Improvement of Immersed Boundary Method for Simulation of Fluid-Structure Interaction

Changhong Hu and Cheng Liu

Research Institute for Applied Mechanics, Kyushu University, Fukuoka, Japan E-mail: hu@riam.kyushu-u.ac.jp, liu@riam.kyushu-u.ac.jp

Highlights:

- An improved immersed boundary (IB) method is proposed to increase the accuracy and stability of fluid-structure interaction computation using Cartesian grid method.
- Bilinear and quadratic reconstruction schemes for surface boundary velocities are proposed.

1. Introduction

In Research Institute for Applied Mechanics (RIAM), Kyushu University, a Cartesian grid based method has been developed by Hu and his colleagues [1-3] for numerical simulation of strongly nonlinear free surface problems, i.e., bow flare slamming, water on deck etc. The wave-body interaction problem is treated as a multi-phase flow problem and solved numerically in a stationary Cartesian grid. The free surface and the body boundary are viewed as inner interfaces which do not depend on the grid. CIP method is applied to improve the calculation accuracy at these interfaces. The free surface and the body boundary are treated as immersed interfaces.

The immersed boundary (IB) method is used to treat fluid-structure coupling. Influence of the body on the flow is represented by adding a forcing term to the right hand of the Navier-Stokes equation. The forcing term is obtained by reconstruction of velocity field on the solid body surface. A significant advantage of such Cartesian grid approach is that the grid generation can be greatly simplified especially for moving boundaries. However, since the grid line usually does not conform to the body surface, it is very difficult to carry out local grid refinement to resolve the boundary layers. To increase the accuracy of IB method is one major challenge for the Cartesian grid method. The velocity reconstruction treatment used in the past numerical method [3] is of first order accuracy. Higher order interpolation scheme is required for improving the accuracy of the reconstruction treatment.

Tracking the moving body surface that is immersed in the fluid is another major challenge for the Cartesian grid method. We have proposed a marker particle method [2] for this purpose. Other existing methods are the panel method [4] and the level set (LS) method [5]. The marker particle method, which is attractive for its simple scheme and easy 3-D extension, employs a series of discrete particles to represent the body surface. However, it is difficult to distribute the particles ideally uniform on the body surface for a stretched grid. For the panel method, due to heavy cost in searching for the adjacent grid nodes and complicity in verifying the intersection cases between a panel and the grid lines, its application to 3-D problems is usually very difficult. The level set method is widely used for capturing moving interface shape, in which the signed distance function is defined in the computational domain. A benefit of LS method is that the judgment of whether a node is inside or outside of the body surface as well as the calculation of the surface normal vectors and intersection points can be straightforwardly. However, the traditional re-initialization procedure of the level set function is very cumbersome and costly for a moving body.

The purpose of this study is to improve the accuracy and stability of our Cartesian grid method. Several new improvements have been made on the IB treatment. A virtual panel method is developed for tracking the body. LS method is applied and a simple mapping strategy is newly developed for re-initialization of the level set function, which

can deal with 2-D or 3-D complex surface geometry of rigid or deformable body easily. For velocity reconstruction on the body surface, both bilinear and quadratic interpolation schemes are implemented. In this extended abstract we simply describe important points of those improvements and present a numerical result to demonstrate the performance. More recently obtained results will be presented at the workshop.

2. Improved Immersed Boundary Method

2.1 Body Surface Tracking Strategy

A modified IB treatment combined with LS method has been developed for a moving body with complex geometrical boundaries. For initialization of the signed distance function, a region partition method is proposed to enhance robustness in the sign calculation, in which a bounding box enclosed a triangle panel is used to reduce redundant calculations. To determine whether a point is inside or outside of the surface, we consider a direct method by using the surface normal vectors as shown in Fig. 1(a). The sign is defined as

$$sign = \begin{cases} -1 & \mathbf{n} \cdot \overline{PA} < 0\\ 1 & \mathbf{n} \cdot \overline{PA} > 0 \end{cases}, \tag{1}$$

where **n** is the normal vector of the segment adjacent to point *P*, *A* is the closest point to *P*. If the surface shape is a sharp corner, e.g., 2-D case shown in Fig. 1(a), special treatment is required. In this case, both segment normal vectors \mathbf{n}_1 and \mathbf{n}_2 are adjacent to vertex *A*, however, \mathbf{n}_1 and \mathbf{n}_2 cannot be used for checking relative position of *P* because $\mathbf{n}_1 \cdot \overrightarrow{PA}$ and $\mathbf{n}_2 \cdot \overrightarrow{PA}$ have different sign. This problem can be solved by firstly obtaining normal vector \mathbf{n}_A at the vortex *A*, then using the vortex normal vector to compute the sign by Eq. (2). By using an angle weighted method, the vertex normal \mathbf{n}_A is derived by

$$\mathbf{n}_{A} = \frac{\sum_{k=1...m} \alpha_{k} \mathbf{n}_{k}}{\|\sum_{k=1...m} \alpha_{k} \mathbf{n}_{k}\|}, \qquad (2)$$

where α_k is the angle containing vertex A. The normal vector on the edges can be evaluated with the average of the normal vectors in two adjacent triangles.

For 3-D case, the commonly used CAD stereolithography (STL) model containing three vertexes coordinate values and the face normal vector are used and a region partition method is proposed to specify the sign of ϕ_P (Fig. 1 (b)).



Fig. 1 Method to determine a grid point P inside or outside of the boundary.

For moving bodies, frequently re-initialization of the signed distance field $\phi(\mathbf{x}_i, t)$ is required. Conventional method is to perform an initialization procedure in each time step, which would be cumbersome and costly. To reduce the computation cost, a simple and efficient re-initialization method is developed by directly mapping the signed distance field from the moving reference mesh to the background mesh. For rigid body, the signed distance field is only calculated for one time, and then a mapping strategy will be used to accomplish the re-initialization procedure. In Fig.2, an example of NACA0030 foil profile is shown.



Fig. 2 Mapping method for re-initialization

2.2 Interpolation Scheme for Velocity

In the IB method, the influence of the body is represented by adding a suitable force on the forcing points as shown by solid circles in Fig.3, which are the points belonging to the fluid phase with at least one neighboring solid point. The velocity vectors of forcing points are reconstructed through interpolation using velocities at surrounding fluid grid points and the nearest body surface velocity. A quadratic interpolation scheme can be as follows.

$$u_f = a_1 + a_2 x + a_3 y + a_4 x y + a_5 x^2 + a_6 y^2.$$
(3)

Six points are required for the quadratic interpolation, within which five stencil points belong to the fluid and one point is on the body surface, as shown in Fig. 3.



(a) Definition of stencils (b) Two cases of the interpolationFig.3 Stencils for quadratic interpolation.

3. Numerical Simulation of an Anguilliform Swimmer

Extensive validation of the improved IB method has been carried out. Here as a numerical example to show the performance, simulation of a 3-D flexible structure (anguilliform swimmer) in fluid with prescribed motion is presented. The Reynolds number is $Re = LU/\nu = 2000$, where L is the fish length, U the swimming velocity and ν the kinematic viscosity of water. The Strouhal number St = 2fA/U is defined to describe the frequency of fish tail's swinging motion, where f and A is the frequency and amplitude of the tail beat respectively. St = 0.2 and 0.7 are used

The kinematic form of anguilliform swimmer can be modeled by a backward traveling wave with the wave amplitude increasing almost linearly from the fish head to the tail. The following equation is often used to describe the lateral undulations of the fish

$$h(x,t) = a(x)\sin(kx - wt), \tag{4}$$

where x indicates the distance from head of fish to the end, a(x) the amplitude function, w the angular frequency, $k = L/\lambda$ the wave numbers of the body undulation, λ the wave length. The slip ratio is defined as $\beta = U/V$, where V = w/k is the wave speed. The amplitude envelop a(x) is given by

$$a(x) = a_{max}e^{x-1},\tag{5}$$

where $a_{max} = 0.089$ is used as the maximum amplitude at the fish tail.

The computational domain is a box with dimensions of $6L \times 3L \times L$. Local refined stretched mesh $800 \times 240 \times 160$ is used. A uniform sub-mesh with constant spacing $\Delta x = \Delta y = \Delta z = 0.0025L$ is used to refine the inner box $1.2L \times 0.2L \times 0.1L$, which is enclosed the fish body.



Fig. 4 Pressure contours and streamlines in mid-plane of the fish

Computed flow pattern and pressure distribution are shown in Fig. 4. In Fig. 5 the vertex structures are shown by using *q*-criterion. The present results are compared well to those obtained by other numerical method [6].



Fig. 5 3-D vortex structure visualized by q-criterion for anguilliform swimmer.

References

- C. Hu and M. Kashiwagi. "A CIP-based method for numerical simulations of violent free surface flows", J Mar Sci Technol. Vol. 9 (2004), pp143–157.
- [2] C. Hu and M. Kashiwagi. "Two-dimensional numerical simulation and experiment strongly nonlinear wave-body interactions", J Mar Sci Technol. Vol.14 (2009), pp200–213.
- [3] C. Hu, 3-D Numerical Wave Tank by CIP-based Cartesian Grid Method, Proc. 25th International Workshop on Water Waves and Floating Bodies, Harbin, China, 9-12 May, 2010, pp.61-64.
- [4] A. Gilmanov, F. Sotiropoulos, and E. Balaras, A general reconstruction algorithm for simulating flows with complex 3d immersed boundaries on Cartesian grids, Journal of Computational Physics 191(2003), pp. 660–669.
- [5] J. Choi, R. Oberoi, J. Edwards, J. Rosati, An immersed boundary method for complex incompressible flows, Journal of Computational Physics 224 (2007), pp. 757–784.
- [6] I. Borazjani, F. Sotiropoulos, Numerical investigation of the hydrodynamics of anguilliform swimming in the transitional and inertial flow regimes, Journal of Experimental Biology 212 (2009), pp. 576–592.