Computation of the Diffraction Transfer Matrix and the Radiation Characteristics in the open source zero-order BEM code NEMOH

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Introduction

Due to the limited rated power capacity by device of the present WEC technologies, it is nowadays well-accepted that commercial exploitation of wave energy involves the installation of a large number of wave energy converters (WECs) in an array. Hydrodynamic interactions can affect the efforts on the devices and modify their total energy production in different ways depending on the layout. Forces due to wave radiation and diffraction are represented by the linear first-order radiation and excitation force coefficients. However, their direct computation for large arrays of bodies ($O \sim 100$) is beyond the capabilities of widely available standard BEM codes.

The interaction theory (IT) developed by [1] enables one to circumvent such limitation. It is based on knowing how an individual isolated device scatters and radiates waves. For this, two hydrodynamic operators known as Diffraction Transfer Matrix (DTM) and Radiation Characteristics (RC) need to be computed. The methodology to calculate them for axisymmetric bodies was provided by [1] and its generalization for arbitrary geometries was derived by [2]. The IT by [1] has been used to study multi-moduled floating offshore structures [3, 4], large fields of ice floes [5], very large floating structures [6] and, recently, wave energy converter arrays [7].

This paper presents a comparison of the hydrodynamic operators DTM and RC of a cylinder computed with the BEM solver NEMOH[1], in which the methodology of [2] has been implemented, and the ones obtained with the alternative approach developed and validated by [7]. In addition, a comparison of the wavefield of a small array of 4 freely floating cylinders computed using both the interaction theory and with a direct NEMOH calculation is shown.

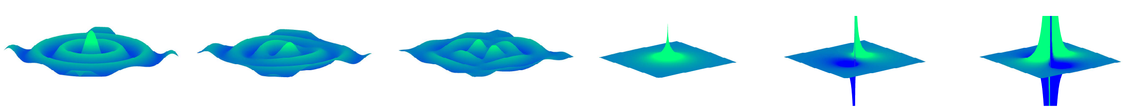
Interaction Theory

In a large array, waves emanating from each body (due to scattering and radiation) will propagate and interact with its neighbours. This will lead to a succession of scattering events which are referred to as multiple-scattering problem [8]. In this context, the different forms of the velocity potential (incident, scattered and radiated) are expressed in the cylindrical reference system local to each body j of the array by means of a superposition of partial cylindrical waves:

$$\phi_j^S = \left(A_j^S\right)^T \psi_j^S; \quad \phi_j^R = \left(R_j^k\right)^T \psi_j^S; \quad \phi_j^I = \left(A_j^I\right)^T \psi_j^I \tag{1}$$

$$\left(\psi_j^S\right)_{nm} = \begin{cases} \frac{cosh\,[k_0(z_j+d)]}{cosh\,k_0 d}\, H_m^{(1)}(k_0 r_j)\, e^{im\theta_j} & n=0 \\ cos\,[k_n(z_j+d)]\, K_m(k_n r_j)\, e^{im\theta_j} & n \geq 1 \end{cases} \qquad \left(\psi_j^I\right)_{lq} = \begin{cases} \frac{cosh\,[k_0(z_j+d)]}{cosh\,k_0 d}\, J_q(k_0 r_j)\, e^{iq\theta_j}, & l=0 \\ cos[k_l(z_j+d)]\, I_q(k_l r_j)\, e^{iq\theta_j}, & l \geq 1 \end{cases} \tag{2}$$

where A_j^S, A_j^I and R_j^k are the complex scattered, incident and radiated vectors of partial waves coefficients respectively. Even though the series are theoretically infinite, for practical computations they need to be truncated. Summations go from $m = -M$ to M and from $n = 0$ to N for outgoing waves indices, and from $q = -Q$ to Q and $l = 0$ to L for incident wave indices. $H_m^{(1)}$ is the Hankel function of the first kind of order m, I_q and K_m are the modified Bessel functions of the first and the second kind of orders q and m respectively and J_q is the Bessel function of the first kind of order q.



(a) $m=0, n=0$ (b) $|m|=1, n=0$ (c) $|m|=2, n=0$ (d) $m=0, n=1$ (e) $|m|=1, n=1$ (f) $|m|=2, n=1$

Figure 1: Partial waves modes. Progressive term $Re\{H_m^{(1)}(r)\}$ (a - c); evanescent term $Re\{K_m(r)\}$ (d - f)

One of the key aspects of the interaction theory by [1] is that the study of the wave scattering on the whole array composed of N_b bodies is undertaken by focusing on one body at a time. If this body is referred to as j and its neighbours as i, the total incident potential to j is expressed as:

$$\phi_j^I = \left(a_j^T + \sum_{\substack{i=1 \\ i\neq j}}^{N_b} A_i^T \mathbf{T}_{ij}\right)\psi_j^S \tag{3}$$

[1]http://lheea.ec-nantes.fr/doku.php/emo/nemoh/start

where a_j are the cylindrical coefficients of either an ambient plane wave (diffraction problem) or the radiated wave by a body i of the array undergoing motion in a degree of freedom k (radiation problem) expressed as $a_j = \mathbf{T}_{ij}^T R_i^k$, A_i are unknown scattered coefficients and $\mathbf{T}_{ij}$ represents the transformation matrix which depends on the relative position between bodies j and i. The latter enables one to express scattered (or radiated) waves from a body i as incident to body j in its local reference system.

The incident and scattered partial waves coefficients by an isolated body can be related as $A_j^S = \mathbf{B}_j A_j^I$ by means of a linear operator known as Diffraction Transfer Matrix $(\mathbf{B}_j)$. Substitution of the partial wave coefficients in (3) into the definition of the DTM leads to a system of equations to be solved for the unknown scattered coefficients:

$$A_j^S = \mathbf{B}_j\big(a_j + \sum_{\substack{i=1\\ i\neq j}}^{N_b} \mathbf{T}_{ij}^T A_i\big) \tag{4}$$

The Diffraction Transfer Matrix (DTM) and the Radiation Characteristics (RC) are calculated with the body in isolation. Two methodologies to compute them are available in the literature [2, 7]. Results obtained with the former, which has been implemented in NEMOH, are presented and compared with the values computed using the latter.

Validations

Diffraction Transfer Matrix

The methodology developed by [2] to find the elements $(\mathbf{B}_j)_{nl}^{mq}$ of the DTM consists of two steps. First, the solution to a diffraction problem where the incident wave is a cylindrical partial wave $\left(\psi_j^I\right)_{lq}$ is found. Then, the scattered potential represented by the source strengths σ_{lqj} is expressed in the base of partial wave functions by means of the Green's function in cylindrical coordinates developed by [9] leading to:

$$B_{0mj} = \frac{i}{2}\, C_0 \, cosh\, k_0 d \times \iint_{S_{Hj}} \sigma_{lqj}(R_j, \Theta_j, \zeta_j)\, J_m(k_0 R_j)\, cosh[k_0(\zeta_j + d)] e^{-im\Theta_j} ds \tag{5}$$

$$B_{nmj} = -\frac{1}{\pi} C_n \times \iint_{S_{Hj}} \sigma_{lqj}(R_j, \Theta_j, \zeta_j)\, I_m(k_n R_j)\, cos[k_n(\zeta_j + d)] e^{-im\Theta_j} ds \tag{6}$$

with S_{Hj} the wetted surface of the body and C_0 and C_n constant coefficients.

The main aim of the alternative procedure derived by [7] is to compute the elements of the DTM using only plane incident waves. As long as a large enough number of pairs of scattered/incident vectors of coefficients is known in advance, the definition of the DTM ($A_j^S = \mathbf{B}_j A_j^I$) can be transformed into a system of equations to solve for its elements. The vectors of incident partial waves are known from an analytical expression [2] whereas the elements of the associated vectors of the scattered coefficients can be derived by means of a Fourier Transform of the scattered potential on the body circumscribing cylinder of radius r_0:

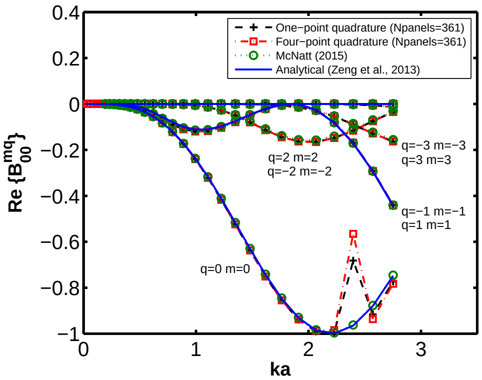
$$a_{0m}^S = -\frac{i}{2\pi}\frac{\omega}{g}\frac{2\, cosh\, k_0 d}{d\left(1 + \frac{sinh\, 2k_0 d}{2k_0 d}\right)}\frac{1}{H_m^{(2)}(k_0 r_0)} \times \int_{-d}^{0}\int_{0}^{2\pi} \phi(r_0,\, \theta,\, z)\, e^{-im\theta} cosh\, k_0(d+z) d\theta dz \tag{7}$$

$$b_{nm}^S = -\frac{i}{2\pi}\frac{\omega}{g}\frac{2}{d\left(1 + \frac{sin\, 2k_n d}{2k_n d}\right)}\frac{1}{K_m(k_n r_0)} \times \int_{-d}^{0}\int_{0}^{2\pi} \phi(r_0,\, \theta,\, z) e^{-im\theta} cos\, k_n(d+z) d\theta dz \tag{8}$$

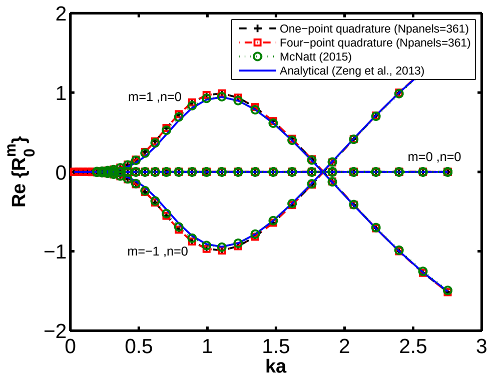
Even if evanescent terms from the scattered potential can be identified using (8), the use of only plane progressive incident waves (with no evanescent components) prevents the calculation of the DTM terms relating incident and scattered evanescent partial waves using this procedure. The DTM elements computed with both methodologies, which make use of different notation conventions and a different scaling of the partial wave coefficients, are related by:

$$\frac{(-1)^{-m}}{(-1)^{-q}}\left(\mathbf{B}^*\right)_{-m,-q}^{Method\,[2]} = \left(\mathbf{B}\right)_{m,q}^{Method\,[7]} \tag{9}$$

Figure 2a shows a comparison of the progressive terms of the DTM computed using both methodologies and with the semi-analytical solution by [10]. A very good agreement of results is observed. The only non-zero DTM terms correspond to pairs of equal incident (q) and outgoing (m) angular modes. This is a particular feature of axisymmetric geometries such as a truncated vertical cylinder.The numerical singularity observed at a ka of approximately 2.3 corresponds to an irregular frequency. The new release of the BEM solver NEMOH will enable its removal.



(a)



(b)

Figure 2: Real part of the DTM (a) and RC (b) progressive terms for a truncated vertical cylinder of $3m$ radius (a), $6m$ draft in a $10m$ water depth. Markers $+$, $\square$ indicate results from NEMOH using method [2] for two different integration schemes.

Radiation Characteristics

The same principle applied for the calculation of the DTM with both methodologies can now be used to obtain the RC vector. With respect to [2], the radiation problem associated with a motion mode k of the body under consideration is solved first. Then, the source strength distribution σ_{jk} is used in conjunction with the Green's function in cylindrical coordinates to express the radiated potential in the base of partial waves leading to expressions (10) and (11) for the RC:

$$R_{0mj} = \frac{i}{2}\, C_0 \, cosh\, k_0 d \times \iint_{S_{Hj}} \sigma_{jk}(R_j, \Theta_j, \zeta_j)\, J_m(k_0 R_j)\, cosh[k_0(\zeta_j + d)] e^{-im\Theta_j} ds \tag{10}$$

$$R_{nmj} = -\frac{1}{\pi}\, C_n \times \iint_{S_{Hj}} \sigma_{jk}(R_j, \Theta_j, \zeta_j)\, I_m(k_n R_j)\, cos[k_n(\zeta_j + d)] e^{-im\Theta_j} ds \tag{11}$$

With regard to [7], the same formulas (7) and (8) are used to express the radiated potential in terms of partial cylindrical wave functions. In this base, the coefficients are known as Radiation Characteristics. Apart from different notation conventions and a different scaling of the partial wave coefficients, the use of two different BEM solvers (methodology by [2] has been implemented in NEMOH whereas WAMIT[2] has been used in conjunction with [7]) results in the following relationship between the Radiation Characteristics:

$$(-1)^{-m} \frac{g}{\omega^2} \left[(a^R_{-mk}) \right]^* = R_{mk} \tag{12}$$

where a^R_{-mk} are the RC in the notation of [7] and R_{mk} in the notation of [2].

Figure 2b shows a comparison of the progressive terms of the surge RC computed using both methodologies and with the semi-analytical solution by [10]. A very good agreement of results is observed. For this mode of motion, it can be observed that only modes $m = 1$ and $m = -1$ are non-zero. This is explained as the wave generated by the motion of a cylinder in surge corresponds to the partial wave shown in Figure 1b.

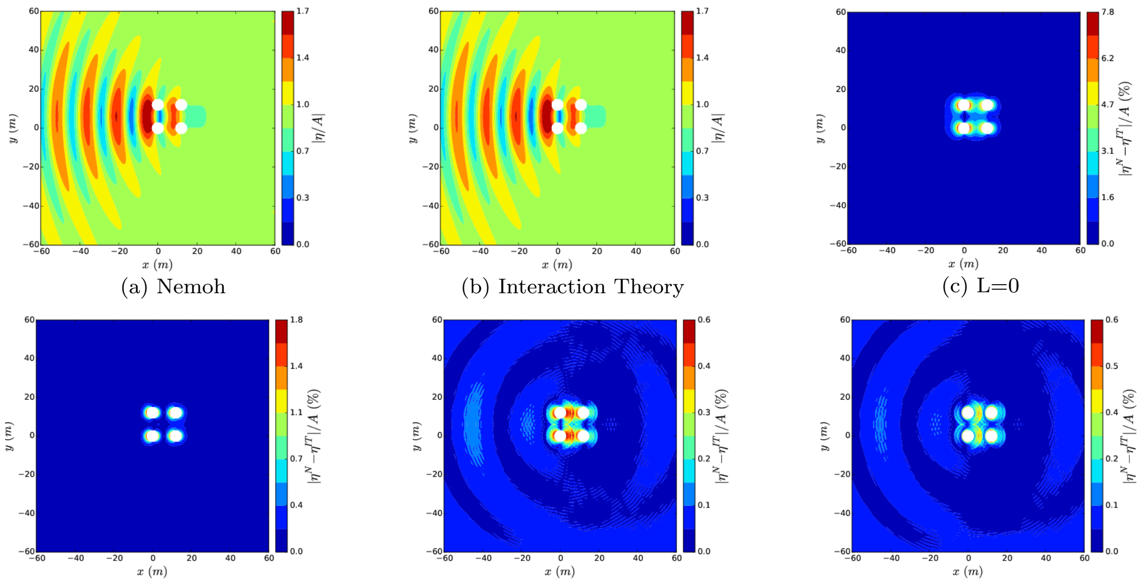
Interaction Theory

The free surface elevation for a small array of 4 freely floating truncated vertical cylinders has been computed by means of the interaction theory and compared with direct calculations using NEMOH (Figure 3) for a regular wave of propagation direction $\beta = 0$ and wavelength $\lambda/a = 10$ with a the radius of the cylinders. The total wave elevation is the sum of incident, scattered and radiated wave elevations and includes the computed body motions. A very good agreement between results can be observed in the whole domain when no evanescent modes are used with the highest differences being located at the vicinity of the bodies. The use of a higher evanescent modes truncation reduces the error at these regions and convergence with the direct computation results by NEMOH is achieved.

Conclusion

The calculation of the DTM and the RC has been implemented in the open source BEM solver NEMOH using the methodology of [2]. Results of the hydrodynamic operators of a truncated vertical cylinder have been compared with the methodology developed by [7] and with the semi-analytical solution by [10] and a very good match has

[2]http://www.wamit.com/



(a) Nemoh (b) Interaction Theory (c) L=0

(d) L=6 (e) L=12 (f) L=18

Figure 3: Magnitude of surface elevation for an array of 4 cylinders of $3m$ radius, $6m$ draft in a $50m$ water depth with a separation distance of 12m. Plots c, d, e, f show the percentage difference between the wave fields computed with the interaction theory (IT) and the direct calculation using NEMOH (N) as a function of the evanescent modes truncation L. Results are normalized by the amplitude of the incident wave (A). Propagation direction is defined from left to right.

been obtained. Wave fields computed with the interaction theory by [1] for a small array of 4 truncated vertical cylinders have been compared to direct computations using NEMOH and a very good agreement has been observed. A decrease of the error at the vicinity of the bodies when the number of evanescent modes is increased has been found as expected.

Acknowledgements

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