

Deformations of an elastic clamped plate in uniform flow and due to jet impact

A.A. Korobkin¹, T.I. Khabakhpasheva¹, S. Malenica²

¹ School of Mathematics, University of East Anglia, Norwich, UK,

e-mail: t.khabakhpasheva@uea.ac.uk, a.korobkin@uea.ac.uk

² Bureau Veritas, Marine & Offshore Division - Research Department, France,

sime.malenica@bureauveritas.com

The present study is concerned with the bending stresses in a clamped elastic plate subject to liquid jet impact. The stresses during the initial impact stage are estimated and compared with the static stresses in the plate placed in the equivalent steady jet flow. It is shown that the static stresses are always smaller than the bending stresses during the early stage of impact for a given speed and thickness of the jet. This implies that if the stresses in the plate during the unsteady impact stage are smaller than the yield stress of the plate material and no plastic deformations in the plate occur, then the plate will behave elastically after the impact, without plastic deformations. Appearance of plastic deformations is treated here as a damage to the plate. It is shown that maximum stresses in the clamped plate always occur at the clamped end of the plate.

In this analysis the jet is two-dimensional and of constant thickness H . The liquid is incompressible and inviscid. An elastic plate is clamped to the flat bottom with another end being free of stresses and shear forces. The jet has a flat front and approaches the plate at a constant speed. Pressure-impulse theory and fully coupled approach is used to evaluate the bending stresses in the plate during the early stage of impact. Decoupled approach is used to evaluate the static stresses in the plate at large times.

The plate is clamped to a flat bottom (Figure 1) and the jet moves at constant speed V along the bottom towards the plate. The jet thickness H can be smaller or greater than the plate length L . The jet flow can be caused by liquid impact onto a rigid wall, by breaking wave impact onto a vertical wall, or it can be a model of long wave propagating along a dry bed. In practical problems, the jet is of variable thickness and the jet (wave) front is not parallel to the wall at the impact time instant. However, the present analysis still can be used if the jet thickness varies slowly and the jet speed is large enough.

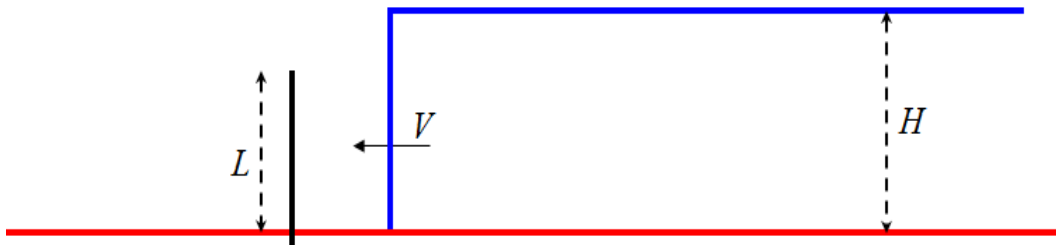


Fig. 1 The configuration of the problem before jet impact.

We shall evaluate the bending stresses in the plate caused by the jet and to compare them with the yield stress σ_Y for the material of the plate. If the induced stresses exceed the yield stress, this is treated as the plate damage. In this case, plastic deformations in the plate occur and the plate cannot return to its initial shape after the hydrodynamics loads disappear. Both the short-term and long-term interactions of the plate with the jet flow are studied. In the long-term analysis, the jet flow and the plate deflections are stationary, the hydrodynamic loads are much smaller than during the initial impact stage but last longer. In the short-term analysis, duration of which is of

the order of the period of the first mode of the plate vibration, hydrodynamic loads are impulsive and the plate is likely to be damaged if the jet speed is large enough. It is shown in the present study that, if the plate was not damaged during the impact stage, it will be not damaged in the steady jet flow.

Coupled problem of jet impact

The two-dimensional problem of elastic plate interaction with the jet flow is considered in the Cartesian coordinate system xOy with the origin at the plate end which is clamped to the bottom. The flow is two-dimensional and potential. The gravity and surface tension effects are not taken into account in this jet impact problem. The plate deflections are described by the Euler beam equation. The plate is of constant thickness h_p and isotropic. The plate-jet interaction is described by the equations of hydrodynamics and elastic theory written in non-dimensional variables. The plate length L is taken as the length scale, VL as the scale of the velocity potential, $\rho LV/T$ as the scale of the hydrodynamic pressure during the impact stage and VT as the scale of the plate deflection. Here $T = 2\sqrt{3}L^2/(h_p c_p)$ is the time scale, $c_p = (E/\rho_p)^{1/2}$ is the so-called bar velocity, E is the Young modulus and ρ_p is the density of the plate material. The non-dimensional parameter $\alpha = \rho L/(\rho_p h_p)$ indicates importance of the added mass of the plate, which is proportional to the product ρL , compared to the structural mass $\rho_p h_p$ per unit length of the plate.

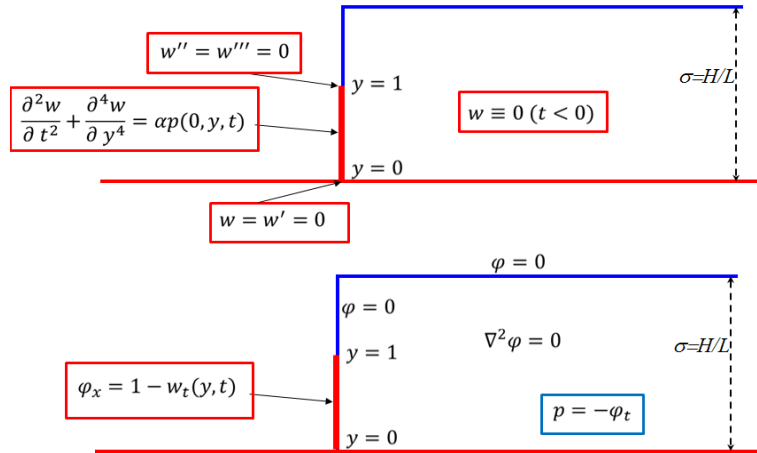


Fig. 2 The structural and hydrodynamic problems of jet impact onto elastic plate.

During the initial impact stage, duration of which is of the order $O(1)$ in the non-dimensional variables, the boundary conditions are linearised and imposed on the position of the liquid boundary just before impact. The flow after the impact, $t > 0$, is described by the velocity potential $\varphi(x, y, t)$. The plate deflection is described by the equation $y = -w(y, t)$. If the jet thickness is smaller than the plate length, $\sigma < 1$, this coupled problem of hydroelasticity has been studied in [1]. In this case the hydrodynamic pressure is applied only along the wetted part of the plate, $0 < y < \sigma$. Note that both the non-linear terms in the equations of motions and boundary conditions are neglected during the early stage of impact. There are two non-dimensional parameters in the problem, α and σ . The plate rigidity does not appear in this formulation but only in the scaling. If the parameter α is small, then $w = O(\alpha)$ and the plate deflection can be neglected in the hydrodynamic problem, see Figure 2. Then the hydrodynamic pressure at leading order is the pressure acting on an equivalent rigid plate. Within this decoupled approximation, when the hydrodynamic loads are independent of the plate deflections, the stresses in the plate can be readily calculated. The parameter α indicates how closely the hydrodynamic and structural problems are coupled.

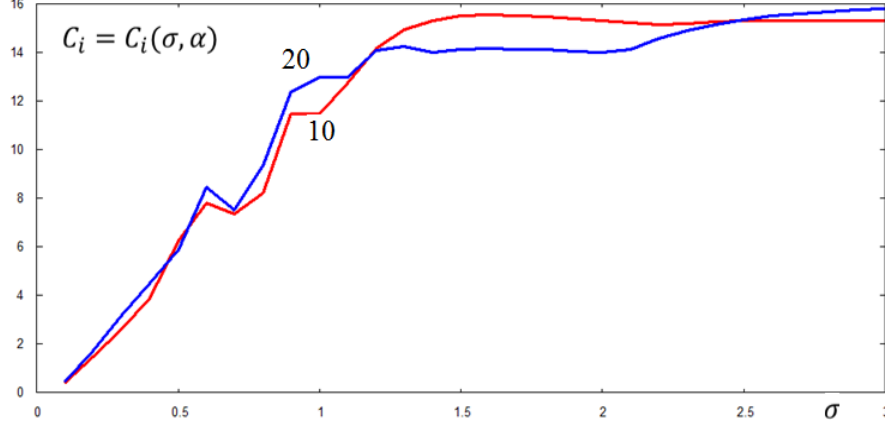


Fig. 3 The function $C_i(\sigma, \alpha)$ computed with 10 and 20 modes for $\alpha = 11.1$ and $0 < t < 2$.

Calculations in this study are performed for aluminium plate of length $L = 3\text{cm}$, thickness $h_p = 1\text{mm}$, density $\rho_p = 2700\text{kg/m}^3$, Young's module $E = 70 \times 10^9 \text{ N/m}^2$, and the yield stress $\sigma_Y = 414 \text{ MPa}$. The liquid density is taken as $\rho = 1000\text{kg/m}^3$. For this conditions we calculate $T \approx 0.6\text{ms}$ and $\alpha \approx 11.1$, which implies that the coupling between the hydrodynamic loads and the plate response is rather strong. The problem of jet impact is solved by the normal mode method [2]. The bending stresses on the surface of the plate are calculated by the formula

$$\sigma(y, t) = -\sqrt{3}\rho_p c_p V w_{yy}(y, t),$$

where $\sqrt{3}\rho_p c_p \approx 23.8 \text{ MPa} \cdot \text{s/m}$ for the aluminium plate. The stresses are proportional to the impact velocity V . The absolute maximum of the stresses σ_{\max} is given by

$$\sigma_{\max} = 23.8VC_i(\sigma, \alpha)(\text{MPa}), \quad C_i(\sigma, \alpha) = \max_{0 < y < 1, t > 0} |w_{yy}(y, t)|,$$

where the jet speed is in meters per second. By using $\sigma_Y = 414 \text{ MPa}$, we find that plastic deformations in the plate occurs when the impact velocity V is greater than

$$V > (17.4/C_i)\text{m/s}.$$

The function $C_i(\sigma, \alpha)$ is shown in Figure 3. It is seen that $C_i < 16$. For given characteristics and dimensions of the plate, the minimum speed of the jet $V_Y^{(i)}(\sigma)$, which leads to plastic deformations, depends only on the non-dimensional jet thickness σ . Here subscript (i) stands for the critical speed calculated during the impact stage. The minimum value of $V_Y^{(i)}(\sigma)$ is estimated as 1.0875 m/s .

Decoupled problem of plate deflection in uniform flow

The stresses in the elastic plate during the impact stage are expected to be high due to impulsive hydrodynamic loading of large intensity. After the impact stage the loads are smaller but they last longer. We do not consider here a periodic vibration of the plate in steady jet. It is possible that the clamped plate can survive the impact loads without plastic deformations, but is deformed plastically later on, when the loads are stationary. In order to investigate is it possible or not, we consider the steady problem of elastic clamped plate placed in steady uniform flow. This problem can be viewed as the jet/plate interaction problem for infinite thickness of the jet. A reason for such an approximation comes from the analysis of hydrodynamic force acting on a rigid plate placed in steady jet flow, it is shown in [3] that the total force is weakly dependent on the jet thickness H

when $H > L$, and the assumption that the maximum bending stress is a monotonic function of the jet thickness. We also assume that the deflection of the elastic plate in a steady uniform flow is small and the hydrodynamic pressures along the plate can be approximated by their values calculated for an equivalent rigid plate. In dimensional variables, which are denoted by primes, the pressure distribution along the plate is given in parametric form by the formulae (see [3])

$$p'(0, y') = \frac{1}{2}\rho V^2 P(\xi), \quad P(\xi) = 1 - \xi^2(1 + \sqrt{1 - \xi^2})^{-2},$$

$$y'/L = \frac{2}{4 + \pi}(2\xi + \xi\sqrt{1 - \xi^2} + \arcsin \xi),$$

where $0 < \xi < 1$, V is the flow velocity and ρ is the liquid density. Here $\xi = 0$ corresponds to the clamped end of the plate, $y' = 0$, where the pressure is equal to the stagnation pressure $\frac{1}{2}\rho V^2$, and $\xi = 1$ corresponds to the free end of the plate, $y' = L$, where the pressure is equal to the ambient pressure taken as zero in this analysis.

Small static deflection $w'(y')$ of the clamped plate is described by the Euler beam equation with the corresponding end conditions

$$EJ \frac{d^4 w'}{dy'^4} = p'(0, y') \quad (0 < y' < L),$$

$$w'(0) = 0, \quad \frac{dw'}{dy'}(0) = 0, \quad \frac{d^2 w'}{dy'^2}(L) = 0, \quad \frac{d^3 w'}{dy'^3}(L) = 0.$$

The pressure is positive, therefore the third derivative of the deflection is negative and the second derivative is positive. The maximum stress is achieved at $y' = 0$, where

$$\sigma_{\max} = \sigma(0) = C_s \rho V^2 (L/h_p)^2.$$

Calculations yield

$$C_s = \frac{3\pi^2/16 + 2}{(\pi/4 + 1)^2} \approx 1.208.$$

Note that the static maximum stress σ_{\max} is independent of the plate rigidity and is proportional to the flow speed squared. The maximum stress σ_{\max} exceeds the yield stress of the plate material, $\sigma_Y = 414$ MPa, for $V > 19.5$ m/s. The obtained value of the jet velocity is about twenty times larger than the minimum speed of the jet $V_Y^{(i)}(\sigma)$, which leads to plastic deformations of the plate during the early impact stage.

We conclude that, if a clamped elastic plate was not damaged by jet impact, it will not be damaged in the later stages of the plate/fluid interaction.

Acknowledgment: Third author acknowledges the support of the National Research Foundation of Korea (NRF) grant funded by the Korea Government (MEST) through GCRC-SOP.

References

1. Korobkin, A.A., Malenica, S. (2007) Steep wave impact onto elastic wall. Proc. 22nd Intern. Workshop on Water Waves and Floating Bodies, Plitvice, Croatia, 4 pp.
2. Korobkin, A.A. (1998) Wave impact on the center of an Euler beam. Journal of Applied Mechanics and Technical Physics. Vol. 39. No. 5, pp. 770-781.
3. Gurevich, M. I. (2014). The Theory of Jets in an Ideal Fluid: International Series of Monographs in Pure and Applied Mathematics (Vol. 93). Elsevier.