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**Wave scattering by semicircular porous breakwater placed on a porous seabed**

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**Highlights**

- Oblique gravity wave scattering by a semicircular porous breakwater located on a porous bed is studied.
- The physical problem is handled for solution using a coupled eigenfunction expansion-boundary element method.
- To understand the effectiveness of the breakwater, reflection and transmission coefficients, wave forces acting on the breakwater are computed and analyzed for various values of wave and structural parameters.

1. Introduction

In recent decades, there is an increasing interests to use porous breakwaters to dissipate a major part of the incoming wave energy for protection of coastal facilities and infrastructures. One such effective structure is the semicircular caisson breakwater which was initially built at Miyazaki Port, Japan in early 1990s. In 1997, a very long semicircular shape breakwater was built in Tianjin port of China and a long submerged semicircular caisson jetties was constructed in the Yangtze River Estuary (see Zhang et al. (2005)). This type of structures mainly consists of two components, namely a bottom slab and an permeable semicircular arc. One of the advantage of this types of structures are that these structures are light in weight which often suitable for soft soil foundation. Moreover, these structures are highly stable against sliding and the overturning moment caused by high incoming wave impact is almost negligible. These structures are often used as offshore-detached breakwaters. To reduce the impact of wave load by energy dissipation, permeable semicircular breakwaters are proposed. Moreover, the permeable breakwaters will decrease the reflection and transmission of the incident wave energy which helps in the reduction of seabed scouring in front of breakwaters. In the present study, scattering of oblique wave by a perforated semicircular breakwater placed on a porous bed is analyzed. The fluid flow is governed by the linearized theory of water waves whilst, flow within the porous bed is governed by the Sollitt and Cross model and Darcy’s law is used to model the flow past the permeable semicircular arc (see Koley et al. (2015a), Koley et al. (2015b)). The mathematical boundary value problem is handled for solution using a coupled eigenfunction expansion-boundary element method. Several results of physical interests are computed and analyzed to study the effectiveness of the semicircular breakwater in scattering of obliquely incident gravity waves.

2. Mathematical formulation

In the present study, the problem is analyzed in the 3D Cartesian coordinate system under the assumption of small amplitude water wave theory. The x-y plane is considered as the horizontal plane and the z-axis is assumed to be positive in the upward direction. The perforated semicircular breakwater, consists of a permeable semicircular arc and an impermeable rigid bottom slab (as in Fig. 1), is placed on a porous sea bed which occupies the region \{-l \leq x \leq r,-\infty < y < \infty\}. For modeling purpose, the physical domain is divided into two regions, namely the inner region \{-l \leq x \leq r\} and the outer region \{-\infty < x < l \cup r < x < \infty\} respectively. The outer region is further divided into two sub-regions, namely \(R_1 = \{x < -l,-h < z < 0\}\) and \(R_2 = \{x > r,-h < z < 0\}\). In presence of the breakwater, the fluid domain within the inner region is divided into three sub-regions: namely the fluid domain outside the breakwater confined within the range \(-h < z < 0\), the fluid domain inside the breakwater and within the porous sea bed, and are referred as regions \(R_3, R_4\) and \(R_5\) respectively. Assuming the fluid motion is simple harmonic in time with the angular frequency \(\omega\) and the incident wave is propagating by making an angle \(\theta\) with the x-axis, the velocity potentials \(\Phi_j(x, y, z, t)\) exist and are assumed to be of the forms \(\Phi_j(x, y, z, t) = \text{Re}\{\phi_j(x, z)e^{ik_jy+\omega t}\}\) with the subscripts \(j\) referring to the fluid domains \(R_j\) for \(j = 1, 2, 3, 4, 5\). Thus, the spatial velocity potentials \(\phi_j\)s satisfy

\[
\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - k_j^2\right)\phi_j = 0.
\]
along with the linearized free surface boundary condition

\[ \frac{\partial \phi_j}{\partial z} = K \phi_j \quad \text{on} \quad z = 0 \quad \text{for} \quad j = 1, 2, 3, \]

where \( K = \omega^2/g \) and \( g \) is the acceleration due to gravity. The bottom boundary condition is given by

\[ \frac{\partial \phi_j}{\partial n} = 0 \quad \text{on} \quad z = d_i(x) \quad \text{for} \quad j = 1, 2, 5 \]

where \( \partial/\partial n \) denotes the normal derivative, with \( d_i(x) \) being given by

\[ d_i(x) = \begin{cases} -h & x < -l \cup x > r \\ -l & \leq x \leq r \end{cases} \]

with \( d_i(x) \) being the bottom rigid bed profile of region \( R_i \). At the interface between the regions \( R_i \) and \( R_j \), the continuity of pressure and mass flux yield

\[ \phi_3 = (m - if)\phi_5, \quad \frac{\partial \phi_3}{\partial n} = \epsilon \frac{\partial \phi_5}{\partial n} \quad \text{on} \quad \Gamma_{b1} \cup \Gamma_{b3}, \]

where \( m \) and \( f \) are the inertial and friction coefficients respectively and \( \epsilon \) is the porosity of the porous seabed. The boundary condition on the perforated semicircular arc is given by

\[ \frac{\partial \phi_3}{\partial n} = -\frac{\partial \phi_4}{\partial n} = ik_0G_0 (\phi_3 - \phi_4) \quad \text{on} \quad \Gamma_3, \]

where \( G_0 \) is the porous-effect parameter as defined in Koley et al. (2015a). At the rigid bottom slab of the breakwater, the no flow condition gives

\[ \frac{\partial \phi_4}{\partial n} = \frac{\partial \phi_5}{\partial n} = 0 \quad \text{on} \quad \Gamma_{b2}. \]

Assuming the two auxiliary boundaries \( \Gamma_1 \) and \( \Gamma_\alpha \) situated at \( x = -l \) and \( x = r \) respectively, the continuity of pressure and normal velocity on \( \Gamma_1 \) and \( \Gamma_\alpha \) yield

\[ \phi_3 = \begin{cases} \phi_1, & \text{on} \quad \Gamma_1, \\ \phi_2, & \text{on} \quad \Gamma_\alpha, \end{cases} \quad \text{and} \quad \frac{\partial \phi_3}{\partial n} = \begin{cases} \frac{\partial \phi_1}{\partial n}, & \text{on} \quad \Gamma_1, \\ \frac{\partial \phi_2}{\partial n}, & \text{on} \quad \Gamma_\alpha. \end{cases} \]

Finally, the far-field boundary condition is given by

\[ \begin{cases} \frac{\partial (\phi_1 - \phi_{inc})}{\partial x} + i q_0 (\phi_1 - \phi_{inc}) = 0, & \text{as} \quad x \to -\infty \\ \frac{\partial \phi_2}{\partial x} - i q_0 \phi_2 = 0, & \text{as} \quad x \to \infty \end{cases} \]

with \( \phi_{inc} \) being the incident wave potential and is given by \( \phi_{inc} = e^{ik_0(x+iy)}f_0(k_0, z) \) with \( q_0 = \sqrt{k_0^2 - k_y^2} \) being the \( x \)-component of the wave number \( k_0 \) satisfying \( \omega^2 = gk_0 \tanh k_0h \) and \( f_0(k_0, z) \) being the vertical eigenfunction in the open water region \( R_f \) for \( j = 1, 2 \).

3. Solution via coupled eigenfunction expansion - boundary element method

The boundary value problem (BVP) in the physical domain is reduced to two different domains namely, the inner and outer regions. The eigenfunction expansion method is employed to derive the velocity potentials in the outer

\[ \text{Fig. 1. Schematic diagram of perforated semicircular breakwater placed on a porous seabed.} \]
region whilst, the BVP in the inner region is converted into a system of integral equations using Green’s integral theorem. Finally, matching the velocity and pressure on the interface boundaries between the inner and outer regions, the system of integral equations are handled for solution numerically.

3.1. Contribution from the outer region

In the outer region \( R_1 \), the spatial velocity potential \( \psi_1 \), satisfying Eq. (1) along with the boundary conditions as in Eqs. (2), (3) and (9) can be expressed in the form

\[
\psi_1(x, z) = e^{i\omega(x + l)}f_0(k_0, z) + \sum_{n=0}^{\infty} R_n e^{-i\omega(x + l)}f_n(k_n, z), \quad \text{for} \; x < -l, \; -h < z < 0, \tag{10}
\]

where the eigenfunctions \( f_n(k_n, z) \)s for \( n = 0, 1, 2, 3... \) are given by

\[
f_n(k_n, z) = \left( \frac{-	ext{igA}}{\omega} \right) \frac{\cosh k_n(z + h)}{\cosh k_nh}.	ag{11}
\]

where \( A \) is the incident wave amplitude, \( q_n = \sqrt{k_n^2 - k_0^2} \) and \( k_n \) satisfies the dispersion relation \( K = -k_n \tan k_nh \) for \( n = 1, 2, ... \). The eigenfunctions \( f_n(k_n, z) \)s satisfy the orthogonal relation given by

\[
\langle f_m(k_m, z), f_n(k_n, z) \rangle = \int_{-h}^{0} f_m(k_m, z)f_n(k_n, z) \, dz = \mathcal{A}_n \delta_{mn}, \tag{12}
\]

where \( \delta_{mn} \) is the Kronecker delta function with

\[
\mathcal{A}_n = -\left( \frac{g^2A^2}{\omega^2} \right) \frac{2k_nh + \sinh (2k_nh)}{4k_n \cosh^2 (k_nh)}.	ag{13}
\]

Now, assuming \( \psi_1 = \tilde{\psi}_1 \) at \( x = -l \) and taking the inner product as in Eq. (12) over the functions \( \psi_1(-l, z) \) and \( f_n(k_n, z) \), and using the identity as in Eq. (12), it is derived that

\[
R_n + \delta_{n0} = \frac{1}{\mathcal{A}_n} \langle \tilde{\psi}_1(z), f_n(k_n, z) \rangle. \tag{14}
\]

Substituting for \( R_n \) from Eq. (14) in Eq. (10) and truncating the series after \( N \)-terms, the normal derivative of the potential \( \psi_1(x, z) \) yields

\[
\frac{\partial \psi_1}{\partial n} \bigg|_{x=-l} = \mathbf{Q}_1 \left[ \tilde{\psi}_1(z) \right] - 2iq_0f_0(k_0, z), \tag{15}
\]

where \( \mathbf{Q}_1 \left[ \tilde{\psi}_1(z) \right] \) is given by

\[
\mathbf{Q}_1 \left[ \tilde{\psi}_1(z) \right] = \sum_{n=0}^{N} \frac{iq_n}{\mathcal{A}_n} \langle \tilde{\psi}_1(z), f_n(k_n, z) \rangle f_n(k_n, z). \tag{16}
\]

Proceedings in a similar manner as in region \( R_1 \), the spatial velocity potential \( \psi_2 \) and its normal derivative in region \( R_2 \) can be expressed as

\[
\psi_2(x, z) = \sum_{n=0}^{\infty} T_n e^{i\omega(x - r)}f_n(k_n, z), \quad \text{for} \; x > r, \; -h < z < 0, \tag{17}
\]

\[
\frac{\partial \psi_2}{\partial n} \bigg|_{x=r} = \mathbf{Q}_2 \left[ \tilde{\psi}_2(z) \right], \tag{18}
\]

with \( \mathbf{Q}_2 \left[ \tilde{\psi}_2(z) \right] \) is given by

\[
\mathbf{Q}_2 \left[ \tilde{\psi}_2(z) \right] = \sum_{n=0}^{N} \frac{iq_n}{\mathcal{A}_n} \langle \tilde{\psi}_2(z), f_n(k_n, z) \rangle f_n(k_n, z). \tag{19}
\]
3.2. Integral equation formulation for inner region

Applying Green’s second identity to the velocity potential $\phi(x, z)$ and the free space Green’s function $G(x, z; x_0, z_0)$ over the domain $\Omega$ bounded by $\Gamma$, the following integral equation is obtained as

$$
-\left( \frac{\phi(x, z)}{z\phi(x, z)} \right) = \int_{\Gamma} \left( \frac{\partial G}{\partial n} - G \frac{\partial \phi}{\partial n} \right) d\Gamma, \quad \text{if} \ (x, z) \in \Omega \text{ but not on } \Gamma \ \text{if} \ (x, z) \text{ on } \Gamma.
$$

(20)

In Eq. (20), the Green’s function $G(x, z; x_0, z_0)$ satisfies the partial differential equation

$$
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) G = \delta(x - x_0) \delta(z - z_0),
$$

(21)

and is given by

$$
G(x, z; x_0, z_0) = -\frac{K_0(kr)}{2\pi}, \quad r = \sqrt{(x - x_0)^2 + (z - z_0)^2},
$$

(22)

where $K_0$ is the modified zeroth-order Bessel function of the second kind with $r$ being the distance from the field point $(x, z)$ to the source point $(x_0, z_0)$. As $r \to 0$, i.e., when the source point $(x_0, z_0)$ coincides with the field point $(x, z)$, the asymptotic behavior of the Bessel function $K_0(kr)$ is given by

$$
K_0(kr) = -\gamma - \ln\left(\frac{k_r}{2}\right),
$$

(23)

where $\gamma = 0.5772$ is known as Euler’s constant. Further, it may be noted that in case of normalized incident waves, i.e., when $\theta \to 0$, $K_0(kr)$ approaches to $-\ln(r)$. Using the boundary conditions (2) - (9) in the regions $R_3$, $R_4$ and $R_5$, the integral equation in Eq. (20) is rewritten as

$$
C\phi_3 + \int_{\Gamma_1} \left( \phi_1 \frac{\partial G_3}{\partial n} + G_3 \frac{\partial \phi_1}{\partial n} \right) d\Gamma + \int_{\Gamma_1} \left( \frac{\partial G_3}{\partial n} - KG_3 \right) \phi_3 d\Gamma + \int_{\Gamma_1} \left( \phi_2 \frac{\partial G_3}{\partial n} + G_3 \frac{\partial \phi_2}{\partial n} \right) d\Gamma
$$

$$
+ \int_{\Gamma_{31} + \Gamma_{33}} \left( (m - if) \phi_3 \frac{\partial G_3}{\partial n} + eG_3 \frac{\partial \phi_3}{\partial n} \right) d\Gamma + \int_{\Gamma_{31}} \left( \frac{\partial G_3}{\partial n} - i\kappa_0 G_0 G_3 \right) \phi_3 d\Gamma + i\kappa_0 G_0 \int_{\Gamma_{31}} \phi_4 G_3 d\Gamma = 0,
$$

(24)

$$
C\phi_4 + \int_{\Gamma_1} \left( \phi_1 \frac{\partial G_4}{\partial n} - i\kappa_0 G_0 G_4 \right) \phi_4 d\Gamma + i\kappa_0 G_0 \int_{\Gamma_{14}} \phi_5 G_4 d\Gamma + \int_{\Gamma_{12}} \phi_4 \frac{\partial G_4}{\partial n} d\Gamma = 0,
$$

(25)

$$
C\phi_5 + \int_{\Gamma_{31} + \Gamma_{33}} \left( \phi_3 \frac{\partial G_5}{\partial n} - G_5 \frac{\partial \phi_3}{\partial n} \right) d\Gamma + \int_{\Gamma_{31} + \Gamma_{33}} \phi_5 \frac{\partial G_5}{\partial n} d\Gamma = 0,
$$

(26)

with $C = 0.5$. By discretizing the entire boundaries of the regions $R_3$, $R_4$ and $R_5$ into a finite number of segments called boundary elements and assuming $\phi$ and $\partial \phi / \partial n$ to be constant over each element (as in Koley et al. (2015b)), the system of integral equations in Eqs. (24)-(26) are converted into a linear system of algebraic equations which are solved to get the unknown quantities of interest.

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References

