Water exit of a light body fully submerged initially

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Highlights:

- The whole process of a buoyant body emerging and departing from water has been simulated.
- A critical density for complete water exit is derived with the slender body theory.

1. Introduction

Water entry and exit has been a topic of wide range of applications in naval architecture, ocean engineering and costal engineering, etc. Relative to water entry, there has been far less work on water exit. During water exit, the wetted surface of the body decreases with time and the liquid may detach from the body eventually. As a result, water exit is less violent and can be taken as a kind of 'loads-off' process, compared to the 'loads-on' one during water entry, which may be part of the reason why water exit is less studied. From the modelling point of view, water exit cannot be easily treated as the reverse process of water entry. For example, it is difficult to use the Wagner theory, commonly used in the water entry, in the water exit problem. However, water exit is also an important engineering problem. In fact, for a ship in rough seas, its bow will repeatedly emerge from water and then hit water surface, which forms a continuous water exit/entry process.

Work on water exit can be divided 'forced water exit' and 'free water exit'. In the forced water exit, Greenhow & Moyo (1997) simulated a 2D circular cylinder by using the boundary-element method (BEM) based on complex velocity potential. They adopted the fully nonlinear boundary condition at the free surface and calculated the vertical rise of the cylinder right before the free surface broke up or the body penetrated through the free surface. Korobkin (2013) studied constant acceleration water exit of a 2D or axisymmetric body initially partially submerged. Boundary conditions on the liquid surface were linearized and imposed on its mean position. The shape of the wetted part of the body was simplified by using the so-called 'equivalent flat-plate approximation'. This model was further applied by Korobkin et al. (2014) to consider a prescribed acceleration which varied in time, and then further extended by Khabakhpasheva et al. (2015) to consider the body whose shape varied in time, which was initially studied by Tassin et al. (2013) by using the von Karman approach. Rajavaheinthan & Greenhow (2015) studied forced constant acceleration exit of 2D bodies by using fully-nonlinear BEM. The bodies with different shapes were initially partially submerged and suddenly moved upwards. However, the simulation terminated before the water detachment from the body. A very recent work by Ni et al (2015) considered the entire process of the water exit of an initially fully-submerged axisymmetric body at constant speed. Two major difficulties, related to free-surface breakup and water detachment from the body, have been solved. Consequently, as the body continued to move up, it would first penetrate through the free surface and would detach from water completely later.

Relative to the forced water exit, the free motion one is more challenging and thus less studied. The fluid force and the motion of the body are coupled, and so the fluid-structure interaction must be solved at each time step, which may cause some difficulties in numerical modelling. Moyo & Greenhow (2000) simulated the free motion of 2D light cylinders by using BEM. Light cylinder with different densities and initial submergence depths were simulated and the free-surface deformation as well as velocity and acceleration of the body were obtained. However the calculation was terminated before the body penetrated through the free surface. In the present work, we shall consider the whole process of water exit of an initially fully submerged axisymmetric buoyant body in free motion.

2. Mathematical Model and Numerical Method

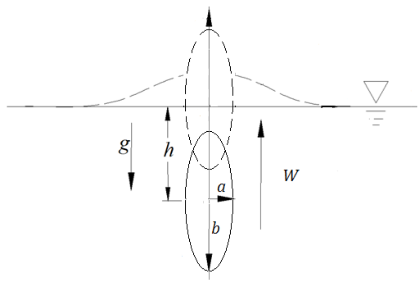

Fig. 1 Sketch of the problem

Fig.1 gives a sketch of the problem, which shows an initially fully-submerged spheroid exiting water under buoyancy, whose major semi axis and minor semi axis are b and a respectively. The distance between the initial body centre and the undisturbed free surface is h. When the body motion is along the z axis, the flow is then axisymmetric. The fluid is assumed inviscid and incompressible, and the flow is irrotational. Thus, a velocity potential Φ can be introduced, which satisfies Laplace's equation

$$\nabla^2\Phi = 0, \tag{1}$$

in the fluid domain. The impermeable boundary condition on the wetted part of the rigid body surface s_b is given by

$$\frac{\partial\Phi}{\partial n} = W(t)\cdot n_z, \tag{2}$$

where $W(t)$ is the vertical velocity of the body, which is a function of time and needs to be found at each time step and n_z is the z component of normal vector $\boldsymbol{n}=(n_r, n_z)$ of the body surface pointing out of the fluid domain. On the free surface s_f, the fully nonlinear kinematic and dynamic boundary conditions can be written below in the Lagrangian framework:

$$\frac{Dr}{Dt}=\frac{\partial\Phi}{\partial r}, \quad \frac{Dz}{Dt}=\frac{\partial\Phi}{\partial z}, \tag{3}$$

$$\frac{D\Phi}{Dt}=\frac{1}{2}|\nabla\Phi|^2-gz, \tag{4}$$

where D/Dt is the substantial derivative following a fluid particle, g is the acceleration due to gravity. Bernoulli equation and constant pressure on the free surface have been used in Eq.(4). The boundary condition at infinity s_∞ is based on the assumption that the fluid there is undisturbed. One has

$$\nabla\Phi\to 0. \qquad \sqrt{r^2+z^2}\to\infty \tag{5}$$

Green's third identity with the Green function and boundary-element method will be adopted to solve Eq.(1) with boundary conditions in Eqs.(2)-(5).

During water exit, the body will break up the free surface when its font emerges from water, and detach from the free surface when its back departs from water. The breakup and detachment of the free surface must be carefully treated in numerical modelling. Following the procedure in Ni et al. (2015), it is assumed that the free surface will break up when the thickness of the water layer right above the top of the body is smaller than a critical distance $\Delta\bar{l}_c$. Before the back of the body detaches from the water, a water column is usually seen to be attached to the body surface. It is assumed when the radius of the water column is smaller than $\Delta\bar{l}_c$, body/liquid detachment will occur in the numerical simulation at the next time step. Ni et al. (2015) have undertaken extensive numerical investigations and found that the results of interest are not sensitive to the choice of $\Delta\bar{l}_c$ when it is sufficiently small. It is taken as 10% of the element size on the body in the present work.

Nondimensionalisation is applied based on the length of the body $L=2b$, the acceleration g due to gravity, and the density of the fluid ρ. Thus we use $\sqrt{L/g}$ for time, $\sqrt{gL}$ for the velocity, $\sqrt{gL^3}$ for the velocity potential and ρgL for pressure respectively. The dimensionless parameters will then be denoted by a bar. Besides, $\lambda=h/L$ is defined as the initial submergence parameter.

2.1 Decoupling of the motion of the body and fluid flow

By Newton's second Law, body motion equation is

$$\bar{m}\frac{d\bar{W}}{d\bar{t}}=\bar{F}+\bar{F}_e, \tag{6}$$

in which the mass of the body $\bar{m}=\bar{\rho}_B\bar{V}_B=\bar{\rho}_B 2\pi\bar{a}^2/3$ with constant body density $\bar{\rho}_B$, $\bar{F}_e$ is the external force exerted on the body, which is $\bar{F}_e=-\bar{m}$ when only the gravity is considered. $\bar{F}$ is the fluid force and can be obtained by integrating the fluid pressure obtained from the Bernoulli equation over wetted body surface:

$$\bar{F}=\bar{F}_z=-\iint_{\bar{s}_b}\left(\frac{\partial\bar{\Phi}}{\partial\bar{t}}+\frac{1}{2}\left|\nabla\bar{\Phi}\right|^2+\bar{z}\right)\cdot n_z d\bar{s}, \tag{7}$$

The term $\partial\bar{\Phi}/\partial\bar{t}$ on the right hand side of Eq.(7) is problematic in direct numerical calculation through difference method with time. We follow the method proposed by Wu & Eatock Taylor (2003). $\partial\bar{\Phi}/\partial\bar{t}$ can be seen as a harmonic function, which satisfies Laplace's equation

$$\nabla^2(\partial\bar{\Phi}/\partial\bar{t})=0, \tag{8}$$

with boundary conditions on the free surface and body surface (Wu, 1998):

$$\frac{\partial\bar{\Phi}}{\partial\bar{t}}=-\frac{1}{2}\left|\nabla\bar{\Phi}\right|^2-\bar{z}, \tag{9}$$

$$\frac{\partial(\partial\bar{\Phi}/\partial\bar{t})}{\partial n}=\frac{d\bar{W}}{d\bar{t}}n_z-\bar{W}\frac{\partial(\partial\bar{\Phi}/\partial\bar{z})}{\partial n}, \tag{10}$$

We introduce two auxiliary functions $\chi_1(\bar{x},\bar{y},\bar{z},\bar{t})$ and $\chi_2(\bar{x},\bar{y},\bar{z},\bar{t})$ (Wu et al., 2004) and let

$$\frac{\partial\bar{\Phi}}{\partial\bar{t}}=\frac{d\bar{W}}{d\bar{t}}\chi_1+\chi_2-\bar{W}\frac{\partial\bar{\Phi}}{\partial\bar{z}}, \tag{11}$$

χ_1 and χ_2 both satisfy the Laplace's equation in the fluid domain and the following boundary conditions,

$$\begin{cases}\dfrac{\partial\chi_1}{\partial n}=n_z, \dfrac{\partial\chi_2}{\partial n}=0 \quad on \quad \bar{s}_b\\ \chi_1=0, \chi_2=-\dfrac{1}{2}\left|\nabla\bar{\Phi}\right|^2-\bar{z}+\bar{W}\dfrac{\partial\bar{\Phi}}{\partial\bar{z}} \quad on \quad \bar{s}_f\end{cases}, \tag{12}$$

The problems for χ_1 and χ_2 can be solved by a similar procedure used for the velocity potential $\bar{\Phi}$. Subsequently the body motion equation can be found

$$(\bar{m}+\bar{m}_a)\frac{d\bar{W}}{d\bar{t}}=-\iint_{\bar{s}_b}\left(\chi_2-\bar{W}\frac{\partial\bar{\Phi}}{\partial\bar{z}}+\frac{1}{2}\left|\nabla\bar{\Phi}\right|^2+\bar{z}\right)n_z d\bar{s}+\bar{F}_e, \tag{13}$$

where $\bar{m}_a=\iint_{\bar{s}_b}\chi_1 n_z d\bar{s}$ which is effectively an added mass (Wu & Eatcok Taylor 2003).

2.2 Slender body theory

When $b/a\gg 1$, we may adopt the slender body theory. The fluid force $\bar{F}$ can be decomposed into the hydrodynamic one $\bar{F}_d$ and the hydrostatic one $\bar{F}_s$:

$$\bar{m}\frac{d\bar{W}}{d\bar{t}}=\bar{F}_d+\bar{F}_s-\bar{m}, \tag{14}$$

When the body is slender, the free surface can be assumed to be undisturbed, and $\bar{\Phi}=0$ on $\bar{z}=0$. Within this framework the hydrodynamic force can be written as (Wu 1998):

$$\bar{F}_d=-\frac{1}{2}\frac{d\bar{m}_a}{d\bar{t}}\bar{W}-\bar{m}_a\frac{d\bar{W}}{d\bar{t}}, \tag{15}$$

where the added mass $\overline{m}_a = \iint_{\overline{S}_b} \psi \frac{\partial \psi}{\partial n} d\overline{s} = \iint_{\overline{S}_b} \psi n_z d\overline{s}$ with $\psi = \Phi / \overline{W}$. The hydrostatic force can be written as

$$\overline{F}_s = -\iint_{\overline{S}_b} \overline{z} n_z d\overline{s} = \overline{V}_d, \tag{16}$$

where $\overline{V}_d$ is the volume of the body below the flat free surface. Substituting Eqs.(15) and (16) into (14), we have

$$(\overline{m} + \overline{m}_a)\frac{d\overline{W}}{d\overline{t}} + \frac{1}{2}\frac{d\overline{m}_a}{d\overline{t}}\overline{W} = (\overline{F}_s - \overline{m}), \tag{17}$$

For a spheroid, one has

$$\overline{V}_d / \overline{V}_B = \begin{cases} 1 & \overline{z}_c < -1/2 \\ 1/2 - 3\overline{z}_c/2 + 2\overline{z}_c^3 & -1/2 \le \overline{z}_c \le 1/2 \end{cases}, \tag{18}$$

where $\overline{z}_c$ is the vertical coordinate of the centre of the body and $\overline{V}_B$ is the volume of the full spheroid defined after Eq.(6).

Based on the slender body theory (Mackie, 1962, Newman, 1977), ψ after Eq.(15) can be written as

$$\psi(\overline{z}, \overline{r}) = -\frac{1}{4\pi}\int_{\overline{z}_c - 0.5}^{-\overline{z}_c + 0.5} \frac{\overline{S}'(\xi) d\xi}{[(\overline{z} - \xi)^2 + \overline{r}^2]^{1/2}}, \tag{19}$$

where $\overline{S}(\xi)$ is the cross-sectional area of the body at $\overline{z} = \xi$ and $\overline{S}'(\xi)$ is its derivative. For the case of a spheroid,

$$\overline{S}(\xi) = \begin{cases} \pi\overline{a}^2[1 - 4(\xi - \overline{z}_c)^2] & (\overline{z}_c - 0.5 < \xi < 0) \\ \pi\overline{a}^2[1 - 4(\xi + \overline{z}_c)^2] & (0 < \xi < 0.5 - \overline{z}_c) \end{cases}.$$

For the axisymmetric problem the added mass can be converted into

$$\overline{m}_a = 2\pi\int_{\overline{l}_b} \psi n_z \overline{r} d\overline{l} = 2\pi\int_{\overline{z}_c - 0.5}^{0} \frac{\psi n_z \overline{r}}{\sqrt{1 - n_z^2}} d\overline{z}, \tag{20}$$

where $n_z = \frac{4\overline{a}(\overline{z} - \overline{z}_c)}{\sqrt{1 + (16\overline{a}^2 - 4)(\overline{z} - \overline{z}_c)^2}}$. Through Eq. (17) one has the acceleration of the body

$$\begin{aligned} \frac{d\overline{W}}{d\overline{t}} &= (\overline{F}_s - \overline{m} - \frac{1}{2}\frac{d\overline{m}_a}{d\overline{t}}\overline{W}) / (\overline{m} + \overline{m}_a) \\ &= (\overline{F}_s - \overline{m} - \frac{1}{2}\frac{d\overline{m}_a}{d\overline{z}}\overline{W}^2) / (\overline{m} + \overline{m}_a) \end{aligned}. \tag{21}$$

Assuming $\Pi = \overline{W}^2$ and noticing $\frac{d\overline{W}}{d\overline{t}} = \frac{d\overline{W}}{d\overline{z}}\overline{W} = \frac{1}{2}\frac{d\overline{W}^2}{d\overline{z}}$, we obtain a linear ordinary differential equation for Π,

$$\frac{d\Pi}{d\overline{z}} = -\frac{d\overline{m}_a / d\overline{z}}{\overline{m} + \overline{m}_a}\Pi + 2\frac{\overline{F}_s - \overline{m}}{\overline{m} + \overline{m}_a}, \tag{22}$$

This can be solved through the standard procedure, from which with the initial condition $\overline{W}(0) = 0$, we have

$$\overline{W} = \sqrt{\Pi} = \sqrt{\frac{2\int_{-\lambda}^{\overline{z}_c}(\overline{F}_s - \overline{m})d\overline{z}}{\overline{m} + \overline{m}_a}} = \sqrt{\frac{2f(\overline{z}_c)}{\overline{\rho}_B + k_s}}, \tag{23}$$

where $f(\overline{z}_c) = \frac{1}{2}\overline{z}_c^4 - \frac{3}{4}\overline{z}_c^2 + (\frac{1}{2} - \overline{\rho}_B)\overline{z}_c + (1 - \overline{\rho}_B)\lambda - \frac{3}{32}$, and $k_s = \overline{m}_a / \overline{V}_B$ is the added mass coefficient.

Substituting Eq.(23) into Eq.(21), we have

$$\begin{aligned} \frac{d\overline{W}}{d\overline{t}} &= \frac{\overline{F}_s - \overline{m}}{\overline{m} + \overline{m}_a} - \frac{d\overline{m}_a / d\overline{z} \cdot \int_{-\lambda}^{\overline{z}_c}(\overline{F}_s - \overline{m})d\overline{z}}{(\overline{m} + \overline{m}_a)^2} \\ &= \frac{\overline{V}_d / \overline{V}_B - \overline{\rho}_B}{\overline{\rho}_B + k_s} - \frac{dk_s / d\overline{z} \cdot f(\overline{z}_c)}{(\overline{\rho}_B + k_s)^2}. \end{aligned} \tag{24}$$

To have the body emerge from water fully, we should have $\overline{W} \ge 0$ at $\overline{z}_c = 1/2$. This gives us a condition to find the critical density $\overline{\rho}_{B,c}$ of the body, below which the body will exit from water completely. By substituting $\overline{W} = 0$ and $\overline{z}_c = 1/2$ into Eq.(23), one can calculate the critical density from $f(1/2) = 0$ to obtain

$$\overline{\rho}_{B,c} = \frac{\lambda}{1/2 + \lambda}. \tag{25}$$

As $\lambda > 1/2$ for an initially fully-submerged body, we have $1/2 < \overline{\rho}_{B,c} < 1$. When $\overline{\rho}_B < \overline{\rho}_{B,c}$, the body will exit the free surface totally. When $\overline{\rho}_{B,c} < \overline{\rho}_B < 1$, the body will exit water partially before it falls down.

3. Results and discussions

We undertake water exit of a body at a small density $\overline{\rho}_B = 0.2$, whose dimensionless minor axis $\overline{a}$=1/8. The submergence parameter $\lambda = 0.55$. According to Eq.(25), one can obtain $\overline{\rho}_{B,c} \approx 0.5238$, and so the body will burst out of the water totally as $\overline{\rho}_B < \overline{\rho}_{B,c}$. The initial total number of elements on the body surface in the meridian plane is taken as $N_b = 40$ and $N_b = 60$ respectively. Elements of equal size $\Delta\overline{l}_b$ are used on the body surface and on the free surface within a prescribed radius from the axis of symmetry. Beyond the prescribed radius, element size increases gradually until it reaches a maximum. As described in section 2.1, the critical distance $\Delta\overline{l}_c$ for the free-surface breakup and the flow detachment is taken as 10% of $\Delta\overline{l}_b$.

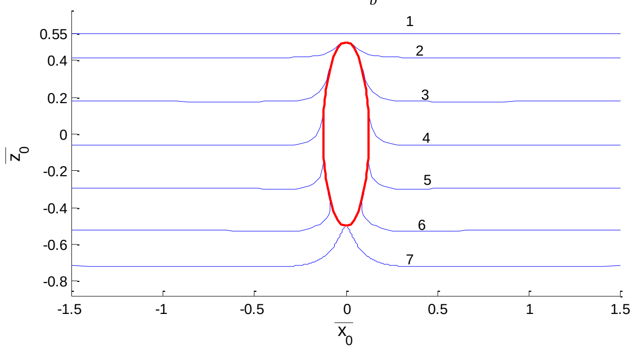

Fig. 2 Free-surface profiles in the $\overline{y}_0 = 0$ plane in the moving system (1): $\overline{t} = 0$, (2): $\overline{t} \approx 0.3045$, (3): $\overline{t} \approx 0.5013$, (4): $\overline{t} \approx 0.6465$, (5): $\overline{t} \approx 0.7763$, (6): $\overline{t} \approx 0.8897$ and (7): $\overline{t} \approx 1.0036$.

Fig.2 provides the free surface at $\bar{y}_0 = 0$ plane in the moving system $O - \bar{x}_0 \bar{y}_0 \bar{z}_0$ with its origin fixed at the centre of the body. The initially stationary body accelerates suddenly under the action of its net vertical force and pushes the water right above upwards and sideway. When the body is close to the free surface, the water layer above the top of the body becomes thinner and thinner. In Fig.2 (2), the very thin water layer has just been removed and the body has just penetrated through the free surface. After that the body continues moving upwards, while the waterline, or the intersection of the free surface and the body surface, stays above the undisturbed free surface. The water detaches from the body at $\bar{t} \approx 1.0036$ in Fig.2 (7). It can be seen a water column below the body has formed at the time of detachment and the free surface is expected to oscillate afterwards under the gravitation. The disturbance will be propagated away in the form of wave radiation.

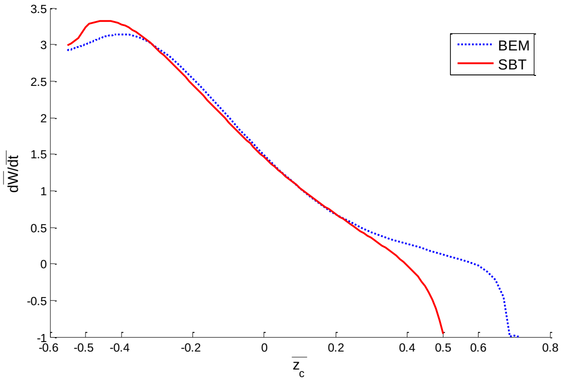

Fig. 3 Acceleration of the body versus the vertical coordinate of the centre of the body

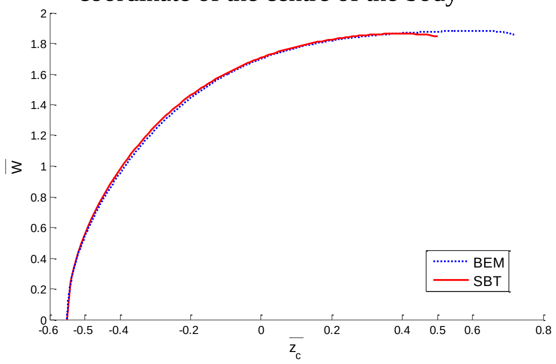

Fig. 4. Velocity of the body versus the vertical coordinate of the centre of the body

We then compare the numerical results of BEM with those from the slender body theory (SBT). Fig.4 presents the variation of acceleration of the body versus the vertical coordinate of the centre of the body, by using SBT and BEM respectively. It can be seen that the results from these two methods agree well, especially in the middle part of the curve which corresponds to post breakup of the free surface. In SBT, the calculation terminates at $\bar{z}_c = 0.5$ when the body departs from the flat free surface. BEM simulation continues because of the rise of the free surface. This is the principal reason why these two curves have a bigger difference at the final stage of water exit. It can be seen that $d\bar{W}/d\bar{t} = -1$ when the body detaches from the water, as the fluid force becomes zero and the external force is gravity only. Fig.4 provides the variation of velocity of the body with its position. The agreement between results from SBT and BEM is even better than that in Fig.3.

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