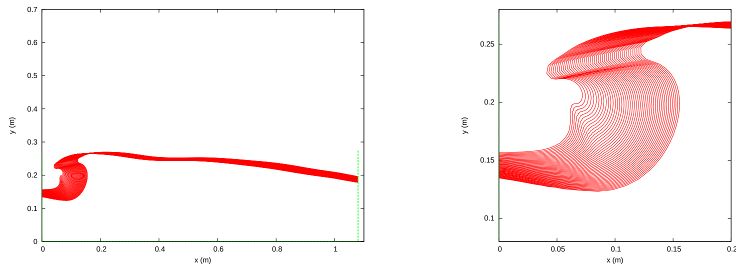
In the present case, at the point where the free surface locally swells, the acceleration of the fluid is much higher than during the previous phase. That phenomenon is identified as a double crest phenomenon but the shape of the wave also reminds the shape of a lobster claw. Experimental observations have been done by Brosset (2015) and confirm the existence of such outgrowth. Further experimental and numerical studies should lead to an explanation of that occurrence since the resulting loadings might be substantially underestimated if such phenomena were ignored. Currently it is not clear whether or not this phenomenon is associated with the forced motion of the tank. A first test consists in restarting a simulation with the solutions (free surface position and distribution of velocity potential) when the free surface reaches its maximum of deformation during the first half cycle of oscillation of the tank knowing that if we force the tank until the end, we would have a double crest. The following figures show the time variations of the fluid energies. At time $t = 1.39s$ the potential energy is maximum. At that time, we extract the free surface position and the corresponding distribution of velocity potential (the initial kinetic energy is slightly not nil).



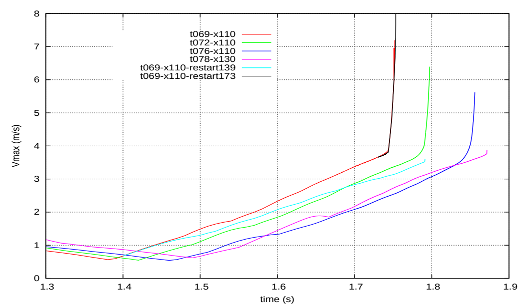




Then we release the dynamical system evolving without any forcing of the tank. The successive free surface profiles are plotted globally (middle figure) and locally at the left wall (right figure). We do not observe the appearance of a "lobster claw". Another test is performed by restarting a simulation with the solutions (free surface position and distribution of velocity potential) early before the appearance of the local growth but during the first phase of moderately accelerated crest. The chosen instant is $t = 1.73s$ and we release the dynamical system evolving without any forcing of the tank. The successive free surface profiles are plotted globally (left figure) and locally at the left wall (right figure).



We do observe the appearance of "lobster claw". As a consequence there is not a strong evidence of the correlation between the forced motion and the appearance of "lobster claw". The next figure compares the time variation of the maximum velocity when either a double crest appears or not. Before the double crest appears, the typical fluid accelerations (slopes of the curves) is the acceleration of gravity $[7m/s^2, 10m/s^2]$. For the three cases where the double crest appears, the fluid acceleration increases with a factor that may reach 35.



When the late restart is performed (at $t = 1.73s$), the corresponding time variation of the maximum velocity matches quite well the original ones starting from rest. More numerical experiments are necessary to confirm that result. Preliminary comparisons are made with the code CADYF, a Navier-Stokes solver developed at Polytechnique Montréal (see Charlot *et al*, 2012). The agreement (not illustrated here) is fairly good. There are hence reasons to think that Potential Theory will be a good frame to describe that new phenomenon.

4) References

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