Added mass and damping of a column with heave plate oscillating in waves

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Introduction

We consider the forced heave motion of a column with a heave plate in the presence of waves. The problem is of relevance to floating offshore wind turbine design, where heave plates are attached to the columns of a semi-submersible in order to improve vertical plane stability and the power output e.g. [5]. Due to the shallow draft of these structures, the heave plates are proximal to the water surface. Numerical studies by [3] have shown that vortices shed by heave plates when executing large amplitude oscillations can disturb an otherwise quiescent free surface. It is of interest then to know how waves on the free surface will alter the hydrodynamic coefficients of the heave plates.

We initially consider a structure that is forced to harmonically oscillate in the vertical direction, Z (heave) in still water. Using Newton's second law, the equation of motion is shown as Eq. 1:

$$F_{33}(t) = (M + A_{33})\ddot{Z} + B_{33}\dot{Z} + K_{33}Z \tag{1}$$

where M and A_{33} are the mass and heave added mass of the body respectively, and B_{33} is the linearized heave damping coefficient. $K_{33} = \rho g A_w$ is the heave hydrostatic restoring coefficient that depends on the water plane area A_w. The non-dimensional hydrodynamic coefficients are defined as:

$$A'_{33} = \frac{A_{33}}{A_{33_{th}}}; B'_{33} = \frac{B_{33}}{2\pi f \cdot A_{33_{th}}} \tag{2}$$

where $A_{33_{th}} = \frac{1}{12}\rho(2D_d^3 + 3\pi D_d^2 z - \pi^3 z^3 - 3\pi D_c^2 z)$ is the theoretical added mass for a column with a disk attached at the bottom [7]. D_c and D_d are the column and disk diameters respectively, and $z = \frac{1}{\pi}\sqrt{D_d^2 - D_c^2}$.

The two dimensionless parameters of relevance to the problem are the Keulegan-Carpenter number $KC = \frac{2 \cdot \pi \cdot Z_0}{D_d}$, and the frequency parameter $\beta = \frac{D_d^2 \cdot f}{\nu}$ introduced by [6]. Here Z_0 is the heave amplitude of oscillation, f is the frequency of oscillation and ν is the kinematic viscosity of the fluid. The damping forces typically have a linear and a quadratic component [8]. By using a linearized damping coefficient, the nonlinear effects are translated into a varying dependence on the coefficients KC and β. A typical forced oscillation experiment in still water can be conducted to evaluate this dependence. When the ambient water is moving such as under the effect of waves, one can resort to a relative velocity formulation, resulting in

$$F_{33}(t) = M\ddot{Z} + A_{33}(\ddot{Z} - \ddot{\xi}) + B_{33}(\dot{Z} - \dot{\xi}) + K_{33}Z + \frac{\pi D_d^2}{4}P_b - \frac{\pi}{4}(D_d^2 - D_c^2)P_t \tag{3}$$

where P_t and P_b respectively represent the wave dynamic pressure acting on the top and bottom of the heave plate, and $\dot{\xi}$ and $\ddot{\xi}$ are the water particle vertical velocity and acceleration respectively. This model is a linearized version of the relative velocity model described in [2].

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Figure 1: Column with a circular heave plate attached to a frame

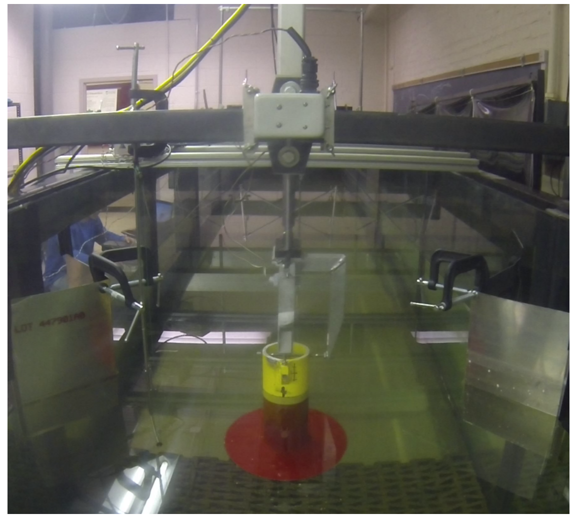

Figure 2: Photo showing the MOOR wave tank with the model and actuator in the foreground. Wave deflectors on either side for still water runs.

Experimental Program

A circular heave plate of diameter 0.25 m and thickness 4.3 mm attached to a column of diameter 0.088 m and draft 0.19 m is considered. This model is a 1:80 scaled version of a demonstration prototype off the coast of Spain reported in [4]. Experiments were performed at the Marine Ocean and Offshore Research (MOOR) wave tank facility at the University of Maine, which is 8 m long and 1 m wide. The water depth for the experiments was kept at 0.7 m. A wedge-shaped plunger type wave maker was installed at one end, and a passive energy absorbing beach at the other end. The wave maker is capable of producing regular waves from 0.5 - 2 s periods and amplitudes ranging from 0.002 - 0.132 m. The beach design was optimized to produce reflection of 5 - 10% over most of the range of testing.

Forced harmonic oscillation of the models in the vertical direction was achieved using a Parker ETH032 linear actuator driven by a 750 W Parker servo motor. Two Omega force sensors were attached by two slender rods to measure the vertical forces (Fig. 1). The heave displacement was measured by a string potentiometer. Output signals were amplified, sampled, and acquired at 1kHz. Using the least squares approach [6], the optimum hydrodynamic coefficients, A_{33} and B_{33}, that minimize the error between the measured force during experiments (F_{exp}) and the heave force (F_{33}) are found. A 32-cycle windowing method described in [1] was used to provide maximum confidence in the added mass and damping evaluation. The first set of experiments were conducted in still water. The model was forced to oscillate over a range of KC values from 0.05 - 1.2 at a frequency of 1 Hz ($\beta = 62251$). To reduce wave reflection from the side walls arising from the structure motion, two triangular wave deflectors were located on each of the tank walls at the heave plate location (Figure 2). This simple device performed satisfactorily as evidenced by visual examination of the sinusoidal nature of the force time histories. The second set of experiments were conducted in waves. The model was forced to oscillate at a frequency of 1 Hz and two KC values of 0.5 and 0.84. The wave frequency was set at 1 Hz to match the heave plate oscillation frequency. The wave steepness varied from $H/L = 0.018 - 0.02$. The phase difference between the wave and

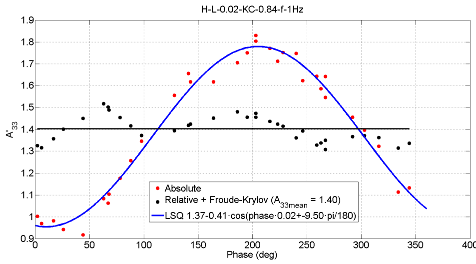

Figure 3: Added mass coefficient vs. phase angle for $H/L = 0.02, KC = 0.84$

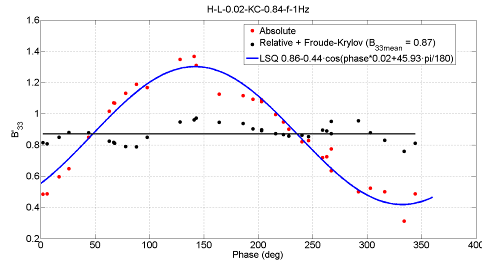

Figure 4: Damping coefficient vs. phase angle for $H/L = 0.02, KC = 0.84$

the platform motion was introduced manually by visual observation of the first three waves from a probe located adjacent to the model. This approach resulted in several runs at different phases ranging over $\alpha = 0^\circ - 360^\circ$.

Results and Discussion

The added mass and damping coefficients for the wave experiments were obtained by two different approaches. In the "Absolute model" approach, Eq. 1 is used in the least squares evaluation. This model is identical to the still water case, and all wave-induced variations were visible in the trends of the coefficients with the phase angle. In the "Relative model" approach, Eq. 3 which incorporates relative kinematics is used. Sample added mass and damping results using the two equation models are shown in Figs. 3 and 4 for different phase angles at $H/L = 0.02$, and $KC = 0.84$. At this KC value, still water added mass and damping values are 1.42 and 0.85 respectively.

When using the absolute model approach, a clear sinusoidal trend is observed with respect to the phase angle. Interestingly, the mean value of this sinusoidal variation matches with the corresponding still water added mass and damping values to within 3%. When the relative flow approach is used, the trend of both coefficients with the phase is much flatter, tending towards a constant value that matches the still water value to within 4%.

The relative phase between the plate and the wave gives rise to an relative change in the KC, although the amplitude of oscillation is kept constant. Thus, we can define a "relative" KC number

$$KC_w = \frac{2 \cdot \pi \cdot A_{rel}}{D_d} \tag{4}$$

A_{rel} is the relative amplitude between the heave plate and the wave, i.e. $Z_{rel}(t) = Z(t) - \eta(t) = A_{rel} \cdot cos(2\pi f t + \alpha)$. This is similar to the relative velocity based KC number mentioned in [2].

Figs. 5 and 6 present the added mass and damping coefficients obtained using the absolute model approach in waves, against KC_w. Results are presented for the cases $KC = 0.84$ and $KC = 0.5$ for a frequency of oscillation of 1Hz and for $H/L = 0.018$ and $H/L = 0.02$. The observed linear trend in the coefficients is remarkable. It can be seen that the added mass and damping coefficients increase as the relative displacement between the plate and the wave particles increases. When compared with the still water coefficients, the added mass coefficients in waves show a steeper linear trend. For small KC, the added mass coefficients in still water are higher. As KC increases, the coefficients in waves become slightly higher than the ones in still water. The damping coefficients in still water and in waves are very similar in slope, with the zero offset showing a difference.

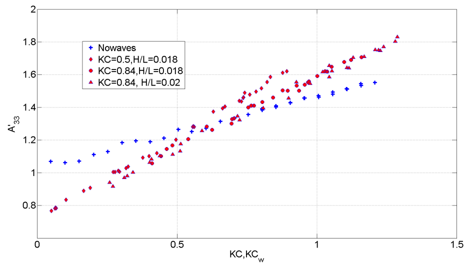

Figure 5: Comparison of the added mass coefficients in waves vs. KC_w with those in still water vs. KC.

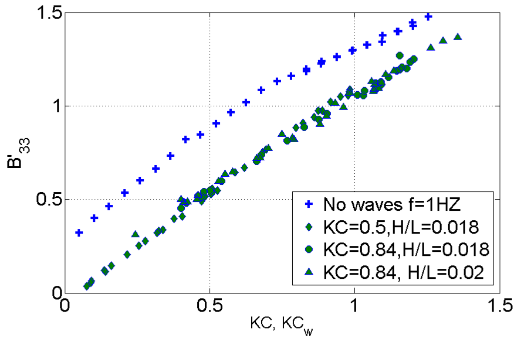

Figure 6: Comparison of the damping coefficients in waves vs. KC_w with those in still water vs. KC.

The results tend to indicate that applying the added mass and damping coefficients obtained from still water experiments for a structure moving in waves may only be agreeable in an averaged sense. For different relative phases of the wave and the motion, large variations could occur. By using a KC that depends on the relative amplitude of motion with respect to the wave, the added mass and damping values are somewhat closer to the still water trends. However, at lower KC values, the added mass coefficients could differ by 30%, which can affect natural frequency estimates. Thus caution needs to be exerted in selection of hydrodynamic coefficients for heave plates oscillating in proximity to the free surface.

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