Upstream waves at ships moving at low subcritical speed

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Introduction

Very long waves of small amplitude are reported, running ahead of the new super-large cruiseferries sailing along the Oslofjord – an inlet in Norway. The waves are characterized by slow in- and outflows of long periods of 30–60 seconds in shallow bays along the fjord, with considerable run-ups and currents up to 1 m/s. The bays are typically filled before they are almost emptied. The wave phenomenon is connected to the passage of the very large ships and may appear several minutes before the ship is passing. Cruiseferries operate many places in the world. With its tonnage of 75 100 GT, Color Magic of Color Line is currently the largest among the cruiseferries. It operates between Oslo and Kiel and cruises the Oslofjord. The volume of Color Magic is almost the double of an older ship sailing these waters, Pearl Seaways of DFDS. While the new upstream waves are associated with the new, large ferries, the phenomenon is almost not noticable for the old, smaller ships.

Upstream waves generated by moving ships are associated with a finite water depth and have been studied, for single layer fluids, in the form of of solitary waves generated by ships moving close to critical speed, see e.g., Wu (1987). In model tank experiments studying finite depth effects, these waves, moving faster than the shallow water speed, were considered as an unwanted, parasittic effect. The observed upstream wave generation by the cruiseferries differ fundamentally from, e.g., a transcritical generation of solitons, however. A main difference is that the cruiseferries move at a speed which is very much less than the shallow water speed. The phenomenon seems to appear when the ship is passing from deeper to shallower water and is thus of transient behaviour. The observations are made in the inner Oslofjord where the bottom is complex, including several shallow regions and rocks as well as trenches, and where the fjord at several places is very narrow.

The observations of the phenomenon at the location of Askholmen islands near the Drøbak Sound in the Oslofjord document the follows. The ship track crosses a shallow region where the depth reduces from 81 m to 39 m. The ship track goes along a rather narrow, 200 m wide trench where shallow, wider regions of complex geometry exist on each side. The upstream waves by Color Magic, observed at the location of Askholmen, are characterized by a short wave train of average period of about 35 seconds, well ahead of the classical ship waves. The operating speed of the ship of 22 knots gives a depth Froude number of $Fr = 0.45$, using the average depth of 60 m as length scale.

Mathematical model

The ship is advancing in steady motion from a deeper region into a shallower region, both of constant depth. The upstream wave formation during this transition is modelled. The ship is modelled by a pressure distribution, determined by the hydrostatic pressure corresponding to the local depth of the ship. Let x_1, x_2 denote horizontal axes where x_1 is directed along the ship motion and x_2 laterally. Let y denote vertical axis. Let t denote time. Linear theory is assumed which means that the kinematic and dynamic conditions at the free surface read:

$$\eta_t = V, \quad \phi_t + g\eta = -p/\rho, \tag{1}$$

where η denotes the free surface elevation, V normal velocity along the free surface, g acceleration of gravity, p the given pressure distribution and ρ density of the water.

The bottom boundary is given by $y = -h_0 + \delta(x_1, x_2)$ where δ denotes the bottom variation. The Laplacian velocity potential ϕ satisfies the condition of zero normal velocity along the bottom and is obtained by the solution of an integral equation. Following Fructus and Grue (2007) the solution of this integral equation expresses the normal velocity along the surface, V, in terms of the velocity potential along the surface, ϕ, and the velocity potential along the bottom, ϕ_b. The relations are obtained using the method of successive approximations and are expressed in terms of the Fourier transform ($\mathcal{F}$) and the inverse transform ($\mathcal{F}^{-1}$) of the quantities. The set of coupled equations read:

$$\mathcal{F}(V) = k\tanh(kh_0)\mathcal{F}(\phi) + \frac{\mathrm{i}\mathbf{k}}{\cosh(kh_0)} \cdot \mathcal{F}(\delta\nabla_H\phi_b) + ..., \tag{2}$$

$$\mathcal{F}(\phi_b) = \frac{\mathcal{F}(\phi)}{\cosh kh_0} - \frac{\mathrm{i}\tanh(kh_0)\mathbf{k}}{k} \cdot \mathcal{F}(\delta\nabla_H\phi) + ..., \tag{3}$$

where even higher order couplings between δ, ϕ and ϕ_b may be found in Fructus and Grue (2007), see also Grue (2015). In (2-3) ∇_H denotes horizontal gradient, $\mathbf{k} = (k_1, k_2)$ wavenumber vector in Fourier space, and $k = |\mathbf{k}|$.

Computations

The horizontal computational domain of length L_1 and width L_2 is discretized by N_1 by N_2 points. The reference water depth h_0 serves as reference length. The ship geometry is represented by a pressure distribution where the hydrostatic pressure corresponds to the local draught of the ship. The integrated volume of this pressure distribution corresponds to the displaced mass of the ship. Simulations of the upstream long wave formation show that rather the volume than the local details of the of the pressure distribution matters. This is different for the downstream ship waves. These appear in the computations but are not of principal interest here. The amplitude distribution of these waves depend crucially on the detailed hull shape (and pressure distribution). The speed has a slow ramp-up phase until a steady value is reached. Initial, transient wave effects are avoided. The ship moves across a smooth bottom change located at $x_{1,A}$ – at this position the bottom slope has is maximum – spanning the width of the computational wave tank. The wave generation taking place ahead of the ship is investigated as the ship is passing by this point. In the concrete observations at Askholmen, the water depth changes from $h_0 + \frac{1}{2}\Delta h$ to $h_0 - \frac{1}{2}\Delta h$ where actual values are $h_0 = 60$ m and $\Delta h/h_0 = 0.7$. The waves are observed about ten water depths downstream of the bottom transition.

Time integration of (1), (2), (3) across the bottom transition, for a Froude number of $U/\sqrt{gh_0} = 0.45$ produces an upstream wave of elevation moving ahead of the ship, see figure 1a. In the computations, wave tank length and width of $L_1 = 150h_0$, $L_2 = 30h_0$, and 540 by 240 points are used. The plot in figure 1a has origin in the mid-position of the ship, and the bottom transition is located at $x_{1,A} = -20h_0$, downstream of the ship. The computations show that the upstream waves are a function of the relative bottom change rather than the bottom slope. The upstream elevation is spanning laterally across the wave tank. A depression is developing laterally from the ship sides, spanning also the across the wave tank. The classical ship waves appear in the computation.

Figure 1b illustrates the generation process of the upstream elevation ($Fr = 0.45$). The upstream wave is seen to fission from the elevation ahead of the ship. The elevation at the ship's bow is steady when the speed and water depth are steady, but becomes unsteady as the ship moves into the shallower water. The elevation is shown in snapshots along the centerplane of the ship, at time instants when the ship has moved distances of $5h_0$, $10h_0$, $15h_0$ and $20h_0$, beyond the transition point $x_{1,A}$. The wave length of the upstream wave is about $14h_0$ and wave height about $0.7 \cdot 10^{-3}h_0$, corresponding to a 4 cm tall and 840 m long wave, for $h_0 = 60$ m.

Figure 1c illustrates the generation process in a narrow channel of width $3.3h_0$, when the ship has moved distances of $10h_0$, $15h_0$ and $20h_0$ beyond the bottom transition. Now the upstream wave height is about 10 times larger than in the wider tank, corresponding to 40 cm. The wavelength is similar as in the previous calculation. Also the wave profiles along the tank walls are shown (figure 1d). One upstream wave, propagating according to the shallow water speed is always formed as the ship moves by the transition. Further, there is always a depression spreading laterally from the position of the ship, moving with the speed of the ship. The distance between the elevation and depression increases steadily. The upstream wave height grows with the Froude number, approximately to cubic power, for Fr in the low sub-critical range, between 0.2 and 0.5, see figure 1e. The wavelength decays with Fr for the calculations shown (figure 1f). Finally, the wave period may be estimated by $T \simeq \lambda/\sqrt{gh} = 43$ seconds, with $\lambda = 14h_0$ and a water depth of 39 m in the shallow region. This period corresponds about to the average period in the observation. The generation process is linear in the displaced mass of the ship.

References

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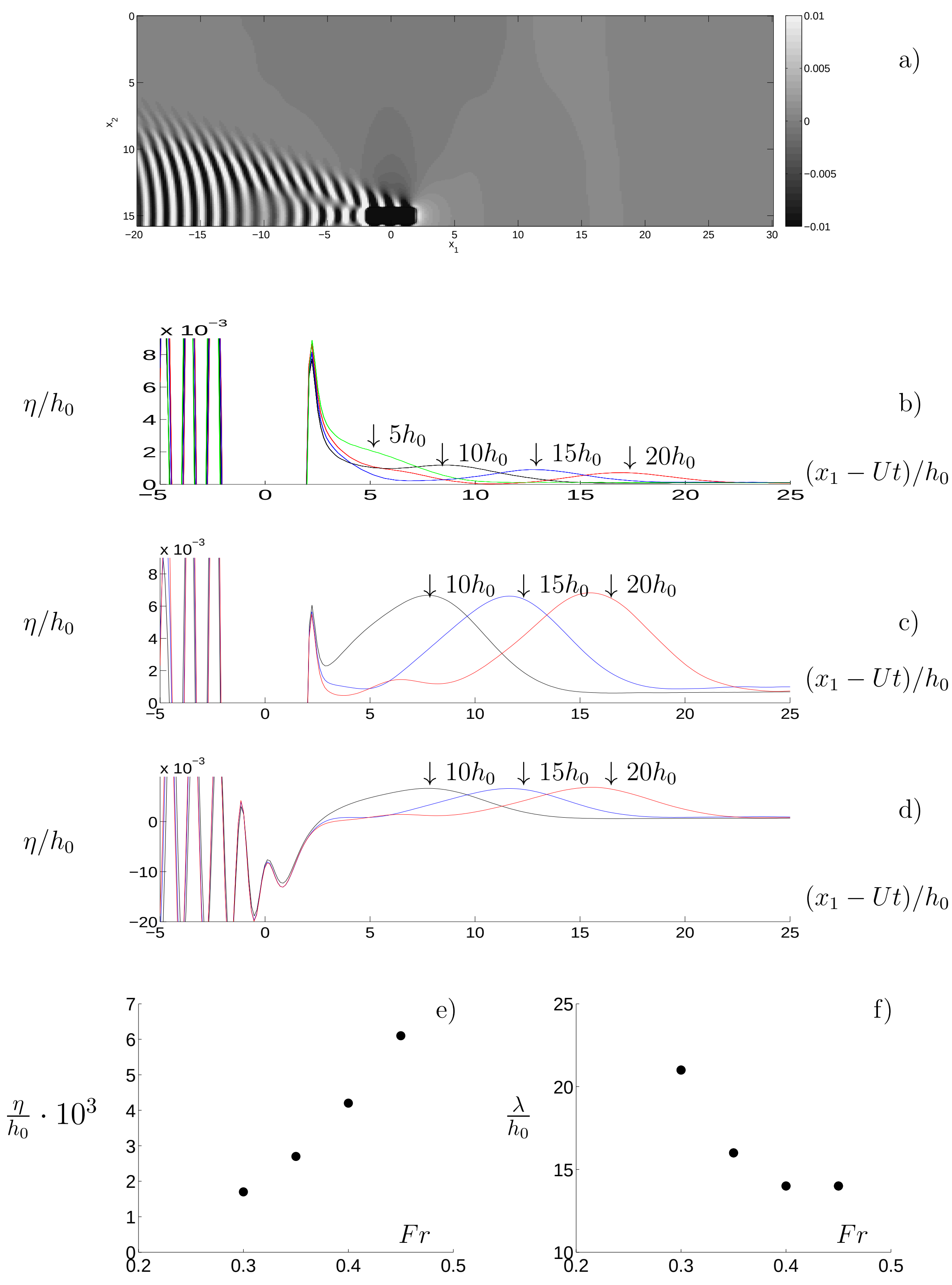

Figure 1: a) Elevation η/h_0 in wave tank of $L_1 = 150h_0$, $L_2 = 30h_0$. $Fr = 0.45$. Bottom transition at $x_{1,A} = -20h_0$. b) Same as a) but elevation along midsection when the ship has moved distances of $5h_0$, $10h_0$, $15h_0$, $20h_0$ beyond $x_{1,A}$. c) Same as b) but for narrow channel of width $L_2 = 3.3h_0$. d) Same as c), but along channel wall. e) Wave height of elevation wave vs. Fr. $L_2 = 3.3h_0$. f) Wave length of elevation wave vs. Fr. $L_2 = 3.3h_0$.