Abstract for 31st IWWWFB, Michigan, USA - April 3-6, 2016

A multi-domain method for the computation of wave loads

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Highlights

- A multi-domain method combining the simple Green function and free-surface Green function is developed.
- The velocity potential and its normal derivative on the control surface are expressed as Fourier-Laguerre series. The external solution provides the $\left[\mathbf{D}^2_{\mathbf{N}}\right]$ operator.
- Irregular frequencies are removed by using the extended BIEs method where the dipole distribution over the internal free surface is expanded into Fourier-Bessel series.

1 Introduction

In order to develop a reliable and efficient tool for the seakeeping problem with forward speed, Ten & Chen [1] proposed a hybrid method combining the simple Green function and free-surface Green function. The fluid domain is divided by a control surface into two subdomains: the internal domain and external domain. The simple Green function is applied in the internal domain where the ship is present, while the free-surface Green function is adopted in the external domain. The method combines the advantages of both the Rankine panel method and that based on the free-surface Green function. The difficulties associated with peculiar properties of complex singularities [2] are solved by the integration of the free surface Green function on the control surface. Furthermore, the internal domain is finite and thus the Rankine panel method is well suited and efficient.

Different from the work by Ten & Chen [1] in which the control surface is in the form of hemisphere, we prefer a cylindrical surface as shown in Figure 1 (a) on which we keep the independence of horizontal and vertical variables. The velocity potential and its radial derivative are represented by a linear combination of base functions composed of Laguerre functions in vertical coordinate and Fourier series in polar angle. The external solution provides the relationship between the velocity potential and its normal derivative on the control surface referred to as the *Dirichlet to Neumann* operator $\left[\mathbf{D}^2_{\mathbf{N}}\right]$. The simple Green function is applied in the internal domain limited by the hull surface, the control surface and the part of free surface between the hull surface and control surface.

This study consists of establishing the $\left[\mathbf{D}^2_{\mathbf{N}}\right]$ operator on the control surface by the boundary integral equation (BIE) in the external domain, and formulating the BIE on the hull surface, part of free surface and the control surface on which unknowns are coefficients of base functions. The point collocation is adopted on the hull surface and free surface, while the collocation of Galerkin type is applied on the control surface. In addition, irregular frequencies caused by the external solution are removed by the extended BIEs [3] via introducing an internal free surface as depicted in Figure 1 (b), and the dipole distribution over the internal free surface is expanded into Fourier-Bessel series.

2 External solution free of irregular frequencies

A 3D Cartesian coordinate system $Oxyz$ is introduced with the xy plane coinciding with the undisturbed free surface and Oz axis orienting positively upwards. We assume the fluid is inviscid, incompressible and flow is irrotational. The normal vector $\boldsymbol{n}$ is defined positively inwards to the fluid. In the external domain, the boundary surfaces include the control surface C, the free surface F_E and the surface at infinity S_∞. Application of Green's identity in the external domain yields:

$$\phi = \iint_{C+F_E+S_\infty} \left(G^F\phi_n - \phi G^F_n\right) \mathrm{d}S = \iint_C \left(G^F\phi_n - \phi G^F_n\right) \mathrm{d}S \tag{1}$$

where G^F denotes the free-surface Green function which can be found in [4]. However, the application of the BIE (1) will result in the occurrence of irregular frequencies. According to the work by Zhu [3], irregular frequencies can be removed by the extended BIEs through introducing an internal free surface I as illustrated in Figure 1 (b). In the present problem, the internal free surface is a circular disc, and then the extended BIEs are expressed as [3]:

$$\begin{aligned} \frac{1}{2}\phi + \iint\!\!\!\!\!\!\!-_{C} \phi G_n^F \mathrm{d}S + \iint_I \mu' G_n^F \mathrm{d}S = \iint_C G^F \phi_n \mathrm{d}S \quad \text{on} \quad C \\ \mu' + \iint_C \phi G_n^F \mathrm{d}S + \iint\!\!\!\!\!\!\!-_{I} \mu' G_n^F \mathrm{d}S = \iint_C \phi_n G^F \mathrm{d}S \quad \text{on} \quad I \end{aligned} \tag{2}$$

where μ' denotes the dipole strength distributed over the internal free surface which can be expanded as Fourier-Bessel series. In addition, the velocity potential ϕ and its normal derivative $\psi = \phi_n$ on the control surface can be expressed as Fourier-Laguerre expansions:

$$\{\phi\,|\psi\}(\varphi, z) = \sum_{k=0}^{\infty}\sum_{l=-\infty}^{\infty}\{\phi_{kl}\,|\psi_{kl}\}\,\mathcal{L}_k(-z)\,e^{\mathrm{i}l\varphi}, \quad \mu'(r,\varphi) = \sum_{p=-\infty}^{\infty}\sum_{q=1}^{\infty}\mu'_{pq}J_p(\alpha_{pq}r/\mathcal{R})\,e^{\mathrm{i}p\varphi} \tag{3}$$

where α_{pq} is the qth zero of J'_p, and $\mathcal{R}$ is the radius of control surface. Multiplying test functions $\mathcal{L}_m(-z)\,e^{-\mathrm{i}n\varphi}$ and $J_u(\alpha_{uv}r/\mathcal{R})\,e^{-\mathrm{i}u\varphi}$ on both sides of the first and second BIEs in (2) and integrating over the control surface and internal free surface by means of Galerkin collocation, we have

$$\begin{aligned} \pi\phi_{mn} + \sum_{k=0}^{\infty}\sum_{l=-\infty}^{\infty}\phi_{kl}\mathcal{H}_{mn,kl}^{CC} + \sum_{p=-\infty}^{\infty}\sum_{q=1}^{\infty}\mu'_{pq}\mathcal{H}_{mn,pq}^{CI} = \sum_{k=0}^{\infty}\sum_{l=-\infty}^{\infty}\psi_{kl}\mathcal{G}_{mn,kl}^{CC} \\ \pi\mathcal{R}^2\frac{\alpha_{uv}^2 - u^2}{\alpha_{uv}^2}J_u^2(\alpha_{uv})\,\mu'_{uv} + \sum_{k=0}^{\infty}\sum_{l=-\infty}^{\infty}\phi_{kl}\mathcal{H}_{uv,kl}^{IC} + \sum_{p=-\infty}^{\infty}\sum_{q=1}^{\infty}\mu'_{pq}\mathcal{H}_{mn,pq}^{II} = \sum_{k=0}^{\infty}\sum_{l=-\infty}^{\infty}\psi_{kl}\mathcal{G}_{uv,kl}^{IC} \end{aligned} \tag{4}$$

with

$$\begin{aligned} \{\mathcal{G}_{mn,kl}^{CC}\,|\mathcal{H}_{mn,kl}^{CC}\} &= \int_{-\pi}^{\pi}\int_{-\infty}^{0}\iint\!\!\!\!\!\!\!-_{C}\{G^F\,|G_n^F\}\,\mathcal{L}_m(-z)\,\mathcal{L}_k(-\zeta)\,e^{\mathrm{i}l\varphi^\star}e^{-\mathrm{i}n\varphi}\mathrm{d}S\mathrm{d}z\mathrm{d}\varphi \\ \mathcal{H}_{mn,pq}^{CI} &= \int_{-\pi}^{\pi}\int_{-\infty}^{0}\iint_I G_n^F\mathcal{L}_m(-z)\,J_p(\alpha_{pq}r^\star/\mathcal{R})\,e^{\mathrm{i}p\varphi^\star}e^{-\mathrm{i}n\varphi}\mathrm{d}S\mathrm{d}z\mathrm{d}\varphi \\ \mathcal{H}_{uv,pq}^{II} &= \int_0^{\mathcal{R}}\int_{-\pi}^{\pi}\iint\!\!\!\!\!\!\!-_{I} G_n^F r J_u(\alpha_{uv}r/\mathcal{R})\,J_p(\alpha_{pq}r^\star/\mathcal{R})\,e^{\mathrm{i}p\varphi^\star}e^{-\mathrm{i}u\varphi}\mathrm{d}S\mathrm{d}\varphi\mathrm{d}r \\ \{\mathcal{G}_{uv,kl}^{IC}\,|\mathcal{H}_{uv,kl}^{IC}\} &= \int_0^{\mathcal{R}}\int_{-\pi}^{\pi}\iint_C\{G^F\,|G_n^F\}\,rJ_u(\alpha_{uv}r/\mathcal{R})\,\mathcal{L}_k(-\zeta)\,e^{\mathrm{i}l\varphi^\star}e^{-\mathrm{i}u\varphi}\mathrm{d}S\mathrm{d}\varphi\mathrm{d}r \end{aligned} \tag{5}$$

Rewriting the extended BIEs in (4) in a concise form yields

$$\begin{bmatrix} \pi E + \mathcal{H}^{CC} & \mathcal{H}^{CI} \\ \mathcal{H}^{IC} & \lambda E + \mathcal{H}^{II} \end{bmatrix} \cdot \begin{Bmatrix} \phi \\ \mu' \end{Bmatrix} = \begin{Bmatrix} [\mathcal{G}^{CC}]\cdot\{\psi\} \\ [\mathcal{G}^{IC}]\cdot\{\psi\} \end{Bmatrix} \tag{6}$$

with $\lambda_{uv} = \pi\mathcal{R}^2\left(1 - u^2/\alpha_{uv}^2\right)J_u^2(\alpha_{uv})$. By rearranging equation (6), we can obtain

$$\left[\mathbf{D}_{\mathbf{N}}^2\right] = \left[\left[\mathcal{G}^{CC}\right] - \left[\mathcal{H}^{CI}\right][T]^{-1}\left[\mathcal{G}^{IC}\right]\right]^{-1}\cdot\left[\left[\pi E + \mathcal{H}^{CC}\right] - \left[\mathcal{H}^{CI}\right][T]^{-1}\left[\mathcal{H}^{IC}\right]\right] \tag{7}$$

with $[T] = \left[\lambda E + \mathcal{H}^{II}\right]$. The $\left[\mathbf{D}_{\mathbf{N}}^2\right]$ operator in equation (7) provides the relationship between the velocity potential and its normal derivative on the control surface: $\psi = \left[\mathbf{D}_{\mathbf{N}}^2\right]\langle\phi\rangle$.

3 Internal problem

In the internal domain, the hull surface H and free surface F are panelized and divided into N_H and N_F elements, respectively, while the control surface C is not panelized. Thus, application of the Green's identity in the internal domain yields

$$\phi = \left(\sum_{\alpha=1}^{N_H} \iint_{H^\alpha} + \sum_{\beta=1}^{N_F} \iint_{F^\beta} + \iint_C \right) \left(G^S \phi_n - \phi G_n^S \right) \mathrm{d}S \tag{8}$$

where G^S represents the simple Green function defined as $G^S = -1/(4\pi r)$. On the control surface in the internal domain C, the velocity potential and its normal derivative are consistent with the external domain, so that they can also be expanded as a double series of base functions as presented in equation (3). Substituting equation (3) into (8) and using the linear free-surface condition yields

$$\begin{aligned} \frac{1}{2}\phi = & -\sum_{l=-\infty}^{\infty} \sum_{k=0}^{\infty} \psi_{kl} \iint_C \mathcal{L}_k(-\zeta) e^{\mathrm{i}l\varphi^\star} G^S \mathrm{d}S - \sum_{l=-\infty}^{\infty} \sum_{k=0}^{\infty} \phi_{kl} \iint_C \mathcal{L}_k(-\zeta) e^{\mathrm{i}l\varphi^\star} G_n^S \mathrm{d}S \\ & + \sum_{\alpha=1}^{N_H} \psi_\alpha \iint_{H^\alpha} G^S \mathrm{d}S - \sum_{\alpha=1}^{N_H} \phi_\alpha \iint_{H^\alpha} G_n^S \mathrm{d}S - \sum_{\beta=1}^{N_F} \phi_\beta \iint_{F^\beta} \left(k_0 G^S + G_n^S \right) \mathrm{d}S \end{aligned} \tag{9}$$

where k_0 is wavenumber, and ϕ_α and ϕ_β denote the velocity potential on the hull surface and free surface, respectively. Applying the BIE (9) on the control surface and integrating the test function $\mathcal{L}_m(-z) e^{-\mathrm{i}n\varphi}$ over the control surface in the sense of Galerkin collocation, the BIE becomes

$$\pi\phi_{mn} = -\sum_{l=-\infty}^{\infty} \sum_{k=0}^{\infty} \left(\mathcal{G}_{mn,kl}^S \psi_{kl} + \mathcal{H}_{mn,kl}^S \phi_{kl} \right) + \sum_{\alpha=1}^{N_H} \left(\psi_\alpha \mathcal{G}_{mn,\alpha}^{CH} - \phi_\alpha \mathcal{H}_{mn,\alpha}^{CH} \right) - \sum_{\beta=1}^{N_F} \phi_\beta \left(k_0 \mathcal{G}_{mn,\beta}^{CF} + \mathcal{H}_{mn,\beta}^{CF} \right) \tag{10}$$

with
$$\left\{ \mathcal{G}_{mn,\alpha(\beta)}^{CH} \middle| \mathcal{H}_{mn,\alpha(\beta)}^{CH} \right\} = \int_{-\infty}^{0} \int_{-\pi}^{\pi} \iint_{H^\alpha(F^\beta)} \left\{ G^S \middle| G_n^S \right\} \mathcal{L}_m(-z) e^{-\mathrm{i}n\varphi} \mathrm{d}S \mathrm{d}\varphi \mathrm{d}z \tag{11}$$

Then, application of the BIE (9) on the hull surface and free surface on the collocation point of each element, we have

$$\frac{\phi}{2} = -\sum_{l=-\infty}^{\infty} \sum_{k=0}^{\infty} \left(\mathcal{G}_{\gamma,kl} \psi_{kl} + \mathcal{H}_{\gamma,kl} \phi_{kl} \right) + \sum_{\alpha=1}^{N_H} \iint_{H^\alpha} \left(\psi_\alpha G^S - \phi_\alpha G_n^S \right) \mathrm{d}S - \sum_{\beta=1}^{N_F} \phi_\beta \iint_{F^\beta} \left(k_0 G^S + G_n^S \right) \mathrm{d}S \tag{12}$$

with
$$\left\{ \mathcal{G}_{\gamma,kl} \middle| \mathcal{H}_{\gamma,kl} \right\} = \iint_C \left\{ G^S \middle| G_n^S \right\} \mathcal{L}_k(-\zeta) e^{\mathrm{i}l\varphi^\star} \mathrm{d}S \tag{13}$$

Combining BIEs (10) and (12), we can obtain a linear equation system with unknowns including the potential on the hull surface ϕ_α, potential on the free surface ϕ_β and expansion coefficients on the control surface ϕ_{kl}. Then, the wave loads exerting on the floating body can be calculated.

4 Results and discussion

Figure 2 demonstrates the surge and heave added mass $(a_{11}, a_{33}) = (A_{11}, A_{33}) / \left(2\pi\rho R_0^3/3\right)$ and damping $(b_{11}, b_{33}) = (B_{11}, B_{33}) / \left(2\pi\rho\omega R_0^3/3\right)$ coefficients of a floating hemisphere with the variation of $k_0 R_0$ where R_0 is the radius of the hemisphere. Comparison is made with the analytical solution by Hulme [5]. In Figure 2, the results calculated by the original BIE deviate significantly from the analytical solution near locations of irregular frequencies associated with the radius of the control surface. Nevertheless, the extended BIEs give the smooth results across irregular frequencies and they are in good agreement with the analytical solution.

Furthermore, the introduction of the control surface enables us to evaluate the second-order wave drift loads using the middle-field formulation [6] due to the fact that we have the coefficients of the Fourier-Laguerre expansions. The expansion coefficients for the total potential are expressed as $(\phi_{kl}, \psi_{kl}) = (\phi_{0kl}, \psi_{0kl}) + (\phi_{7kl}, \psi_{7kl})$, where ϕ_{7kl} are obtained by solving the diffraction problem, and ψ_{7kl} are achieved through the $\left[\mathbf{D}_\mathbf{N}^2\right]$ operator. Finally, the wave drift force can be calculated analytically by integrating products of Laguerre functions and Fourier series over the control surface.

5 Conclusions

A multi-domain method combining the simple Green function and free-surface Green function is proposed. The free-surface Green function is adopted in the external problem to provide the $[\mathbf{D}_{\mathbf{N}}^2]$ operator on the control surface. The matrices associated with the internal problem in (11) independent on wave frequencies can be evaluated only once for a given ship geometry and different frequencies. Furthermore, irregular frequencies are present and dependent on the radius of the control surface, and they can be removed by adopting the extended BIEs in which the dipole distribution over the internal free surface is expanded into Fourier-Bessel series.

References

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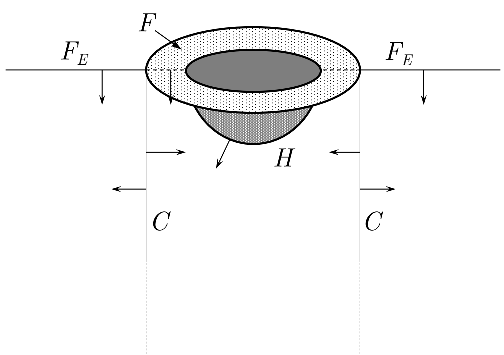
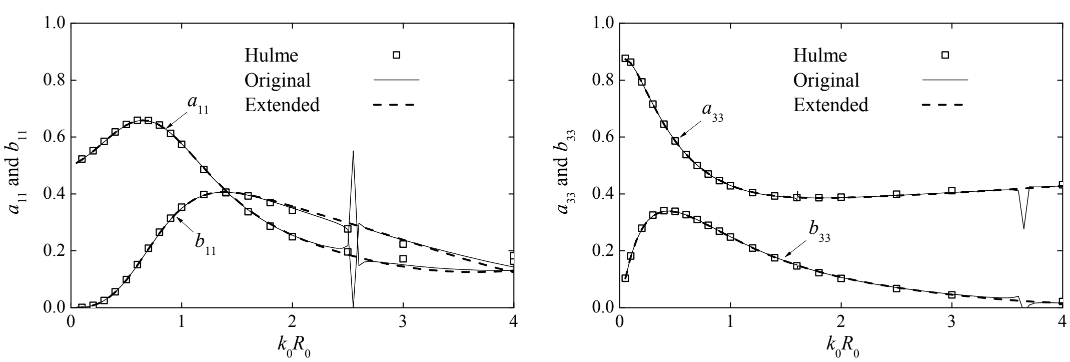


(a) (b)

Figure 1: Schematic of a fluid domain divided by a cylindrical control surface



(a) (b)

Figure 2: Surge and heave added mass and damping of a floating hemisphere varying with k_0R_0.