Highest waves created by a fast ship

Chao Ma, Yi Zhu, Huiyu Wu, Jiayi He, Chenliang Zhang, Wei Li, Francis Noblesse
State Key Lab. of Ocean Eng., School of Naval Architecture, Ocean & Civil Eng.
Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration
Shanghai Jiao Tong University, Shanghai, China; noblfranc@gmail.com

Highlights and main conclusions

• At *high Froude numbers*, constructive interference among the *divergent* waves created by sources and sinks distributed over the hull surface of a monohull ship, or a catamaran, steadily advancing in calm water of large depth results in *highest* waves along ray angles located inside the cusps of the Kelvin wake.
• These highest waves are much *shorter* than the transverse and divergent waves along the cusps of the wake. E.g., for Froude numbers $1.5 < F$, the waves along the cusps of the Kelvin wake of a ship are more than ten times longer than the highest waves created by the ship.
• The highest waves created by a fast ship are also significantly shorter than the *ship length*. E.g., the wavelength of the highest waves created by a catamaran is equal to the distance between the twin bows of the catamaran if $1 < F$.
• The ray angles of the highest ship waves are only *weakly* influenced by the hull shape. Similarly, the wavelength of the highest waves is not significantly affected by the hull shape. Thus, the location (ray angle) and the wavelength of the highest ship waves mostly are *kinematic* flow features.
• The ray angles where the highest-waves are found, and the corresponding wavelengths, are determined via *simple analytical relations* in terms of the Froude number and, for a catamaran, the distance between the twin bows of the catamaran. These relations hold for *all monohull ships and catamarans*.
• These *general results* about the highest waves created by a fast ship widely expand the classical results about far-field ship waves obtained by Kelvin in 1887 for a ship modeled as a *one-point* wavemaker.

1. Kelvin's 1-point wavemaker ship model and wave-interference effects

The farfield waves created by a monohull ship of length L, or a catamaran with two identical demi-hulls of length L separated by a lateral distance S, that advances at constant speed V along a straight path in calm water of large depth are considered. The Froude number F based on the ship length L, and the Froude number F_S based on the spacing S between the twin hulls of the catamaran, are defined as

$$F \equiv V/\sqrt{gL} \equiv F_S\sqrt{s} \text{ and } F_S \equiv V/\sqrt{gS} \equiv F/\sqrt{s} \text{ where } s \equiv S/L \tag{1}$$

and g denotes the gravitational acceleration.

Well-known basic features of ship waves have been explained by Kelvin in 1887 via a classical analysis in which a ship is modeled as a one-point wavemaker. This simple model is adequate to explain main features of ship waves, notably that a ship wake contains transverse and divergent waves within a wedge with half angle $\psi^K \approx 19°28'$ from the track of the ship. However, within the framework of linear potential flow theory, considered by Kelvin and here, the flow around a ship hull is represented via a hull-surface distribution of sources and sinks, as well known. Kelvin's one-point wavemaker model cannot account for interference among the waves created by sources and sinks distributed over a ship hull surface. In fact, longitudinal interference among the divergent waves created by sources and sinks distributed over the bow and stern regions of a ship hull surface, as well as lateral interference among the divergent waves created by sources (or sinks) distributed over the port and starboard sides of a hull surface or the twin hulls of a catamaran, are shown in [1-3] to have very important effects on the appearance of far-field ship waves at high Froude numbers. Indeed, constructive interference among divergent waves created by sources and sinks distributed over a ship hull surface results in highest waves along rays that are located inside the cusps $\psi = \pm\psi^K$ of the Kelvin wake at high Froude numbers.

2. Highest ship waves and related apparent wake angles

Specifically, the systematic numerical computations, for seven ship hulls that correspond to broad ranges of main hull-shape parameters (beam/length, draft/length, beam/draft, waterline entrance angle) at ten Froude numbers within the range $0.6 < F \le 1.5$, reported in [2] shows that the highest waves created by a monohull ship are found at ray angles $\psi = \pm\psi^M$ with

$$\psi^M \approx \psi^K \text{ for } F \le F_K^M \approx 0.573 \tag{2a}$$

$$\psi^M \approx \arctan(0.116/F^2) \text{ for } F_K^M \le F \le 0.85 \tag{2b}$$

$$\psi^M \approx \arctan[0.08\,(1+0.6/F)/F] \text{ for } 0.85 \le F \tag{2c}$$

The apparent wake angle ψ^M, associated with the highest waves created by a monohull ship, given by (2) is equal to the Kelvin cusp angle ψ^K at low Froude numbers $F \le F_K^M$. However, the wake angle ψ^M is a rapidly decreasing function $\psi^M(F)$ of the Froude number F for high Froude numbers $F_K^M < F$.

Similar computations are reported in [3] for catamarans with two identical demi-hulls, Froude numbers $F_S \equiv V/\sqrt{gS}$ within the range $0.4 \leq F_S \leq 3.5$, hull spacings $s \equiv S/L$ within the range $0.2 \leq s \leq 0.8$, and seven demi-hulls that correspond to a broad range of main hull-shape parameters. These systematic computations show that the highest waves created by a catamaran are found along two sets of ray angles $\psi = \pm\psi^i$ and $\psi = \pm\psi^o$ with $\psi^i \leq \psi^o \leq \psi^K$. The inner wake angle $\psi^i(F_S, s)$ is approximately given by the relations

$$\psi^i \approx \psi^K \ \text{ for } \ F_S \leq F_{SK}^i \approx 0.35 + 0.135/s - 0.01/s^2 \tag{3a}$$

$$\psi^i \approx \psi_*^i \approx \arctan(0.2/F_S) \ \text{ for } \ F_S^i \approx 0.13 + 0.47/s \leq F_S \tag{3b}$$

$$\psi^i \approx \psi_*^i + 50\,[0.47 + (0.13 - F_S)\,s]^2 \ \text{ for } \ F_{SK}^i \leq F_S \leq F_S^i \tag{3c}$$

Similarly, the outer wake angle $\psi^o(F_S, s)$ is given by the relations

$$\psi^o \approx \psi^K \ \text{ for } \ F_S \leq F_{SK}^o \approx 1.1 + 0.04/s \tag{4a}$$

$$\psi^o \approx \psi_*^o \approx \arctan(0.37/F_S) + 0.02\,(0.64/s^2 - 1) \ \text{ for } \ F_S^o \approx 1.14 + 0.06/s^2 \leq F_S \tag{4b}$$

$$\psi^o \approx \psi_*^o + 22\,[0.6 + (11.4 - 10\,F_S)\,s^2]^3 \ \text{ for } \ F_{SK}^o \leq F_S \leq F_S^o \tag{4c}$$

The angles ψ^i and ψ_*^i in (3) and the angles ψ^o and ψ_*^o in (4) are expressed in degrees.

The inner and outer wake angles ψ^i and ψ^o are equal to the Kelvin cusp angle ψ^K for 'slow' catamarans at Froude numbers $F_S \leq F_{SK}^i(s)$ or $F_S \leq F_{SK}^o(s)$. However, the inner and outer wake angles ψ^i and ψ^o are decreasing functions $\psi^i(F_S, s)$ and $\psi^o(F_S, s)$ of the Froude number F_S for 'fast' catamarans at Froude numbers $F_{SK}^i < F_S$ or $F_{SK}^o < F_S$. Moreover, the relation (3b) shows that the inner wake angle ψ^i does not depend on the hull spacing s, i.e. only depends on F_S, for 'very fast' catamarans at Froude numbers $F_S^i(s) \leq F_S$. Likewise, the hull spacing s only has a very small (almost negligible) influence in the relation (4b), and the outer wake angle ψ^o therefore mostly depends on F_S for 'very fast' catamarans at Froude numbers $F_S^o(s) \leq F_S$. For Froude numbers within the transition ranges $F_{SK}^i < F_S < F_S^i$ or $F_{SK}^o < F_S < F_S^o$, the relations (3c) and (4c) show that the wake angles $\psi^i(F_S, s)$ and $\psi^o(F_S, s)$ depend on both the Froude number F_S and the hull spacing s.

[3] shows that the waves along the inner wake angle $\psi = \psi^i$ are significantly higher than the waves along the outer wake angle $\psi = \psi^o$ for high Froude numbers F and/or large hull spacings s, i.e. for fast and/or wide catamarans. Specifically, [3] show that the ratio $r \equiv a^i/a^o$ of the amplitudes a^i and a^o of the divergent waves at the inner and outer wake angles $\pm\psi^i$ or $\pm\psi^o$ is larger than 3, 4 or 5 if

$$F_3 \approx 1.1 - 0.2/s + 0.065/s^2 \leq F \ \ , \ \ F_4 \approx 1.5 - 0.5/s + 0.16/s^2 \leq F \ \ , \ \ F_5 \approx 2 - 1/s + 0.4/s^2 \leq F \tag{5}$$

The relations (5) define alternative boundaries $F_n(s)$ of a fast and/or wide catamaran regime where the highest waves are most likely found along the inner ray angles $\pm\psi^i$. Moreover, the inner wake angle ψ^i is given by (3b) if $F_S^i \leq F_S$ or equivalently if

$$F^i \approx 0.13\sqrt{s} + 0.47/\sqrt{s} \leq F \tag{6}$$

This relation defines the boundary $F^i(s)$ of a class of fast and/or wide catamarans for which lateral interference effects between the twin hulls of the catamarans are dominant. In fact, the relation (3b) corresponds to the highly-simplified model of a catamaran as a two-point wavemaker (two point sources at the twin bows of the catamaran) considered in [1].

The shape of the demi-hulls of a catamaran is found in [3] to have a notable influence upon the ratio r of the *amplitudes* of the dominant waves found along the inner and outer ray angles $\pm\psi^i$ and $\pm\psi^o$. However, the hull shape (for a monohull ship as well as a catamaran) only has a weak influence upon the wake angles ψ^M, ψ^i and ψ^o. Indeed, the relations (2) for monohull ships and the relations (3) and (4) for catamarans do not involve the hull shape. Thus, the angles $\pm\psi^M$, $\pm\psi^i$ and $\pm\psi^o$ of the highest waves created by fast monohull ships or catamarans mostly correspond to a *kinematic* flow feature that is little affected by the hull shape. E.g., the waves created by a monohull ship with beam/length ratio $2b$ are (approximately) twice as high as the waves due to a similar ship with beam/length ratio b, but the highest waves created by the wide ship and the thin ship (at the same Froude number) are found along nearly the same ray angles.

3. Wavelengths of the highest waves created by fast ships

The wavelengths λ^M, λ^i and λ^o of the highest ship waves found along the ray angles $\pm\psi^M$, $\pm\psi^i$ and $\pm\psi^o$ are also of practical interest and are considered, and compared to the wavelength $\lambda^C = 4\pi F^2/3$ of the waves at the cusps $\pm\psi^K$ of the Kelvin wake and the (nondimensional) ship length $\ell \equiv 1$, in [4] and below. It is easily shown [4,5] from the dispersion relation and elementary stationary-phase considerations that waves with wavelength λ are found at a ray angle ψ given by the basic relation

$$\tan\psi = \sqrt{\lambda'(1 - \lambda')}/(2 - \lambda') \ \text{ where } \ \lambda' \equiv \lambda/\lambda_0 \ \text{ and } \ \lambda_0 \equiv 2\pi F^2 \tag{7}$$

is the wavelength of the longest (transverse) waves created by a ship along its track $\psi = 0$. The ray angle ψ defined by (7) is real for $0 \le \lambda' \le 1$. Moreover, ψ increases from 0 to the Kelvin angle ψ^K for $0 \le \lambda' \le 2/3$, and decreases from ψ^K to 0 for $2/3 \le \lambda' \le 1$. The 'short' waves $0 \le \lambda' \le 2/3$ and the 'long' waves $2/3 \le \lambda' \le 1$ correspond to the divergent waves and the transverse waves, respectively, in the Kelvin wake. The wavelength λ^C at the cusps $\psi = \pm\psi^K$ of the Kelvin wake corresponds to $\lambda' = 2/3$.

For the range $\lambda \le \lambda^C$ that corresponds to divergent waves, considered hereafter, (7) yields

$$\lambda^D = \lambda_0 \, (8 \tan^2\psi)/(1 + 4 \tan^2\psi + \sqrt{1 - 8 \tan^2\psi}) \le \lambda^C = 2\lambda_0/3 \tag{8}$$

The relation (8) determines the wavelength λ^D of the divergent waves found at a ray angle ψ. This relation, with ψ taken as $\psi^M(F)$, $\psi^i(F_S, s)$ or $\psi^o(F_S, s)$ determines the wavelengths $\lambda^M(F)$, $\lambda^i(F, s)$ and $\lambda^o(F, s)$ of the highest waves found along the ray angles $\pm\psi^M$ for a monohull ship or the inner and outer ray angles $\pm\psi^i$ and $\pm\psi^o$ for a catamaran, for which the relations (1) between F and F_S can be used.

The relations (2c), (3b) and (4b), as well as Fig.1, show that the highest-waves angles ψ^M, ψ^i and ψ^o are small for large Froude numbers. The relation (8) then yields

$$\lambda^D \approx 4 \lambda_0 \tan^2\psi = 8\pi F^2 \tan^2\psi = 6 \, \lambda^C \tan^2\psi \tag{9}$$

This approximation and the relations (2c), (3b), (4b) yield the high Froude number approximations

$$\lambda^M \approx 0.16 \, (1 + 0.6/F)^2 \approx 0.16 \quad , \quad \lambda^i \approx s \quad , \quad \lambda^o \approx 3.44 \, s \qquad \text{for } 1 \ll F \tag{10}$$

The wavelengths λ_0 and λ^C of the waves created by a ship at its track $\psi = 0$ or at the cusps $\psi = \pm\psi^K$ of the Kelvin wake *increase* (rapidly) as the Froude number F increases, and indeed are larger than the ship length for $0.4 \le F$ or $0.49 \le F$, whereas the wavelengths of the highest waves created by a monohull ship or a catamaran *decrease* as F increases. The high Froude number limit 0.16 of the highest-waves wavelength λ^M is approximately equal to the (nondimensional) beam of common monohull ships and is significantly smaller than 1 (the nondimensional ship length). The high Froude number limit s of the inner highest-waves wavelength λ^i is also smaller than 1 for a typical catamaran with hull spacing $s < 1$. The high Froude number limit $3.44 \, s$ of the outer highest-waves wavelength λ^o is greater than 1 for a 'wide' catamaran with hull spacing $0.29 < s$. However, the waves at the inner wake angles $\pm\psi^i$ dominate the waves at the outer wake angles $\pm\psi^o$ for fast and wide catamarans, as already noted. Thus, the highest waves created by high-speed monohull ships and catamarans are significantly shorter than the ship length.

4. Ray angles and wavelengths of the highest ship waves

The left side of Fig.1 shows the Kelvin cusp angle $\psi^K \approx 19°28'$, the region $\psi \le \psi^{min} \approx 6°23'$ where divergent waves are more than twenty times shorter than the transverse waves at the track $\psi = 0$ of a ship, and the three wake angles ψ^M, ψ^i and ψ^o related to the highest waves created by monohull ships (thick solid black line) or catamarans with hull spacings $s = 0.2$ (solid and dashed green lines), $s = 0.35$ (red lines) or $s = 0.5$ (blue lines). This figure also depicts the Froude number $F^i(s)$ that is defined by (6) and determines the fast and/or wide catamaran regime where the simple two-point wavemaker model considered in [1] is accurate, as well as the three Froude numbers $F_3(s)$, $F_4(s)$ and $F_5(s)$ that are defined by (5) and correspond to the values $r = 3, 4, 5$ of the ratio $r \equiv a^i/a^o$ of the amplitudes a^i and a^o of the waves at the inner and outer rays $\psi = \pm\psi^i$ and $\psi = \pm\psi^o$. The wavelengths $\lambda^M(F)$, $\lambda^i(F, s)$ and $\lambda^o(F, s)$ of the highest waves created by monohull ships (thick solid black line) or catamarans with $s = 0.2$ (solid and dashed green lines), $s = 0.35$ (red lines) or $s = 0.5$ (blue lines) are depicted on the right side of Fig.1,

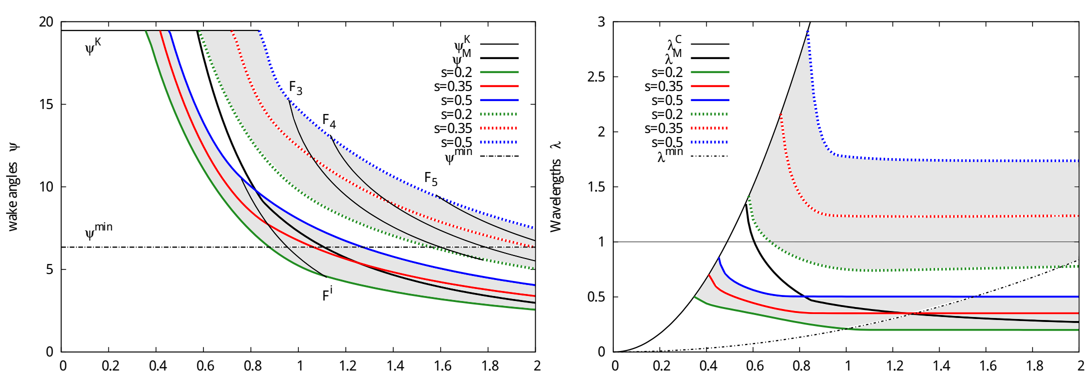

Figure 1: Ray angles ψ^K, ψ^M, ψ^i, ψ^o, ψ^{min} (left) and wavelengths λ^C, λ^M, λ^i, λ^o, λ^{min} (right) related to the highest waves created by monohull ships or catamarans with hull spacings $s = 0.2$, 0.35 and 0.5.

where the wavelength λ^C at the cusps of the Kelvin wake and the wavelength $\lambda^{min} = \varepsilon\,\lambda_0$ with $\varepsilon = 0.05$ are also shown. The two shaded regions in Fig.1 (left and right sides) correspond to the inner or outer wake angles ψ^i or ψ^o for catamarans with hull spacings $0.2 \leq s \leq 0.5$.

Fig.1 shows that the wake angle ψ^M related to the highest waves created by a monohull ship decreases, rapidly within the approximate range $F_K^M \approx 0.573 \leq F \leq 0.85$, from the Kelvin angle ψ^K as the Froude number F increases within the range $F_K^M \leq F$. The corresponding wavelength λ^M similarly decreases, rapidly for $F_K^M \leq F \leq 0.85$, from the wavelength λ^C at the cusps of the Kelvin wake toward the high-speed limit $\lambda^M \approx 0.16$ given by (10). Fig.1 also illustrates how the inner and outer wake angles ψ^i and ψ^o related to the highest waves created by catamarans decrease from the Kelvin angle ψ^K as F increases beyond the thresholds defined by (3a) and (4a). The wavelengths λ^i and λ^o similarly decrease from the cusp wavelength λ^C to the high speed limits $\lambda^i \approx s$ and $\lambda^o \approx 3.44\,s$ given by (10). Except for relatively small values of the Froude number, the wavelengths λ^M and λ^i are significantly smaller than the ship length $\ell = 1$. Lastly, Fig.1 shows that the wavelength λ^M for monohull ships, as well as the wavelength λ^i for catamarans with hull spacing $s \approx 0.35$, are smaller than the wavelength $\lambda^{min} = 0.05\,\lambda_0$ for Froude numbers greater than approximately 1.3, i.e. for moderately large Froude numbers.

5. Hull-surface source/sink distributions and 2-point wavemaker models

The relations (2), (3) and (4) for the angles ψ^M, ψ^i and ψ^o of the highest ship waves are obtained in [2,3] via parametric numerical computations for a realistic flow model based on hull-surface distributions of sources and sinks. These relations and the relation (8) that determines the corresponding wavelengths λ^M, λ^i and λ^o therefore provide realistic theoretical predictions of the angles ψ^M, ψ^i and ψ^o and the related wavelengths λ^M, λ^i and λ^o for monohull ships and catamarans. These wake angles and the related wavelengths are compared in [4] to the corresponding approximate results obtained in [1] via an elementary analysis based on a highly-simplified flow model that approximates a hull-surface distribution of sources and sinks as a two-point wavemaker.

The comparison given in [4], to be presented at the Workshop, shows that wave-interference effects and the resulting highest waves for fast monohull ships or catamarans can be largely understood and usefully analyzed via two complementary basic two-point wavemaker models associated with longitudinal and lateral interference. Indeed, the two-point wavemaker approximation of a hull-surface distribution of sources and sinks used in the elementary analysis given in [1] provides basic insight and moreover yields reasonable, although not accurate, approximations to the wake angles and the wavelengths of the highest waves created by monohull ships or catamarans. Specifically, the two-point wavemaker model of a monohull ship yields reasonable approximations at moderately large Froude numbers, for which longitudinal interference effects are dominant. The two-point wavemaker model of a catamaran is accurate at Froude numbers greater than about 1, for which lateral interference effects are dominant. However, the elementary two-point wavemaker model of a catamaran considered in [1] is not accurate for F smaller than about 0.9, for which longitudinal interference effects cannot be ignored. The simple analysis of ship waves in deep water considered in [1] is expanded for the more general, and considerably more complicated, case of ship waves in uniform finite water depth in [7,8].

References

[1] F. Noblesse, J. He, Y. Zhu, L. Hong, C. Zhang, R. Zhu, C. Yang (2014) Why can ship wakes appear narrower than Kelvin's angle?, Eur. J. Mech. B Fluids 46:164-171
[2] C. Zhang, J. He, Y. Zhu, C-J. Yang, W. Li, Y. Zhu, M. Lin, F. Noblesse (2015) Interference effects on the Kelvin wake of a monohull ship represented via a continuous distribution of sources, Eur. J. Mech. B Fluids 51:27-38
[3] J. He, C. Zhang, Y. Zhu, L. Zou, W. Li, F. Noblesse (2016) Interference effects on the Kelvin wake of a catamaran represented via a hull-surface distribution of sources, Eur. J. Mech. B Fluids, 56:1-12
[4] C. Ma, Y. Zhu, J. He, C. Zhang, H. Wu, W. Li, F. Noblesse (2016) Wavelengths of the highest waves created by fast monohull ships or catamarans, J. Ocean Eng. 113:208-214
[5] J. He, C. Zhang, Y. Zhu, H. Wu, C.-J. Yang, F. Noblesse, X. Gu, W. Li (2015) Comparison of three simple models of Kelvin ship wake, Eur. J. Mech. B Fluids 49:12-19
[6] F. Noblesse, C. Zhang, J. He, Y. Zhu, C.J. Yang, W. Li (2015) Observations and computations of narrow Kelvin ship wakes, J. Ocean Engineering & Science, 1:1-14
[7] Y. Zhu, J. He, C. Zhang, H. Wu, D. Wan, R. Zhu, F. Noblesse (2015) Farfield waves created by a monohull ship in shallow water, Eur. J. Mech. B Fluids 49:226-234
[8] Y. Zhu, F. Noblesse (2016) Highest waves created by a monohull ship or a catamaran in shallow water, *submitted*